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TRANSIENT ANALYSIS OF EXCAVATIONS IN SOIL

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Abstract

A finite element approach is used to assess the transient stability of excavation in elasto-plastic soils. Stability is shown to be a function of the rate of excavation, the soil permeability and the drainage path lengths. Sequential excavation has been modelled rigorously in the finite element analysis, together with the transient effects through a fully coupled Biot formulation. Results are presented which demonstrate the effects of the rate of excavation and permeability on stability. Both drained and undrained behaviour of the problem are retrieved as special cases of the transient analysis, and comparisons made with classical solutions where available.

INTRODUCTION

With the development of the finite element method, the ability to numerically model complicated soil structures has become possible. Geotechnical construction is one such area that has been investigated extensively by the finite element method. The work presented in this paper is concerned with one particular area of geotechnical construction, that of excavation. As with all numerical modelling the aim is to model as closely as possible the true "in the field" behaviour. Initially the prediction of the critical depth of an excavation was the main concern, usually in terms of the factor of safety using a fixed mesh. Later the process of sequential construction was investigated. This was successfully implemented into numerical models by a number of authors (e.g. Ghaboussi and Pecknold [7], Brown and Booker [5], Ho [11]).

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A number of papers have recently addressed the problem of sequential soil excavation. For example Hsi and Small [13] simulated a fully coupled elastic porous material, and Borja *et al* [4] presented drained analyses in which the results were apparently independent of the number of excavation stages. Borja [3] later extended these total stress analyses to include pore pressures by considering the limits of undrained and drained behaviour.

Many of these procedures modelled 'steady-state' conditions, however it is known that the stability of an excavation in a saturated soil is a time dependent process. Any changes in the total normal stress is initially resisted by pore-pressures, which then dissipate with time. This was recognised by Osaimi and Clough [14] who developed a coupled analysis with the inclusion of a sequential construction sequence; but at best could only model a non-linear elastic soil. Banerjee and Kumbhojkar [1] felt that only a fully coupled, transient, anisotropic soil model could be used with confidence in common soils. An anisotropic soil model was therefore developed and incorporated into an F.E program capable of performing geotechnical construction.

In both cases it was shown that the rate of dissipation of the pore fluid was greatly affected by the value of the permeability of the soil. It is also not uncommon for the loading sequence or excavation velocity to vary, which would also have an effect on the drainage conditions at any moment in time.

A simple soil model may be more illustrative if only dealing with a particular aspect of a problem e.g. stability (Yong *et al* [18]), therefore a simple transient excavation model has been developed and was used to carry out parametric studies on the effect of the soil permeability and excavation rate on the final stability of a vertical excavation in a frictional soil. A simple elastically, perfectly plastic stress-strain law was used assuming zero plastic volume change at failure.

NUMERICAL SOLUTION TECHNIQUE

The meshes used were made up of 8-node quadrilateral elements for the solid phase and 4-noded elements for the fluid phase (Zienkiewicz [19]). Non-linear elasto-plastic behaviour was assumed with the shear strength defined by a Mohr-Coloumb failure envelope with parameters c' and ϕ'. Each time step of the transient behaviour is treated as a pseudo-static analysis, with the violating stresses (according to the failure criteria) being redistributed in an iterative manner using a visco-plastic algorithm (Smith and Griffiths [15]). *Reduced* integration was used throughout the mesh, the use of which when predicting limit loads for 8-node quadrilaterals, especially in plane strain is known to give satisfactory results (see e.g. Griffiths [8]).

Excavation Force Simulation

The excavation was simulated by the removal of elements and the application of the forces generated by this action to the new boundary. The boundary forces at the i^{th} stage of an excavation are given by:

$$F_i = \int_{V_i} B^T \sigma_{i-1} dV - \int_{V_i} N^T \gamma dV \qquad (1)$$

where the first term is the nodal internal resisting force vector due to the stresses in the removed elements, and the second term the reversal of the nodal body-load forces (of the removed elements) assuming that γ (the body-load due to gravity) is acting downwards (negative in this case), see Ghaboussi and Pecknold [7] or Holt [12].

The total stress in equation (1) was obtained by adding the effective stress computed at the Gauss points from the solid phase of the analysis (8-node), to the pore pressures interpolated from their nodal values of the fluid phase (4-node).

