Finite Element Modeling of Settlements on Spatially Random Soil

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Abstract: The effect of a random and spatially correlated soil stiffness on the total settlement under the center of a uniformly loaded flexible-strip footing has been studied. Random field theory has been combined with the finite-element method to perform "Monte Carlo" simulations of the settlement problem with a variable Young's modulus and a constant Poisson's ratio. The soil Young's modulus field has been simulated with a fixed mean, standard deviation, and spatial correlation structure using the local average subdivision (LAS) method. The results of parametric studies have been compared with the deterministic quantities to gauge the effect of the standard deviation, correlation structure, and geometry on the settlement behavior. The results indicate a modest increase in expected settlement for presumptive ranges of soil stiffness variability.

INTRODUCTION

In conventional geotechnical analyses, the soil properties are assumed to be characterized by some fixed value that is determined from laboratory testing of soil samples or in-situ measurements. Analyses then proceed with this value in a deterministic manner yielding a solution that is dependent on the quality of the characteristic value that has been used. Natural soil is often extremely nonhomogeneous in properties due to its composition and deposition. These properties vary from point to point and the variations may be great even if the soil is "homogeneous" under a soil classification system.

The incorporation of random fields (Vanmarcke 1984) into engineering analyses allows the modeling of the spatially random soil, taking into account the correlation structures of the properties. The settlement and stress responses of shallow foundations on a random soil medium have previously been investigated by Baecher and Ingra (1981), Righetti and Harrop-Williams (1988), Zeitoun and Baker (1992), and Paice et al. (1994).

FINITE ELEMENT MODEL

The model that has been used for the present study assumes plane strain conditions. The soil medium is underlain by a rigid stratum and is loaded at the center of the soil surface, as shown in Fig. 1 (the regions of lower Young's modulus are represented by darker grays; higher Young's modulus regions are represented by lighter grays). The finite element program used is similar to that published by Smith and Griffiths (1988) and uses meshes of square four-node elements. The global stiffness matrix of the problem has been formed using algebraic closed-form element stiffnesses (Griffiths and Mustoe 1995) based on the "K-G" selective reduced integration (SRI) formulation that splits the element stiffness matrix into bulk modulus, K, and shear modulus, G, partitions. The use of the closed-form element stiffnesses greatly reduces the time required for the forming of the global stiffness matrix, and the "K-G" SRI allows the study of incompressible problems (e.g., undrained clay) in which Poisson's ratio approaches the value of 0.5 without the numerical locking observed when using exact integration.

SIMULATION OF YOUNG'S MODULUS FIELD

For the current study, the soil has been treated as a linear elastic body with a spatially random Young's modulus and a constant Poisson's ratio. The latter assumption has been taken due to little being currently known about the variability of the Poisson's ratio, which is difficult to measure (Baecher and Ingra 1981; Zeitoun and Baker 1992; Paice et al. 1994). The coefficient of variation (σ_E/μ_E) of the Young's modulus has been quoted as having a range from 2% (0.02) to 42% (0.42) with a recommended value of 30% (Lee et al. 1983).

The lognormal distribution has been adopted for the generation of the Young's modulus fields; the use of this distribution being consistent with the knowledge that the Young's modulus may not have a negative value. Accepting the use of this distribution, then, the Young's modulus field is obtained through the transformation

$$E_i = \exp(\mu_{\ln E} + \sigma_{\ln E} g_i) \quad (1)$$

where E_i = Young's modulus assigned to ith element; g_i = local average of standard Gaussian random field, g, over domain of ith element; and $\mu_{\ln E}$ and $\sigma_{\ln E}$ = mean and standard deviation of logarithm of E (obtained from "target" mean and standard deviation μ_E and σ_E).

