NOTES

A chart for estimating the average vertical stress increase in an elastic foundation below a uniformly loaded rectangular area

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In spite of the considerable assumptions involved, stress distribution charts based on elastic theory are still used by engineers to estimate stresses induced in soil masses by surface loading. Although there are limited data comparing calculated and measured stress increments, vertical stress components have been predicted quite reliably by this method. The chart presented here enables the average vertical stress increment beneath a corner of a uniformly loaded rectangular area to be estimated. The results are based on numerical integration of existing solutions for the rectangle problem, and should reduce the need for sublayers when calculating consolidation settlements.

Key words: stress distribution, elasticity, design graph, rectangular footing, consolidation.

En dépit des nombreuses hypothèses impliquées, les abaques de distribution des contraintes fondés sur la théorie de l'élasticité sont toujours utilisés par les ingénieurs pour évaluer les contraintes induites par des charges de surface dans les massifs de sol. Bien qu'il y ait peu de données comparatives entre des accroissements de contrainte calculé et mesuré, les composantes verticales des contraintes ont été prédites de façon assez fiable par cette méthode. L'abaque présenté ici permet l'évaluation de l'accroissement de contrainte verticale moyenne sous le coin d'une surface rectangulaire chargée uniformément. Les résultats sont basés sur l'intégration numérique des solutions existantes pour le problème du rectangle, et ils devraient réduire le nombre de cas où il est nécessaire de considérer des sous-couches dans les calculs de tassement.

Mots clés: distribution des contraintes, élasticité, abaque de calcul, semelle rectangulaire, consolidation.

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Introduction

When computing the consolidation settlement of a footing resting on a compressible soil layer of finite thickness, it is often necessary to assume a stress distribution that takes account of the reduction of vertical stress with depth. Footings which are 'small' relative to the thickness of the compressible layer may be dealt with by assuming a simple spread of load, or more rigorously, by using an integrated version of Boussinesq's point load solution for an elastic half-space. 'Large' footings, on the other hand, may not require such a treatment as the stresses induced may approximate to 'oedometer' conditions.

This technical note enables the average stress increment beneath a corner of a uniformly loaded rectangular footing to be obtained as opposed to the stress increment at a specific depth, as is the case with most previously published charts. This is shown to be more conservative than taking the mid-depth stress for certain combinations of loading geometry and layer thickness. The method also avoids the necessity of considering the compressible soil as a series of sublayers.

Derivation

The consolidation settlement of a loaded area may be estimated from [1]:

$$[1] \quad \Delta_{\mathrm{ULT.CONS}} = \sum_{i=1}^{n} m_{v_i} H_i \sigma_i$$

where n = the number of sublayers, m_{v_i} = the coefficient of volume compressibility in the ith sublayer, H_i = the thickness of the ith sublayer, and σ_i = the vertical stress increment at the middle of the ith sublayer.

In Fig. 1 it is seen that this process involved replacing the smoothly varying stress distribution by a 'staircase,' which assumes a constant stress increment over each sublayer. This enables oedometer-based properties, such as m_v, to be applied to each sublayer.

The stress increment in the middle of each sublayer is conveniently obtained using charts that assume that the soil is linear elastic. Typical of these are the solutions of Steinbrenner (1936) and Fadum (1948) for rectangular loaded areas. Many other solutions exist, and a compre-

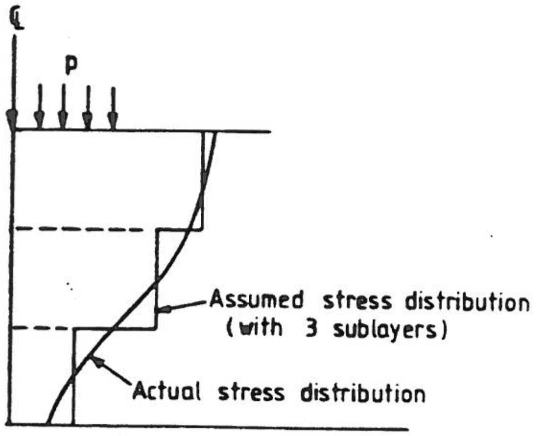

FIG. 1. Sublayer approach.

hensive collection of these may be found in Poulos and Davis (1974).