It should emphasized that for correct simulation of the sequential excavation process, both of the terms in equation (1) must be used to avoid step-sized dependent errors in the final values of the stresses or displacements.

Transient Formulation

Biot [2] formulated a theory in which the soil skeleton is treated as a two phase material coupled by the conditions of compressibility and continuity. Formulations of the Biot equation have been developed which may be used in a Finite element analysis, and have been described in previous work, e.g. Smith and Griffiths [15].

In the case of an excavation, the loading is time dependent, so an incremental formulation was used in the following work producing the matrix version of the Biot equation at the element level presented below:

$$\begin{bmatrix} \mathbf{K_m} & \mathbf{C} \\ \mathbf{C}^T & -\theta\Delta t\mathbf{K_p} \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{r} \\ \Delta\phi \end{Bmatrix} = \begin{Bmatrix} \Delta\mathbf{F} \\ \Delta t\mathbf{K_p}\phi_o \end{Bmatrix} \quad (2)$$

or in the abbreviated form:

$$\mathbf{K_e}\Delta\mathbf{w} = \Delta\mathbf{p} \quad (3)$$

where

$\mathbf{K_m}$	element solid stiffness matrix
$\mathbf{K_p}$	element fluid stiffness matrix
$\mathbf{C}$	element coupling matrix
$\Delta\mathbf{r}$	change in nodal displacements
$\Delta\phi$	change in nodal excess pore-pressures
ϕ_o	'old' excess pore-pressures
$\Delta\mathbf{f}$	change in nodal forces
Δt	calculation time step
θ	time stepping parameter (=1 in present work)

This is similar to the formulation used by Griffiths *et al* [9] in the study of transient earth pressures. By using the fully implicit approach ($\theta = 1$) an unconditionally stable time stepping algorithm is achieved. The advantage over an explicit ($\theta = 0$) method is that the left hand matrix $\mathbf{K_e}$ is constant for linear problems provided the computation time step remains unaltered.

ALGORITHM VERIFICATION

When testing the algorithm the main source of interest is to see if the algorithm is able to correctly monitor the pore-pressure dissipation and if the sequential excavation has any effect on the long term displacements recovered for single or multi staged excavations.

For this, a 1-D example was used, together with a large value of the cohesion, producing an elastic response.

The analysis was performed on the single column of elements in Figure 1 and compared with the results obtained from reported closed form solutions (Terzaghi [17]). The soil was assumed to be initially stress free and have the following drained properties:

$$
\begin{aligned}
E' &= 1.0 \text{ kPa} \\
\nu' &= 0.0 \\
\phi' &= 0.0^\circ \\
\psi' &= 0.0^\circ \text{ (non-dilative)} \\
c' &= 10^{15} \text{ kPa} \\
K_o &= 1.0 \\
k_x/\gamma_w, \quad k_y/\gamma_w &= 1.0 \text{ m}^4/(\text{kN.s}) \text{ (Permeability Coefficient)}
\end{aligned}
$$

The top element was removed and the column was allowed to drain for different values of the non-dimensional time factor T_v given by:

$$T_v = c_v t/H \tag{4}$$

where t is the time, H is the drainage path and c_v the coefficient of consolidation in the vertical direction as defined by:

$$c_v = \frac{k_y(1-\nu')E'}{(1-2\nu')(1+\nu')\gamma_w} \tag{5}$$

Thus for this example $c_v = 1.0$ m^2/s.

For different values of the time factor the distribution of non-dimensional excess pore-pressures throughout the column is shown in Figure 2. The comparison with Terzaghi's [17] one-dimensional theory was exact. Having seen that the algorithm can successfully model the pore-pressure aspect of the problem a final validation would be to test the displacement prediction. Again the mesh in Figure 1 was used, this time with the shaded region being removed in one, two and three stages. The solution for the final displacements at any point after a chosen excavation may be found according to one-dimensional theory by:

