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Geological influence on the spatial variability of soils

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ABSTRACT

An evaluation of the spatial correlation length of two geological formations known as mudflows and residual soils has been conducted using standard penetration test $(N_1)_{60}$ values. The spatial correlation length is an important property in reliability-based design, in addition to the mean and standard deviation of soil parameters, but it is rarely estimated in geotechnical projects. Reported results in this paper from both geologies show the geological origin's importance in the spatial variability analysis. Residual soils are more likely to display isotropic spatial correlation lengths horizontally when compared with the mudflows. The results show that the random field represents more accurately the mudflows' soil variability when the residual soils must be complemented with shear strength tendency analyses.

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KEYWORDS
Spatial correlation length; geology; SPT; RFEM

1 Introduction

Geotechnical engineering projects face daily uncertainties, particularly when they have to deal with soil properties, which can be highly variable. According to Elkateb, Chalaturnyk, and Robertson (2003), soil heterogeneity can be classified into two main categories: lithological heterogeneity and inherent spatial soil variability. The former is usually related to the variation of layers, the inclusion of different materials and human misinterpretation of the soils' geological origin. Inherent spatial soil variability, measured by the autocorrelation function where the most important parameter is the correlation length (θ), is influenced by the geological processes that result in a stochastic dependency of soil properties (Firouzianbandpey et al. 2015).

Soils' inherent variability has been traditionally analysed through different statistical methods, such as first-order and second-order reliability methods (FORM and SORM) and the Monte Carlo (MC) simulation method. Statistical models can be integrated with the traditional limit equilibrium and finite element models in geotechnical designs. However, recently, soil inherent variability has been more accurately modelled as a random field due to the development of numerical techniques, such as the Local Average Subdivision (LAS) (Fenton and Vanmarcke 1990). Combining the LAS method with finite elements allows for the Random Finite Element Method (RFEM) in geotechnical engineering (Griffiths and Fenton 1993; Fenton and Griffiths 1993). However, RFEM requires the definition of the spatial correlation lengths in addition to the mean and the standard deviation of soil parameters to generate a realistic soil model, which may represent a greater difficulty to perform these analyses.

The correlation length (θ) describes the distance over which the spatially random values tend to be correlated (Vanmarcke 1977). Spatial variability in geotechnical engineering has been described by a number of authors (e.g., Baecher and Christian 2003; Basarir et al., 2010; Cai et al. 2017; Cao and Wang 2013; Cheng et al. 2018; Chiasson et al. 1995; El Haj, Soubra, and Fajoui 2019; Fenton 1999; Firouzianbandpey et al. 2014; Gambino and Gilbert 1999; Jha 2014; Leung and Lo 2018; Luo et al. 2018; Papaioannou and Straub 2012; Ravi 1992; Salgado, Ganju, and Prezzi 2019; Soulié, Montes, and Silvestri 1990; H. Zhu and Zhang 2013; Zijun & Yu, 2013; Zhang et al., 2020a) According to the literature review, it is evident the difficulty in obtaining an accurate θ is due to the amount and quality of the field data and the difficulty of the autocorrelation function adjustment in the near-zero correlation values. Also, the range for the horizontal correlation length is broad with values between 0.14 m and 80 m as reported by Ching et al. (2018). Therefore, due to the broad range, it is important to determine the factors that influence the correlation length magnitude for future analyses.

More recently and to a lesser extent, the SPT has been implemented in spatial correlation analyses (e.g., Oguz, Huvaj, and Griffiths 2018; Zhang and Chen 2012). However, none have analysed the geological origin's influence on the correlation length estimation. The SPT implementation to obtain the spatial variability can be used to evaluate the variability of the soil's undrained soil mechanical properties. The above is due to the SPT's affinity with the undrained parameters evidenced throughout the different correlations available in the literature. According to the above, this paper investigates how geological origin influences the correlation length magnitude through statistical trends and the correlation of $(N_1)_{60}$ values for two geologies: mudflows (soils formed from ancient landslides) and residual soils (formed from the weathering of in situ rock).

Taking into account the geological origin as an 'input variable' will allow researchers to determine the most appropriate

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field research and models for the probabilistic geotechnical designs (e.g. Zhang et al., 2020b). The knowledge of the site local geology will allow us to have the previous conception about the possible horizontal correlation magnitude and thus be able to plan the soundings in a more consistent way for the future statistical analyses. The above will reduce, to some extent, the gap between the designs and the field behaviour of the geostructures (Fookes 1997; Viviescas, Osorio, and Cañón 2017).