Considering the case of rectangular loads, the Steinbrenner solution is quoted by Harr (1977) in the form

[2] $\sigma = pI$

where σ = the vertical stress increment at depth z below a *corner* of a uniformly loaded rectangle supporting a stress p, and

[3] $$I = \frac{1}{2\pi} \times \left[\frac{mn}{\sqrt{1+m^2+n^2}}\, \frac{(1+m^2+2n^2)}{(1+n^2)(m^2+n^2)} + \arcsin \frac{m}{\sqrt{m^2+n^2}\sqrt{1+n^2}}\right]$$

= a dimensionless influence factor

If L and B are the side lengths of the rectangle, then

[4] $$m = \frac{L}{B} \quad \text{and} \quad n = \frac{z}{B}$$

The average stress increase beneath a corner of a uniformly loaded rectangle must be a function of the average influence factor over the depth of interest; thus,

[5] $\sigma_{av} = pI_{av}$

where

[6] $$I_{av} = \frac{1}{H}\int_0^H I\, dz$$

Values of I_{av} have been obtained by numerical integration of [3] and are given in Fig. 2 in the form

[7] $I_{av} = f(a, b)$

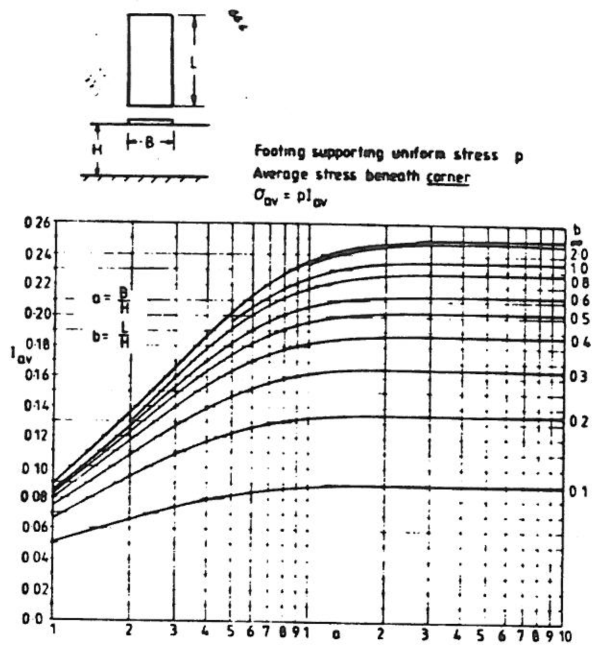

FIG. 2. Influence factors for the average stress beneath a corner of a uniformly loaded rectangular area.

where

[8] $$a = \frac{B\,(\text{or } L)}{H} \quad \text{and} \quad b = \frac{L\,(\text{or } B)}{H}$$

Figure 2 is used in the same way as Fadum's chart, except the full soil thickness is entered into the dimensionless variables instead of a specific depth. Average stress increments occurring beneath locations other than a corner are found using superposition in the normal way.

Average stress in a layer at depth

Although Fig. 2 assumes that the loaded area rests directly on the compressible layer, the average stress in a layer at depth may also be found.