$$\Delta H = m_v \Delta\sigma_v H_o \tag{6}$$

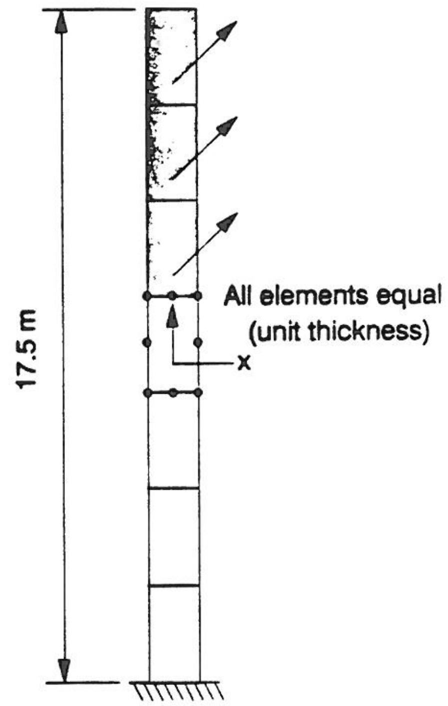

Figure 1 **Mesh used for elastic validations.**

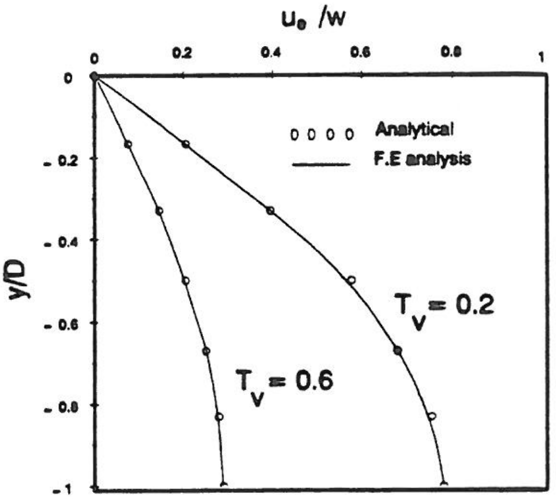

Figure 2 **Excess pore pressure through soil column.**

Where H_o is the height of the soil column (after excavation), $\Delta\sigma_v$ the change in vertical stress and m_v the coefficient of compressibilty where:

$$m_v = 1/E'_o \tag{7}$$

and the one-dimensional modulus may be expressed as:

$$E'_o = \frac{E'(1-\nu')}{(1+\nu')(1-2\nu')} \tag{8}$$

Figure 3 shows the heave at point **X** (Figure 1) due to the described excavation procedures, giving again exact agreement with the theoretical solution. Furthermore the figure also shows that the final displacements after dissipation of the excess pore-pressures is independent of the number of excavation stages. This is in agreement with the stipulation for a suitable excavation model (see Ghaboussi and Pecknold [7], Chow [6]).

2-D TRANSIENT EXCAVATION

The mesh in Figure 4 represents an 8m wide strip of soil (accounting for symmetry) which was gradually excavated until failure occurred as defined by a large increase in the displacements at the crest of the cut.

The assumed soil properties were as follows:

$$\begin{aligned} E' &= 10^5 \text{ kPa} & \psi' &= 0.0^\circ \\ \nu' &= 0.43 & \gamma' &= 20.0 \text{ kN/m}^3 \\ \phi' &= 15^\circ & K_o &= 0.75 \\ c' &= 16^\circ & k_x/\gamma_w = k_y/\gamma_w &= 10^{-1} \text{ m}^4/(\text{kN.s}) \end{aligned}$$

These particular values of c', ϕ' and γ' were chosen because they lead to a critical depth under drained conditions of exactly 4.0m as given by Taylor's [16] charts.

The result produced by the program was 10% higher, giving a critical depth of 4.4m. The result indicated that the chosen value of permeability was sufficiently high, and the excavation rate sufficiently low to closely approximate drained conditions. The discrepancy between the computed value and Taylor's value could be partly explained by the crude mesh which used excavation lifts of 0.4m. Failure occurred in the lift