2 Characteristics of the analysed geology

We analysed geology's influence on the spatial variability of soils. A geology with abrupt changes and a stationary environment was selected. Therefore, mudflows and residual soils from the east side of Medellin city in El Poblado's neighbourhood were analysed. These soils are commonly found in tropical and high mountain geological environments.

Mudflows are soils formed by previous landslides and subjected to transportation and particle sorting that can lead to tremendous uncertainties in the geotechnical shear strength properties (Zhao and Zhang 2014; Zhao et al. 2013). On the other hand, residual soils are materials formed directly from the weathering of in situ rocks. The main feature of this geological unit is an advanced weathering process favoured by the climatic and topographical conditions, without ever being transported (Mitchell and Soga 2005). Based on the Deere and Patton (1971) classification system, the residual soils in this study are classified as having an IC and IC-2A state of weathering.

3 SPT data

The same SPT equipment was implemented in this study to obtain each of the N values to reduce the uncertainty, to a certain degree, of the field measurements and data. Then, we correct the N-value from the SPT according to ASTM D1586 (ASTM 2011) standards to obtain the $(N_1)_{60}$. Table 1 summarizes the collected information for each geology (number of boreholes, $(N_1)_{60}$, Unified Soil Classification System (USCS), maximum analysis depth, and the analysed area).

We previously performed cluster analyses to identify and remove outliers from SPT N-values caused by random factors, such as rock fragments and variations in the state of weathering (Viviescas, Osorio, and Griffiths 2019). Outlier removal may prevent erroneous probabilistic designs due to missing correlation length calculations. The SPT data for mudflows and residual soils are shown in Figure 1. The spatial distribution of the mudflows project's boreholes is shown in Figure 2 and for the residual soils is shown in Figure 3.

4 Correlation length determination

Random fields associated with soil properties are usually defined by the correlation length (θ). θ is an important property that defines the distance within which values are significantly correlated (Fenton and Griffiths 2008). Of the various correlation functions available in the literature, the Markovian (exponential) and Gaussian (Squared exponential) are widely used in geotechnical engineering (e.g. Liu et al., 2019, Ching et al. 2018; Fenton and Vanmarcke 1990; Firouzianbandpey et al. 2015; Griffiths and Fenton 1993; Honjo and Setiawan 2007; Li, Zhang, and Li 2015; Oguz, Huvaj, and Griffiths 2018; Phoon and Kulhawy 1999; Vanmarcke 1977; Zhang and Chen 2012; Zhu and Zhang 2013; Ali et al. 2017). Therefore, the Markovian and Gaussian auto-correlation functions were employed in this paper to determine θ as shown in Eq. 1 and Eq. 2.

Markov

$$\rho(\tau) = exp\left\{-\frac{2|\tau|}{\theta}\right\} \tag{1}$$

Gaussian

$$\rho(\tau) = exp\left\{-\pi\left(\frac{\tau}{\theta}\right)^2\right\} \tag{2}$$

where $|\tau|$ is the absolute distance between the analysed data.

To estimate θ, the following procedure was followed:

1.The respective N-value corrections and standardizations were made according to the ASTM D1586 (ASTM 2011) that recommends that the field measured SPT-N value should be standardized. The general equations for the SPT-N standardization are as follows (McGregor and Duncan 1998):

$$N_{60} = (C_B C_C C_E C_R C_{BF} C_S C_A) N_{field} \tag{3}$$

$$(N_1)_{60} = C_N N_{60} \tag{4}$$

where C_B = Borehole diameter correction factor,C_C = hammer cushion correction factor, C_E = SPT Equipment Energy ratio (60%)/60%, C_R = rod length correction factor, C_{BF} = blow count frequency correction factor, C_S = liner correction factor, C_A = anvil correction factor, C_{field}= Field SPT -N Value and C_N = overburden correction factor

2.After the SPT-N standardization, and considering that the same SPT equipment was implemented, a cluster analysis is performed to remove those outliers that do not represent the soil's mechanical behaviour as an uncertainty decrease method, as was explained above.

3.Then, the vertical and horizontal distance matrices were defined for each borehole on each project.

Table 1. Numbers of SPT data obtained and soil index properties average results for each geological soil.