As I_{av} represents the 'area' of the stress distribution divided by the total depth, the average influence factor for layer 2 in Fig. 3 is given by

[9] $$I_{av} = \frac{H_2 I_{av_2} - H_1 I_{av_1}}{H_2 - H_1}$$

Consolidation settlements

Equation [1] allows for the possibility of both m_v and σ varying with depth. Indeed, for a normally consolidated soil, m_v reduces significantly with depth as the consolidating pressure is increased. For such soils, settlement calculations require a sublayer approach, as suggested in Fig. 1, to take account of this. Figure 2

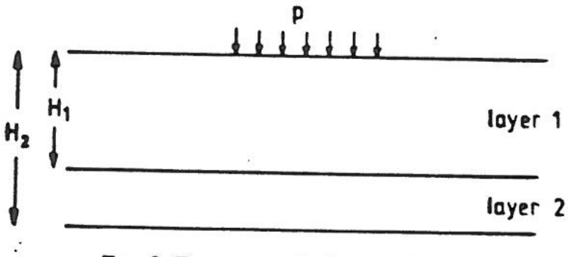

FIG. 3. Treatment of a layer at depth.

could still be used, however, to estimate the average stress in each sublayer from [9].

Average stress values from Fig. 2 are more useful when m_v can be considered essentially constant for the range of stresses to which the soil is subjected. This assumption may be reasonable for stiff, overconsolidated soils in which the loading increment does not cause the preconsolidation pressure to be exceeded.

In such cases, [1] simplifies to

[10] $\Delta_{\mathrm{ULT,CONS}} = m_v H p I_{av}$

enabling consolidation settlements beneath a corner to be estimated in a single calculation without the need for sublayers.

Discussion

The chart in Fig. 2 should be of value in cases where the stress reduction with depth would normally be dealt with by sublayers. It is accepted, however, that in many cases, the stress occurring at the mid-depth will be sufficiently representative of the layer as a whole. Figures 4 and 5 compare the average influence factor from Fig. 2 with the mid-depth influence factor obtained from Fadum (1948). Figure 4 indicates that if one of the dimensions of the rectangle is small relative to the layer thickness ($a = 0.1$), mid-depth values are unconservatively low. For larger loads ($a = 10$), the stresses tend to 'oedometer' conditions and the mid-depth value is quite acceptable. Figure 5 shows the ratio of mid-depth to average influence factors beneath a corner of a square footing for a range of sizes. For $B/H > 0.4$, the factors are virtually indistinguishable, but for smaller B/H ratios the mid-depth value becomes increasingly unconservative.

From a standpoint of classical elasticity, the values of I_{av} produced in Fig. 2 are approximate because they were obtained by applying Boussinesq's half-space theory to a layer of finite extent. Analysis has shown (Poulos and Davis 1974; Perloff 1975) that when a stiff layer underlies a more flexible layer, the stresses in the upper layer are higher than would be indicated by Boussinesq's solution. For cases where strip loads are supported by an elastic material underlain by a rigid, horizontal rough layer, for example, the charts of Poulos

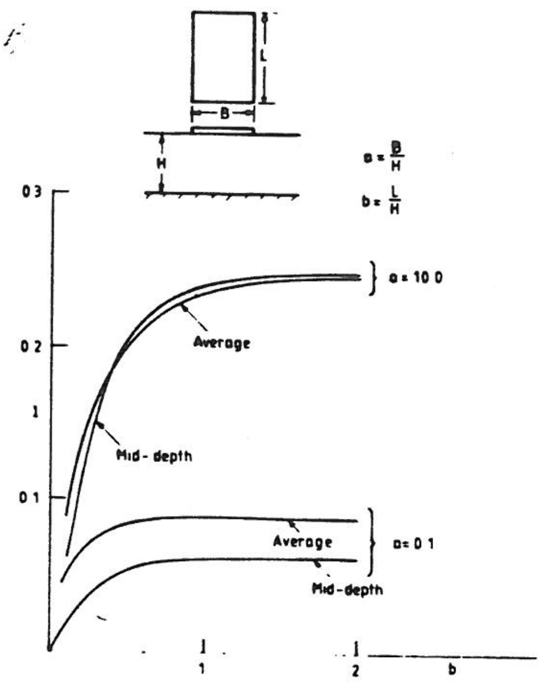

FIG. 4. Comparison of the average and mid-depth influence factors.