The local average subdivision (LAS) technique (Fenton 1990; Fenton and Vanmarcke 1990) generates local averages g_i, which are derived from the random field g having zero mean, unit variance, and a spatial correlation controlled by the scale of fluctuation, θ_E. The scale of fluctuation is a measure of the distance, beyond which the points of the random field are effectively uncorrelated. As the scale of fluctuation goes to infinity, g_i becomes equal to g_j for all elements i and j—the field of Young's moduli tends to become highly correlated and uniform on each realization. At the other extreme, as the scale of fluctuation goes to zero, g_i and g_j become independent for all $i \neq j$—the field of Young's moduli tends to become highly uncorrelated and varies rapidly from point to point. For all of the analyses an exponential correlation structure has

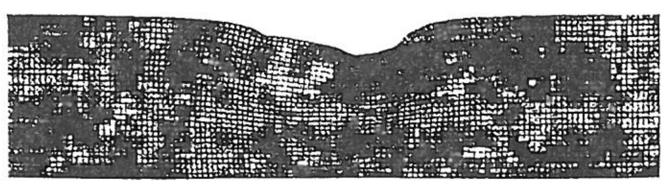

FIG. 1. Typical Displaced Finite Element Mesh Overlain by Young's Modulus Field ($B/h = 0.5$, $\nu = 0.2$, $\sigma_E/\mu_E = 0.42$, $\theta_E = 4.0$ m; Displacement Magnification = 2.5×10^5)

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been used, i.e., the positive correlation between points in the field decays exponentially with increasing distance.

For the present study, the soil was represented by a two-dimensional (2D) isotropic random field and, thus, the out-of-plane variation of the soil Young's modulus has not been taken into account. This limitation on the analysis implies that the scale of fluctuation in this direction is infinite (the soil's Young's modulus is constant in this direction). This is clearly a deficiency of the current model, but it is believed that the general trends of the behavior of the soil medium are still valid. Earlier studies (Paice et al. 1994) made the assumption of a random field that was symmetrical about the footing centerline. In the current study this assumption was not used and the whole domain has been analyzed, although it was found that the statistics of the symmetric and nonsymmetric analyses were in close agreement.

PARAMETRIC STUDIES

For the analyses, parametric studies were performed for a range of problem geometries and soil parameter values. To obtain a sufficiently large number of results for the calculation of the output statistics, it was decided that each "Monte Carlo" analysis would consist of 2,000 realizations of the Young's modulus field. The output statistics are then obtained from the ensemble of solutions calculated by the finite element method program. In all analyses the soil is assumed to be isotropic with a mean Young's modulus of $\mu_E = 1.0 \times 10^6 \text{kN/m}^2$.

The values of Poisson's ratio used were

$$\nu \in \{0.0, 0.2, 0.4\} \qquad (2)$$

but results are only presented for $\nu = 0.2$.

The coefficient of variation (σ_E/μ_E) was varied over the following range:

$$0.0 \leq \sigma_E/\mu_E \leq 1.0 \qquad (3)$$

where a value of σ_E/μ_E equal to zero indicates a deterministic analysis based on a constant Young's modulus of μ_E assigned to all elements in the finite element mesh. The scale of fluctuations (θ_E) considered was as follows:

$$\theta_E \in \{0.25 \text{ m}, 1.0 \text{ m}, 4.0 \text{ m}, 16.0 \text{ m}\} \qquad (4)$$

Parametric studies of the flexible-strip footing carrying a unit uniformly distributed load were performed to determine the effect of the coefficient of variation (σ_E/μ_E) and the scale of fluctuation (θ_E) on the total vertical settlement under the center of the footing.

For each case in the parametric studies, statistics were obtained relating to the vertical settlement under the center. The statistics are presented in the form of an influence coefficient (I) that is obtained in the same manner as Poulos and Davis (1974)

$$I = \frac{\rho \pi E}{ph} \qquad (5)$$

where the Young's modulus, E, is taken as μ_E. The principle of superposition has been used to calculate the values of the influence coefficient at the center of the loaded width; i.e., B is equal to half the width and ρ is equal to half the central settlement. A value of $h = 15.0$ m has been used for all of the analyses that are presented here.

RESULTS

Influence Coefficient

Fig. 2 shows the variation of the mean of the influence coefficient, μ_I, with the coefficient of variation, σ_E/μ_E where $\mu_E = 1.0 \times 10^6 \text{kN/m}^2$, for a Poisson's ratio of 0.2 and scale of fluctuation $\theta_E \in \{0.25 \text{ m}, 1.0 \text{ m}, 4.0 \text{ m}, 16.0 \text{ m}\}$. For all scales of fluctuation, as the coefficient of variation approaches zero the value of μ_I tends to the deterministic value of 1.25, which agrees with the chart of Poulos and Davis (1974). As the coefficient of variation increases from zero, the value of μ_I increases for all θ_E.