	Number of data			Soil index properties average results				
Location – Geology	Borings	$(N_1)_{60}$	Max depth (m)	LL (%)	PL (%)	γ (g/cm^3)	USCS	Reference
Poblado's mudflows	17	132	16.6	61.9	41.7	1.65	MH	Viviescas and Osorio (2015)
San Diego's residual soil	19	146	14	46.1	30.5	1.78	ML	Present study

Where LL = Liquid Limit, PL = Plastic Limit, γ = Moist unit weight, USCS = Unified Soil Classification System

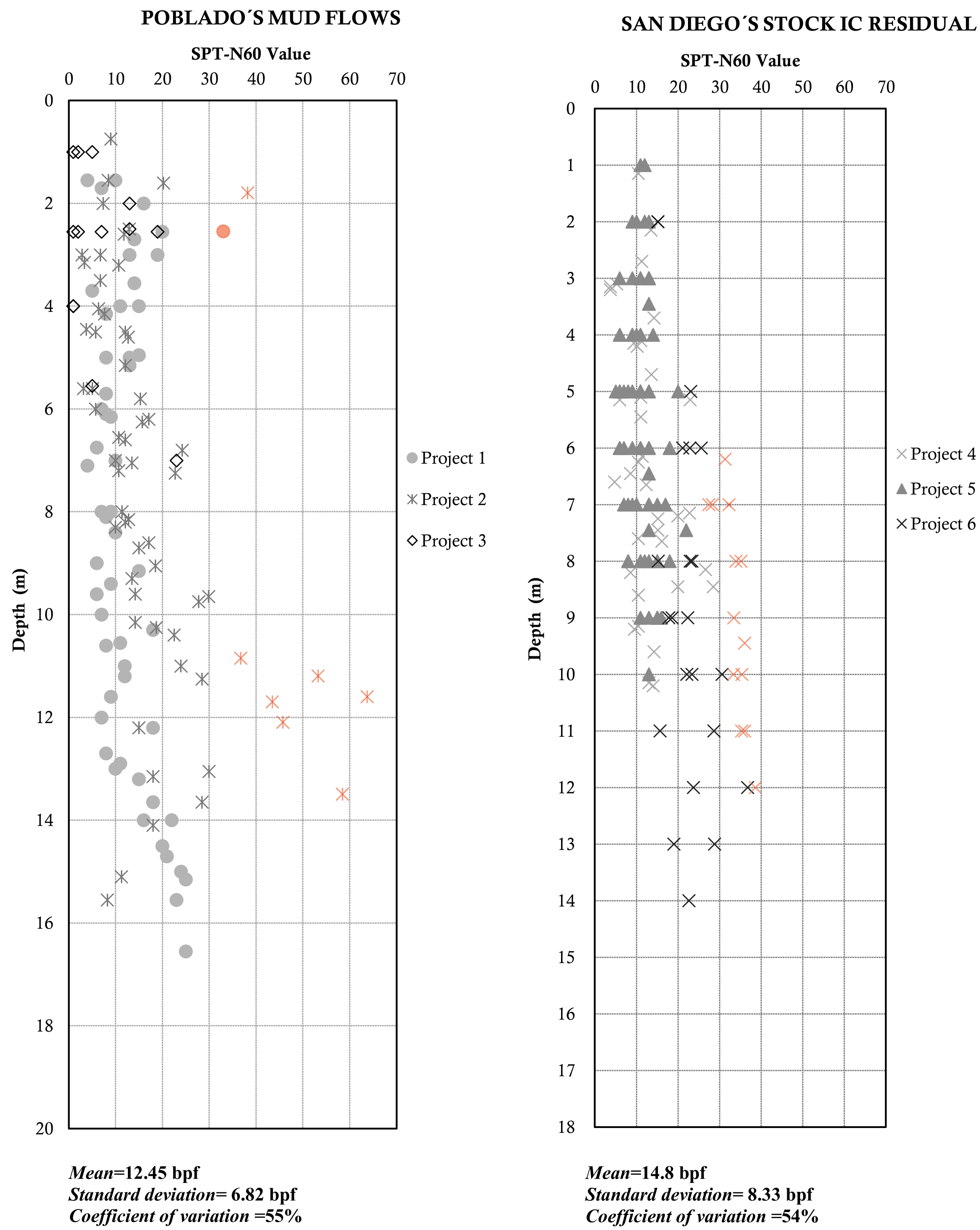

Figure 1. $(N_1)_{60}$ values for Poblado's mudflows and San Diego´s residual soils. (The values in red correspond to the outliers removed from the analyzes).

4.For the horizontal analysis, one borehole is set as fixed, and the correlation between this and the remaining boreholes at the same depth was obtained. For the vertical analysis, the first $(N_1)_{60}$ value was taken as the fixed point to start obtaining the variation of each vertical distance's correlation coefficient.

5.The data present important trends in the vertical axis presents as it was evidenced by Viviescas, Osorio, and Griffiths (2019). Therefore, a general regression was initially performed on each of the boreholes.