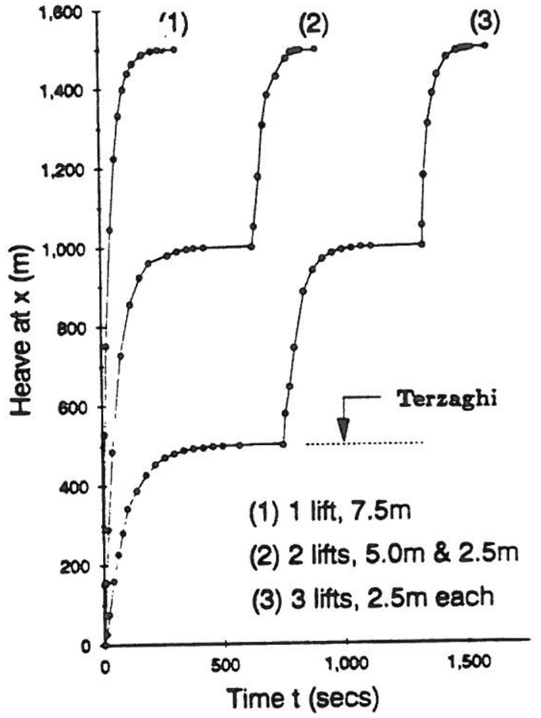

Figure 3 Heave at point X with time for varying lifts.

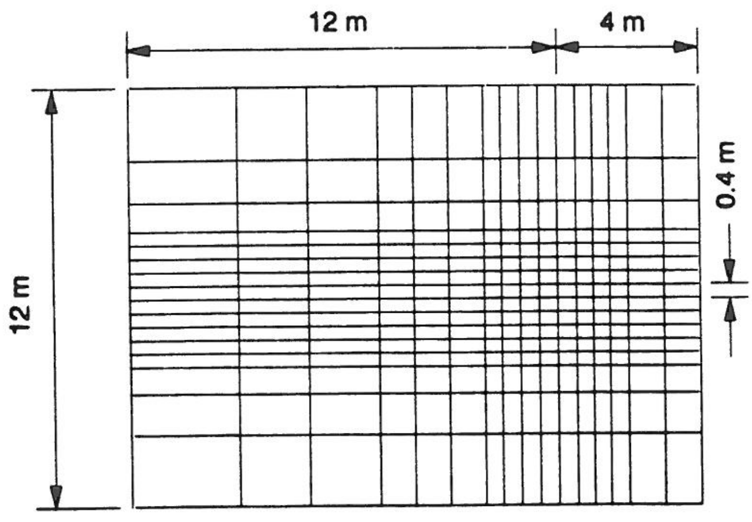

Figure 4 Original mesh for elasto-plastic excavation analyses.

immediately following a depth of 4m. A finer division of the soil in that area leads to a closer approximation to the classical solution as will be shown later in the paper.

The deformed mesh at failure shown in Figure 5 gives the predicted failure mode, being that of a 'wall' failure as was expected for this problem (Terzaghi [17], Taylor [16]).

Influence of the soil permeability

Intuitively, if the soil has a very low permeability, greater excess pore-pressures will still remain after an excavation (and period of drainage) tending towards a so called 'undrained' condition. To see the true effect of the permeability alone, the excavation velocity, or the rate at which successive lifts are removed must be kept constant for all tests. The concept of a constant excavation velocity is taken account of by approximating it to a step function. For example a 2.0m lift left to drain for 2 seconds, followed by a 1.0m lift left to drain for 1 second represents an excavation velocity of 1m/s, see Figure 6.

Using the mesh of Figure 4, a constant excavation velocity of 1m/s was implemented to remove a 4.0m wide strip of soil. The analysis was performed with different values of the permeability coefficient K which was varied in the range:

$$10^{-10} \leq K_{x,y} \leq 10^{-1} \qquad m^4/(kN.s) \tag{9}$$

Figure 7 shows that the lower the permeability of the soil the higher the value of the critical depth obtained. This was to be expected due to the retention of the pore fluid following unloading effects during excavation. The upper limit of the critical depth was found to be 5.6m for permeabilities of 10^{-7} $m^4/(kN.s)$ or less.