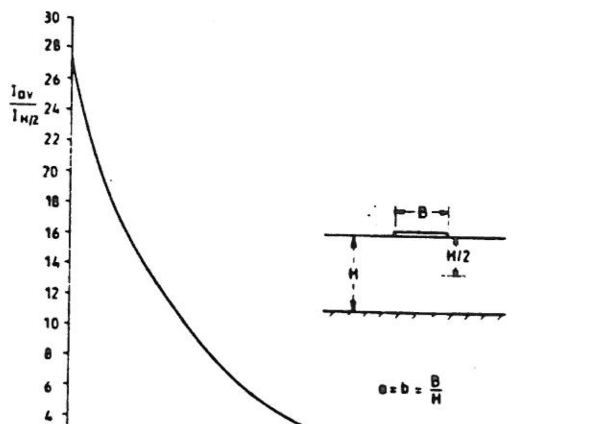

FIG. 5. Comparison of average and mid-depth influence factors for square footings.

(1967) could be used in conjunction with a sublayer approach.

Experimental evidence is inconclusive, however, and the half-space solutions are customarily used in engineering practice. Furthermore, the cases for which Fig. 2 would be most useful are when the footing size to soil depth ratio is relatively small; in such cases, any discrepancy would tend to be less significant.

Conclusions

A chart has been presented, which gives the average stress increase beneath a corner of a uniformly loaded rectangle. The chart assumes the soil to be a linear elastic half-space and has been obtained by numerical integration of Steinbrenner's (1936) expression.

The chart can be used in cases where the side length of a rectangular load is less than about 0.4 of the soil layer thickness. For larger loads, no real benefit would be obtained from using the chart as the mid-depth stress in a single sublayer analysis would be sufficiently accurate.

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Comparison of observed and calculated slip surface in slope stability calculations

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This study presents a reanalysis of four documented test embankment failures. Previous analysis of the embankments was done using the simplified Bishop method, for both total and effective stress analyses. The present work makes use of an optimization procedure called SSOPT, which searches a slope for the critical slip surface, without predefining the shape of this surface. This procedure yields reasonable factors of safety and a good estimation of the expected slip surface. This is not always the case for the Bishop-type analyses.

Key words: slope stability, slip surface, analysis and observation.

Cet article présente une réanalyse de quatre ruptures de remblais expérimentaux présentées dans la littérature. Les analyses antérieures de ces remblais avaient été faites au moyen de la méthode de Bishop simplifiée, tant pour les analyses en contraintes totales que pour celles en contraintes effectives. La présente étude fait appel à une procédure d'optimisation appelée SSOPT, qui recherche la surface de rupture critique dans un talus sans que la forme de cette surface ait été spécifiée. Cette procédure conduit à des facteurs de sécurité raisonnables et à une bonne définition de la surface de rupture probable. Ceci n'était pas toujours le cas pour les analyses du type Bishop.

Mots clés: stabilité des pentes, surface de rupture, analyse et observation.

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Introduction

Slope stability calculations are essentially a problem in the calculus of variations (Baker and Garber 1978), namely the determination of the slip surface that yields the minimum factor of safety. This problem may be written formally as follows:

$$[1] \quad F_s = \min_{y(x)} f\{y(x)\} = f\{y_c(x)\}$$

where F_s is the minimum factor of safety; $y(x)$ is a potential slip surface; $y_c(x)$ is the critical slip surface; and $f\{y(x)\}$ is a functional (rule of correspondence) that assigns a value of F to every potential slip surface $y(x)$.

In general there are two approaches to the approximate solution of [1]:

(i) Methods in which $y(x)$ is assumed to be of a particular geometry, for example a straight line through the toe of the slope (Culmann 1866), a circular arc (Fellenius 1936; Taylor 1948; Bishop 1955), a log spiral (Spencer 1969; Frohlich 1953), or an arc of a cycloid (Ellis 1973). These restrictions on the shape of the slip surfaces make the minimization process of [1] much