6.Given the SPT values at x_i and the measure depth (y_i), the correlation function at each separation, or lag distance (k), can be determined by (adapted from Zhang and Chen (2012)):

$$\rho(k) = \frac{\sum_{i=1}^{n-k} (y_i - \bar{y}_i)(y_{i+k} - \bar{y}_{i+k})}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2} \tag{5}$$

where k is the lag of the data (k =0,1, ...,k −1), n is the total number of y measurements, and $\bar{y}_i$ and $\bar{y}_{i+k}$ are the trend values at point x_i and x_{i+k}.

Eq. 3 allows, in conjunction to the obtained SPT tendency with depth, to remove the influence of the data trend in the obtained correlation function.

7.Finally, the correlation coefficient (ρ) versus the lag or absolute distance (τ) was plotted for both vertical and horizontal directions. Then, an exponential and squared exponential goodness of fit was performed to obtain the correlation value

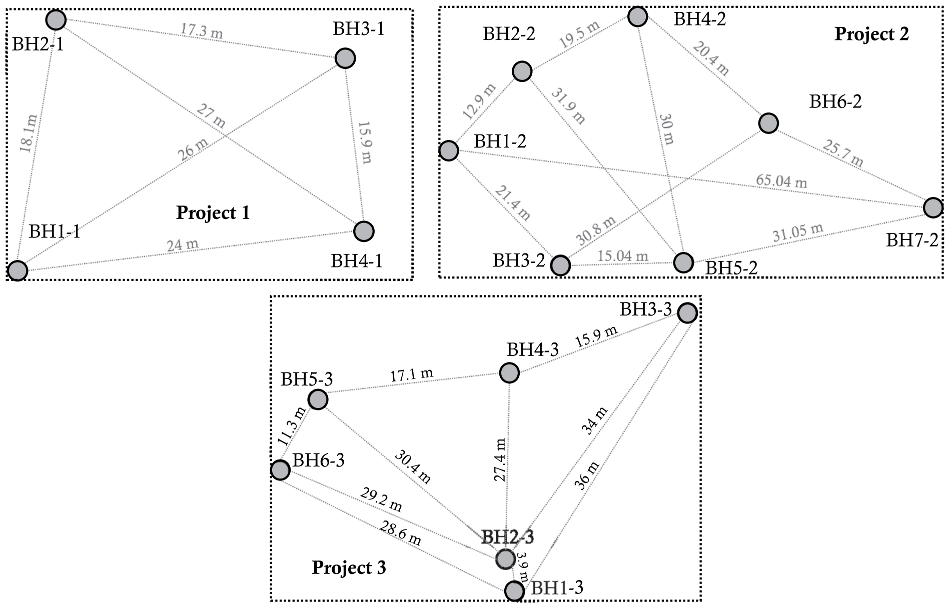

Figure 2. Spatial distribution of the boreholes in Project 1, 2 and Project 3 – mudflows.

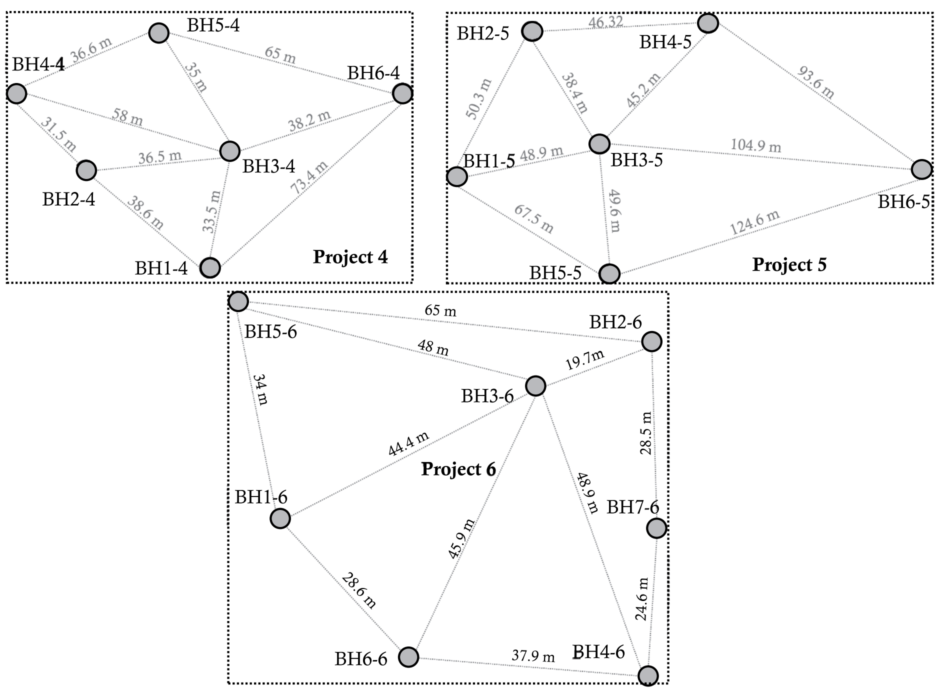

Figure 3. Spatial distribution of the boreholes in Project 4, Project 5 and Project 6 – residual soils.