To investigate more closely the transient region between the upper and lower limits ('drained' or 'undrained'), the finer mesh given in Figure 8 was employed for further analyses, and the step size of the permeability coefficient reduced by a factor of ten from the previous range. Again the upper limit for the critical depth was found to be 5.6m, however the lower limit was reduced to 4.2m now less than 5% in error with the analytical solution. By plotting the results in the same manner as before but putting a 'best fit' line through the points, the effect of reducing the permeability on the critical

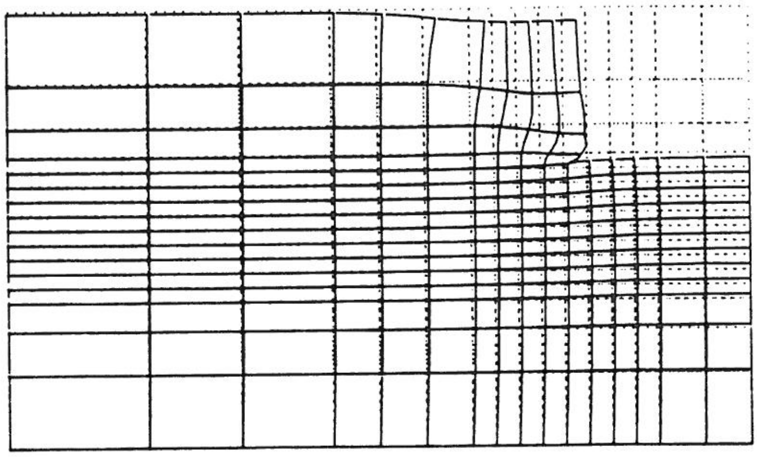

Figure 5 Deformed mesh at failure after dr: ined excavation.

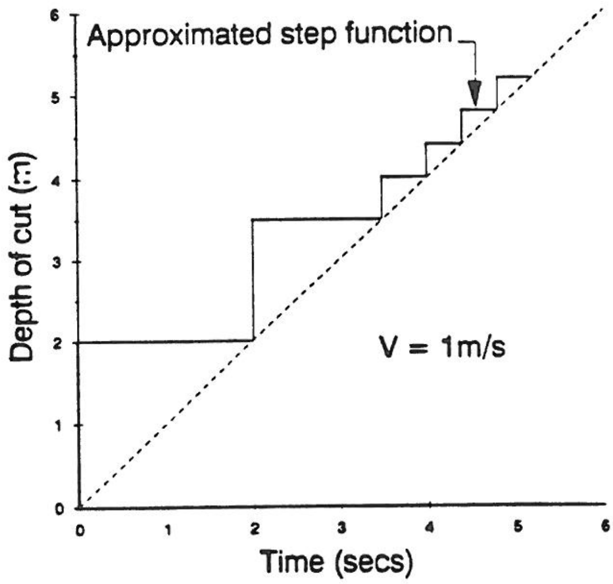

Figure 6 Step function to simulate constant rate of excavation.

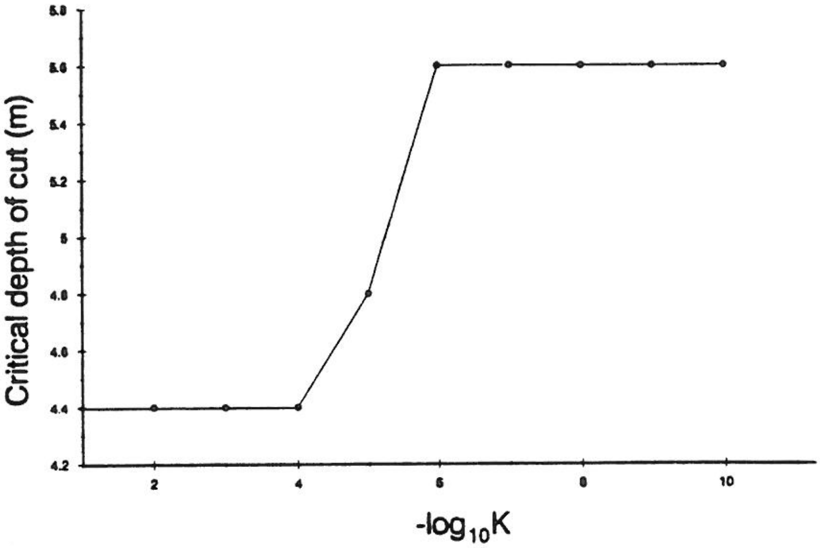

Figure 7 Effect of permeability on critical depth.