At a coefficient of variation of 0.42, the recommended upper value by Lee et al. (1983), the increase in the value of μ_I is modest for θ_E up to 16.0 m, between 1.08 and 1.15 times the deterministic value. For a higher coefficient of variation of 1.0, the value of μ_I shows a significant increase equal to 1.43 → 1.71 times the deterministic value for $\theta_E = 0.25 \rightarrow 16.0$ m.

Fig. 3 shows the variation of the standard deviation of the influence coefficient, σ_I, for the same analyses that are shown in Fig. 2. Again, as σ_E/μ_E approaches zero the analyses become deterministic for all scales of fluctuation (a standard deviation of zero implies no variability and, therefore, a deterministic analysis). As the coefficient of variation increases, the standard deviation of the influence coefficient rapidly increases for all scales of fluctuation. This rapid increase shows the degree of uncertainty in the influence coefficient and indicates

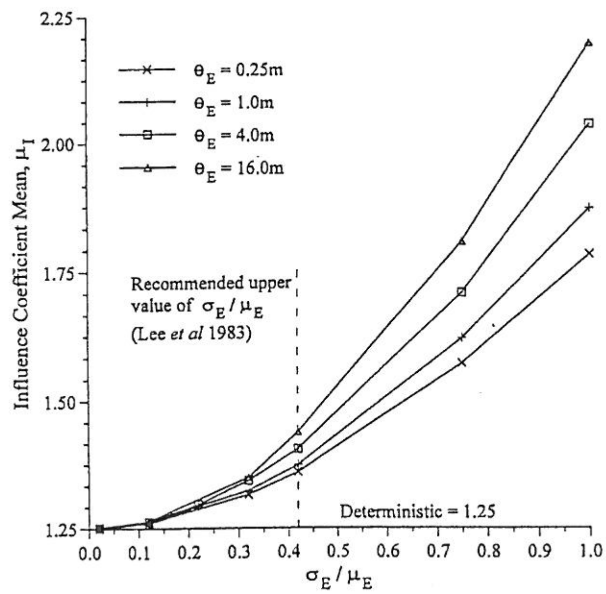

FIG. 2. Variation of μ_I with σ_E/μ_E for $B/h = 0.5$, $\nu = 0.2$

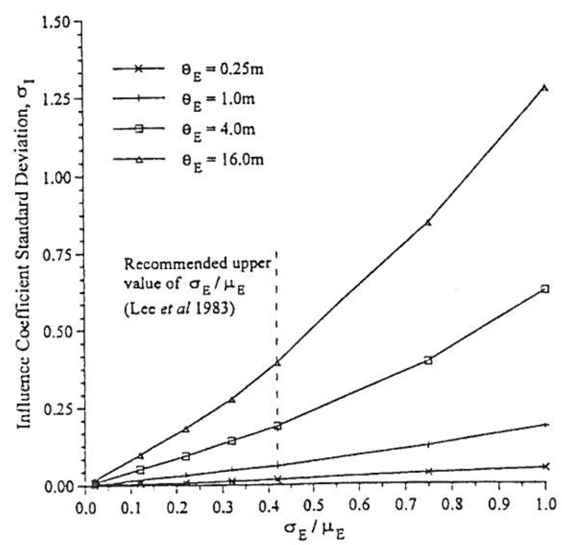

FIG. 3. Variation of σ_I with σ_E/μ_E for $B/h = 0.5$, $\nu = 0.2$

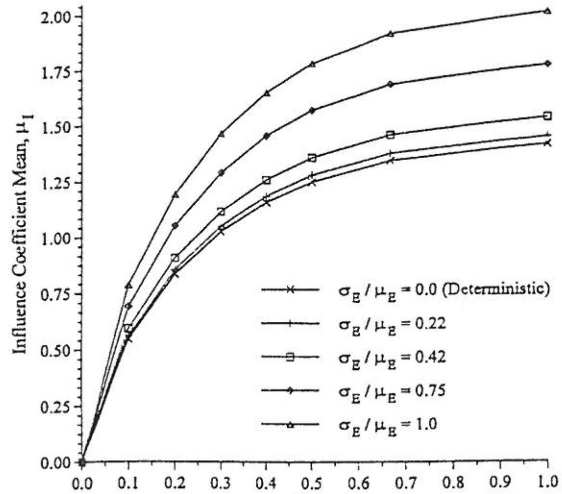

FIG. 4. Variation of μ_I with B/h for $\nu = 0.2$, $\theta_E = 0.25$ m, and $\sigma_E/\mu_E \in \{0.0, 0.22, 0.42, 0.75, 1.0\}$