on each project and direction (θ_H = horizontal and θ_V = vertical correlation length) according to Eq. 6 and Eq. 7:

$$\exp\{-a|\tau|\} = \exp\left\{-\frac{2|\tau|}{\theta}\right\} \text{ and a } = \frac{2}{\theta} \text{ then } \theta = \frac{2}{a} \tag{6}$$

$$exp\{-b|\tau|^2\} = exp\left\{-\pi\left(\frac{\tau}{\theta}\right)^2\right\} \text{ and b } = \frac{\pi}{\theta^2} \text{ then } \theta = \sqrt{\frac{\pi}{b}} \tag{7}$$

Where a and b is the exponential and squared exponential goodness of fit coefficients

4.1 Spatial correlation length (horizontal direction)

According to the borehole distribution in Figure 2 and Figure 3, three projects for each geology, identified as Project 1 (P1), Project 2 (P2) and Project 3 (P3) were analysed for the mudflows and Project 4 (P4), Project 5 (P5) and Project 6 (P6) for the residual soils. The results of the horizontal spatial correlations for

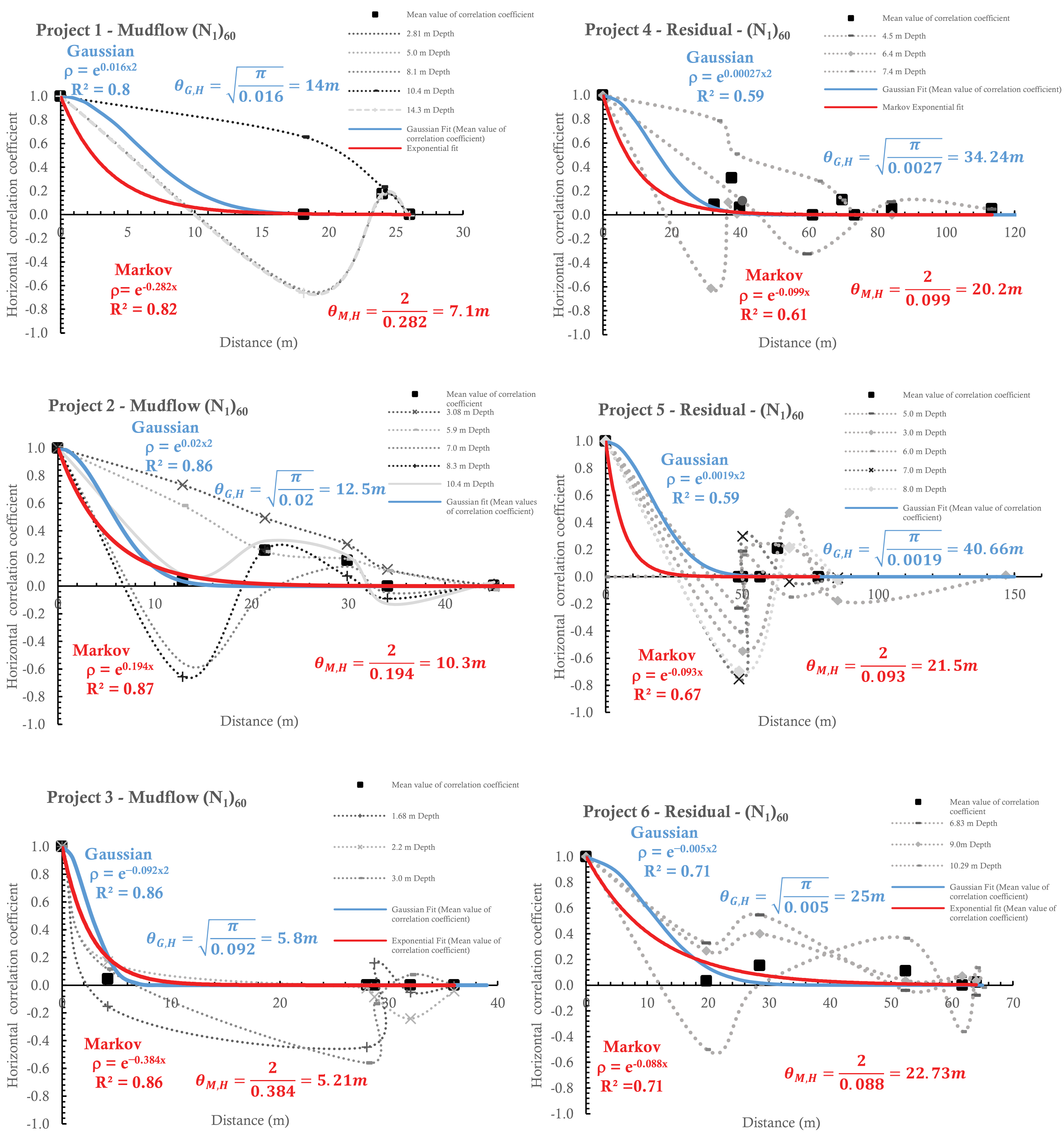

Figure 4. Horizontal spatial correlation for mudflows and residual soils from $(N_1)_{60}$ results. Where $\theta_{M,H}$ = Markovian horizontal correlation length and $\theta_{G,H}$= Gaussian horizontal correlation length.