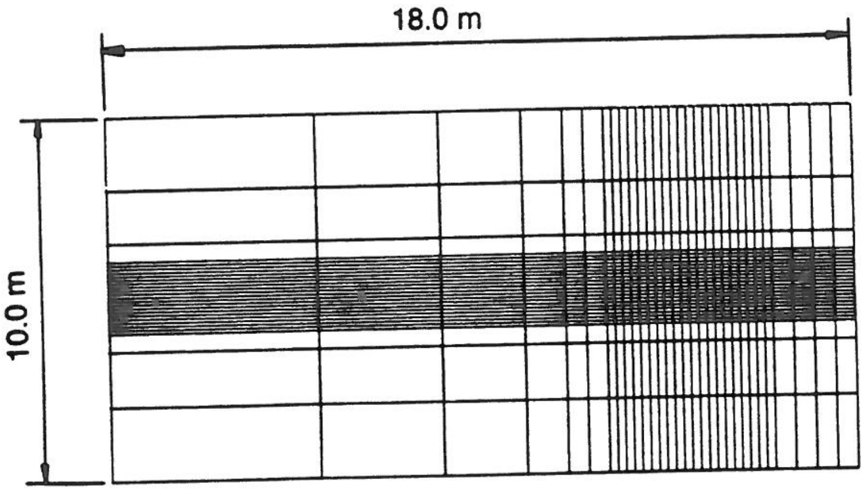

Figure 8 Refined mesh for elasto-plastic excavation analyses.

depth can be traced as shown in Figure 9

Influence of the excavation velocity

Until now, concentration has focussed on parametric studies relating to permeability effects. However for any value of the permeability the soil will eventually reach a drained condition after an excavation. Therefore, whether the soil is going to be drained or undrained depends on how long it is left to drain before any subsequent lifts take place. If the speed of excavation were to be reduced, we would expect to observe drained conditions with less permeable soils due to the longer drainage times between each excavation step.

To check this, the excavation analyses were repeated using the fine mesh of figure 8 with two new excavation velocities given by 0.2 m/s and 0.02 m/s. A very similar distribution in terms of the drained and undrained limits was observed as shown in Figure 10, however as the excavation rate was reduced, the curves were shifted to the right, with drained conditions being observed at increasingly lower permeabilities.

Figure 11 summarises the results in the form of a chart which gives the critical depth as a function of permeability and excavation velocity. For the particular case considered, it is shown that the drained and undrained limits are obtained by changing the permeability by approximately three orders of magnitude assuming a constant excavation velocity.

CONCLUSION

The work undertaken was to develop a fully transient program for the analysis of excavations. This was to allow the transition between the states of drained and undrained behaviour to be investigated. An algorithm for carrying out such an analysis has been presented in this paper. The transient capability of the program was in the form of a Biot consolidation algorithm with a visco-plastic algorithm to account for the material non-linearity. The algorithm was validated against one-dimensional theory for both pore-pressure and displacement prediction, and was used to predict the failure depth and mechanism for a two-dimensional stability problem. The influence of the mesh refinement was also demonstrated.

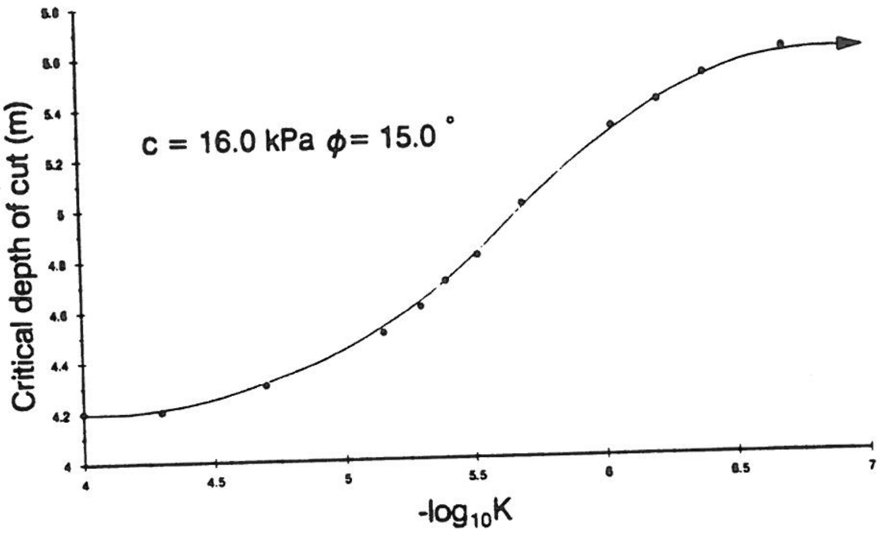

Figure 9 'Best-fit' approximation to transition zone.