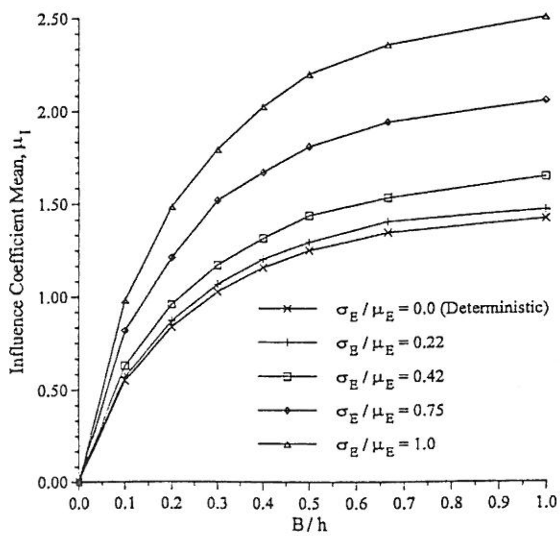

FIG. 5. Variation of μ_I with B/h for $\nu = 0.2$, $\theta_E = 16.0$ m, and $\sigma_E\mu_E \in \{0.0, 0.22, 0.42, 0.75, 1.0\}$

the observed scatter of the central settlements and could be interpreted in the context of reliability-based design.

Influence of Width-to-Depth Ratio

Figs. 4 and 5 show the variation of μ_I with B/h and σ_E/μ_E for $\nu = 0.2$ and $\theta_E \in \{0.25 \text{ m}, 16.0 \text{ m}\}$ in the form of Poulos and Davis (1974). In both figures, $\sigma_E/\mu_E = 0.0$ represents the deterministic value of the influence coefficient. Figs. 4 and 5 again show that as the coefficient of variation increases the value of the mean influence coefficient also increases for all geometries within the range of the study. The lines corresponding to $\sigma_E/\mu_E = 0.42$ indicate the possible settlement response that could occur at the recommended upper value stated by Lee et al. (1983). The lines corresponding to $\sigma_E/\mu_E = 0.75$ and 1.0 demonstrate the marked increase in the possible settlements if the coefficient of variation of the soil Young's modulus is higher than the recommended upper value.

CONCLUDING REMARKS

In the present paper, random field theory has been combined with the finite element method to study the settlement response of a flexible-strip footing founded on a spatially random soil. Parametric studies have been carried out for a number of coefficients of variation and scale of fluctuation to investigate the effect of the variability of the soil Young's modulus and the spatial correlation on the observed settlements.

For Young's modulus variances up to the recommended upper limit of $\sigma_E/\mu_E = 0.42$, the spatial correlation structure had little effect on the expected settlement that was observed to be around 12% higher than the deterministic value based on the mean Young's modulus value. In the unlikely event of higher Young's modulus variances well above the recommended range, the influence of spatial correlation becomes much more pronounced and the expected settlement can considerably exceed the deterministic value. In practice, predicted settlements on such highly variable materials would require a corresponding high factor of safety.

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APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

B = width of flexible strip footing;
E = Young's modulus (elastic modulus);
E_i = Young's modulus assigned to ith element;
G = shear modulus;
g = standard Gaussian random field;
g_i = local average of g over ith element;
h = depth of soil layer to rigid underlying stratum;
I = edge settlement influence coefficient;
K = bulk modulus;
p = load/unit area applied to flexible strip footing;
ρ = settlement under edge of flexible strip footing;
ν = Poisson's ratio;
μ_E, σ_E = Young's modulus mean and standard deviation;
σ_E/μ_E = coefficient of variation of Young's modulus;
μ_I, σ_I = influence coefficient mean and standard deviation; and
θ_E = scale of fluctuation of Young's modulus.