both geologies are shown in Figure 4. It is shown that the best auto-correlation method on the horizontal direction for both geologies is the Markovian function (higher R^2). The results of the mudflows' horizontal correlation length show a better auto-correlation fit with a θ_H of 5.2 m and 10.3 m. The residual soils' results have shown that the θ_H is up to three times greater than the mudflows' horizontal length, with values around 20.2 m and 22.7 m. The above correlates with the soil's geological origin because soils that have not been transported in their geological history should have greater correlation lengths.

4.2 Spatial correlation length (vertical direction)

The results of the vertical spatial correlations for both geologies are shown in Figure 5. The results for both geologies show that the Gaussian auto-correlation gives a better fit (without

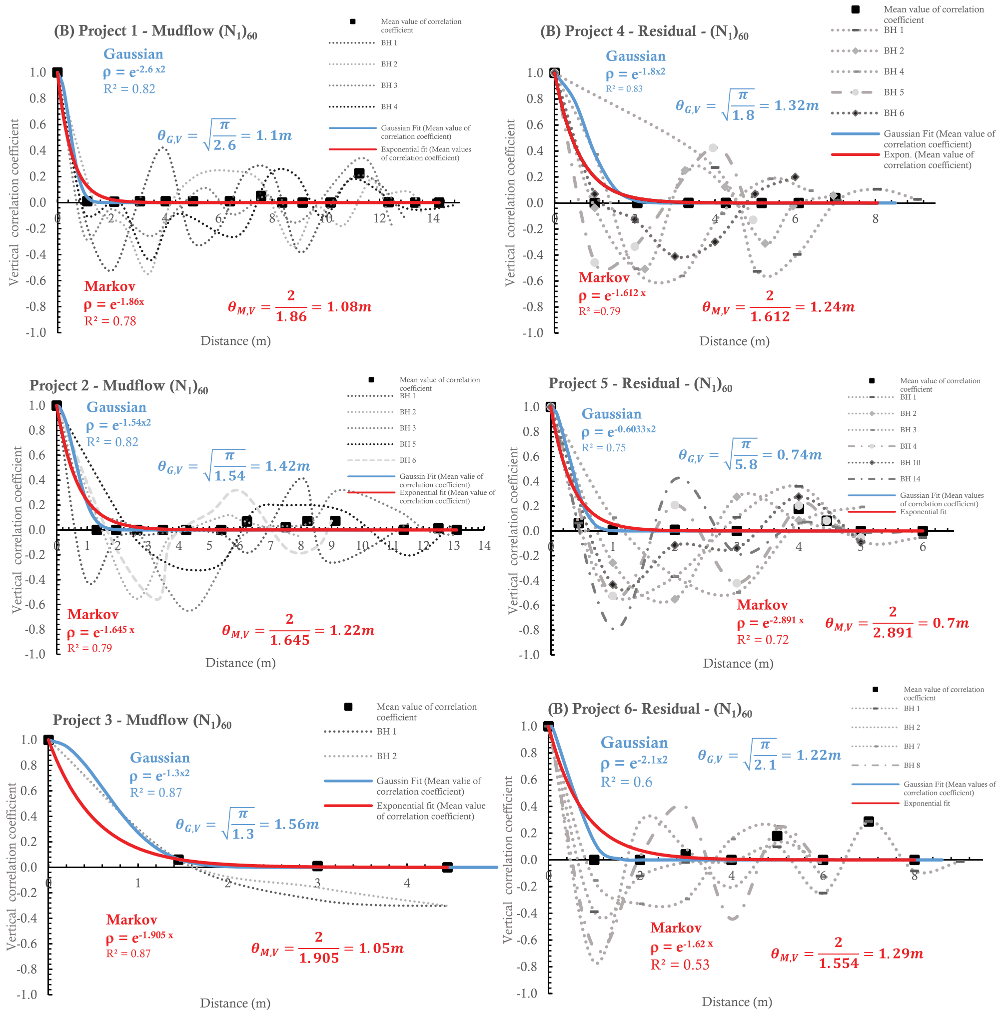

Figure 5. Vertical spatial correlation for mudflows and residual soils from $(N_1)_{60}$ results. Where $\theta_{M,V}$ = Markovian vertical correlation length and $\theta_{G,V}$= Gaussian vertical correlation length.