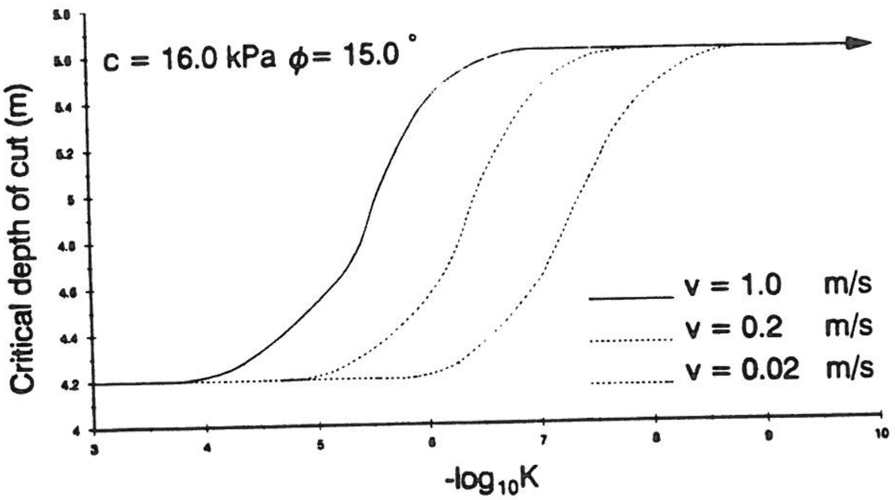

Figure 10 Effect of excavation velocity on critical depth.

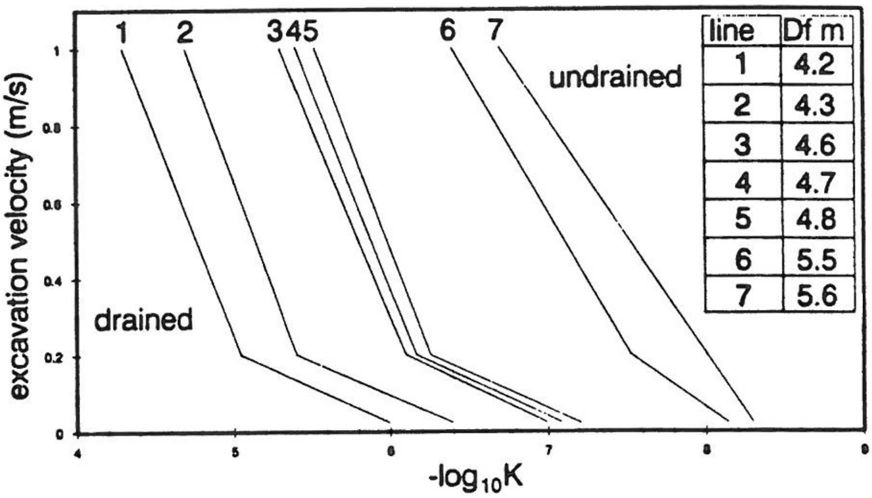

Figure 11 Summary of results for soil properties $\phi' = 15^o, c' = 16$ kPa.

The influence of the two main dependent variables in an excavation were studied. Due to the retention of the pore fluid when a low permeability was enforced, an idealised undrained condition was seen and an increase in the critical depth was observed. The distribution of the critical depth took the form of an 'S' curve. As with the permeability, a change in the excavation rate led to varying values of the critical depth. Slower excavation rates producing larger drainage periods between subsequent lifts, and hence a final drained condition even in the case of relatively low values of the permeability.

It should be emphasised that the results presented herein were based on a simple elasto-plastic soil model which assumed no plastic volume change at failure. The resulting undrained stress paths approximate to 'medium' dense materials; however if the soil under investigation exhibited significant dilative or contractive tendencies, the 'simple' model would not be suitable and a more sophisticated soil model would be required. Of particular interest would be a soil which exhibited contractive tendencies during shear. Such a material could experience a rapid loss of strength during undrained loading. In the context of excavation analysis, this might lead to the undrained case being more critical than the drained case. Studies are continuing in this area, and will be the subject of a subsequent publication.

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