much difference with the Markovian function). The results of the mudflows' vertical spatial correlation show a θ_H of 1.1 m and 1.56 m and those of residual soils' show a θ_V of 0.74 m and 1.32 m. Taking into account the previous results, it can be shown that the θ_V has similar values for both geologies. The above may indicate that θ_V is mainly influenced by the vertical effective stress regardless of the soil's origin.

5 Summary of results and discussion

It is shown that the best auto-correlation method is the Markov in the horizontal direction and the Gaussian for the vertical direction for both geologies. According to the results in Table 2,

Table 2. Summary of the vertical and horizontal correlation lengths.

θ(m)		Horizontal (Markov)		Vertical (Gaussian)	
Mudflows	Project	θ_H(m)	R^2*	θ_V(m)	R^2*
	P1	**7.1**	0.82	**1.1**	0.82
	P2	**10.3**	0.87	**1.42**	0.82
	P3	**5.21**	0.86	**1.56**	0.87
Residual	P4	**20.2**	0.61	**1.32**	0.83
	P5	**21.5**	0.67	**0.74**	0.75
	P6	**22.73**	0.71	**1.22**	0.6

* R^2 was adjusted for nonlinear functions fits according to the sum of squares of the residuals and the total sum of squares.

the $\theta_V$and $\theta_V$values for both geologies fell within the range of the reported values (Ching et al. 2018; Phoon and Kulhawy 1999; Stuedlein et al. 2012). However, $\theta_H$in residual soils is up to three times the mudflows' horizontal length. The geological influence in the θ_H magnitude is related to the geological processes that formed the soils. Materials with abrupt changes (mudflows) will present lower θ_H compared with stationary soils (residual soils). Therefore, soils formed by previous landslides will result in a more heterogeneous material compared with soils that were never transported and with changes in the state of weathering with depth than the residual soils.

The obtained values of the vertical correlation length, such as those presented in the literature, present values around $\theta_H \approx$ 0.5 to 1.5 m (Phoon and Kulhawy 1999; Stuedlein et al. 2012). The above may indicate that θ_V is mainly influenced by the vertical effective stress regardless of the soil's origin. Therefore, the estimation of the horizontal correlation length is the most important parameter for the random field's 1Based on previous results, a graphical model verification of the geotechnical properties' spatial correlation lengths was performed using a random finite-element method (RFEM) software (Griffiths and Fenton 1993; Fenton and Griffiths 1993). RFEM combines an elastoplastic finite-element analysis with the Local Average Subdivision method (LAS) to generate a random field of the

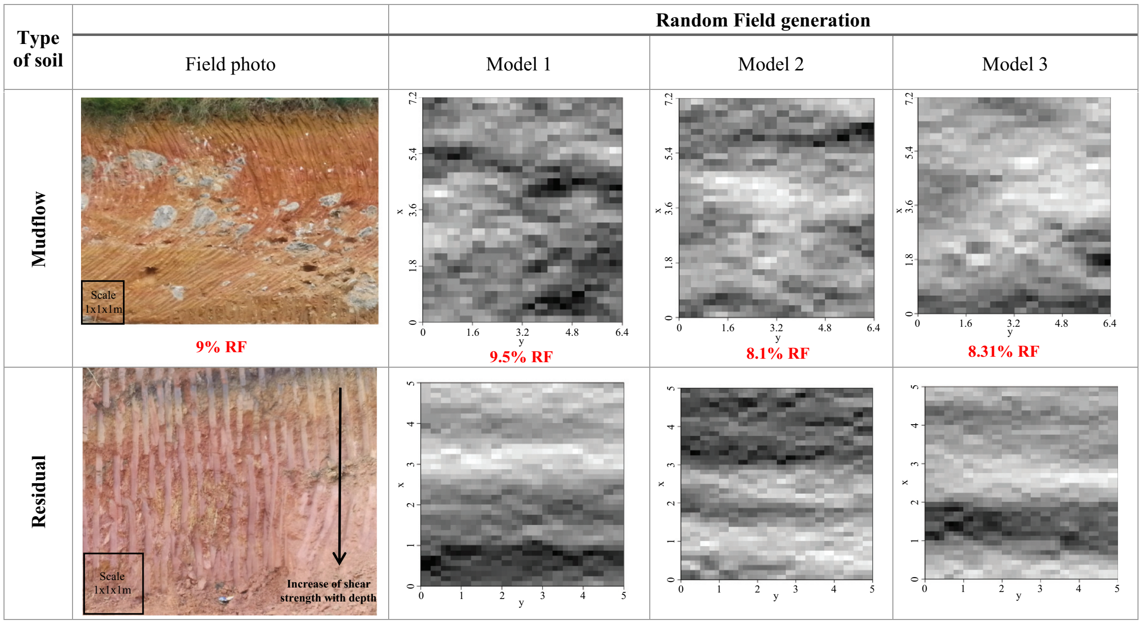

Figure 6. Graphic spatial correlation length comparison according to three LAS random models and field photos for mudflows and residual soils. (where RF = Rock fragments).

soil properties (Fenton and Vanmarcke 1990). Considering the variability of the geotechnical properties, the LAS randomly defines finite elements with greater resistance (black elements) and with lower resistance (white elements). The results in Figure 6 show a comparative analysis between a field photo of the analysed soils versus three random field models.

The comparison between the field photos and the LAS models on mudflows was made through a pixel count corresponding to the rocks in both the photos and the LAS models (where the rock corresponds to the dark pixels). Then, the rock pixel count was compared with the image's total number of pixels to obtain the percentage of rock pixels. Figure 6 shows that the random models present rock percentages almost identical between the field photo and the LAS model with 9% of rock fragments. The random model also shows that those higher resistance elements (rock fragments) in mudflows are randomly distributed similarly to the field sites.

However, the random models of residual soils show that, although the correlation length is obtained correctly, the RFEM model in Figure 6 shows a concentration of rigid zones at any depth, which is inconsistent with the residual soil's behaviour. The inconsistencies are because the LAS model implemented does not consider the Shear Strength Tendency (SST), which is consistent with sedimentary soils. Lately, LAS models have been combined with a linear SST (e.g. Zhu et al. (2017), which differs from the reported SST in Residual soils (Viviescas, Osorio, and Griffiths 2019). Therefore, the challenge lies in modelling the residual soils by presenting functions other than linear.

6 Conclusions

The SPT-N values were used to obtain the spatial correlation length due to the SPT's affinity with the undrained parameters. Determining spatial correlation lengths can be difficult due to SPT uncertainties and the auto-correlation model sensitivity. However, the results show that, in spite of the SPT data uncertainties, there is a clear difference between the correlation lengths of residual soils and mudflows. The residual soils have a lower spatial variability than the mudflows because the formation occurs *in situ* with the weathering processes, varying with depth. On the other hand, the mudflows are the result of landslides that mix materials with different degrees of weathering.

From these results, it is shown that the geological context is an important input characteristic for geotechnical practice and research. The prior knowledge of the local geology will allow us to have a previous conception about the possible horizontal correlation magnitude, which will help to adequately plan the sounding distribution in a more consistent way for the statistical analysis, as it was suggested by Ching et al. (2018).

Random fields more accurately represent the variability of mudflow soils, where the LAS models show that the horizontal correlation length is an essential variable. The above is because RFEM models predict similar percentages of rock fragments to those that appear on the field photos, as shown in Figure 6. However, the random fields of residual soils must be complemented with no linear shear strength tendency analyses to adequately determine resistance changes with depth due to these materials' intrinsic weathering processes. Therefore, a geological characteristic can be used to build a random field through an image processing algorithm to obtain θ through an inverse calculation of the random field according to the input parameters' probability density function.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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List of symbols

θ_V Blows per foot normalized to a 60% of the theoretical free – fall hammer energy of the SPT test.

SD Standard Deviation

R^2Coefficient of determination

$(N_1)_{60}$ Correlation function

$\rho(\tau)$ Absolute distance between points

$|t|$ Correlation length

θ Gaussian horizontal correlation length

$\theta_{G,H}$ Gaussian vertical correlation length

$\theta_{G,V}$ Markovian horizontal correlation length

$\theta_{M,H}$ Markovian vertical correlation length

$\theta_{M,V}$ lag of the data

k Total number of n measurements

y Trend values at point $\bar{y}_i$

x_iTrend values at point$\bar{y}_{i+K}$

x_{i+k} Variable x at position t

BH Bore Hole

$X(t')$ Horizontal correlation length

θ_H Vertical correlation length

θ_V Exponential goodness of fit coefficient

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