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# THE EFFECT OF PORE-FLUID COMPRESSIBILITY ON FAILURE LOADS IN ELASTO-PLASTIC SOIL

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## SUMMARY

The influence of pore-fluid compressibility on failure loads in certain soil mechanics problems is investigated. The results are tested using a finite element approach treating the soil as an elasto–plastic material. A limiting value for the pore-fluid compressibility to give satisfactory solutions and numerical stability is suggested.

## INTRODUCTION

A method described by Naylor[1,2] for modelling the undrained behaviour of soils using finite elements suggested that a 'large' fluid bulk modulus should be added to the soil's effective stress–strain matrix.

For a 2-D problem, this is given by

$$\mathbf{D} = \mathbf{D}' + \begin{bmatrix} K_e & K_e & 0 & K_e \\ & K_e & 0 & K_e \\ & & 0 & 0 \\ \text{symmetrical} & & & K_e \end{bmatrix} \tag{1}$$

This assumes that the stress tensor is given in the following order

$$\boldsymbol{\sigma}' = [\sigma'_x \ \sigma'_y \ \tau_{xy} \ \sigma'_z]^{\mathrm{T}} \tag{2}$$

If $E'$ and $\nu'$ are the effective elastic parameters, then equation (1) leads to total stress parameters given by

$$E = E'\left[\frac{3(1-2\nu') + E'/K_e}{2(1+\nu')(1-2\nu') + E'/K_e}\right] \tag{3}$$

and

$$\nu = \frac{1}{2} - \frac{(1-2\nu')E'/Ke}{4(1+\nu')(1-2\nu') + 2E'/Ke} \tag{4}$$

## TRIAXIAL TEST

Consider a normally consolidated soil subjected to a consolidated undrained triaxial test. It is easily shown[3] that for a cohesionless soil in which $\Delta\sigma_3 = 0$, the deviator stress at failure is given by

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$$D_f = \frac{\sigma_3(K_p - 1)}{1 + \bar{A}_f(K_p - 1)} \tag{5}$$

Pore pressures are generated from Skempton's[4] equation

$$\Delta u = B(\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)) \tag{6}$$

and

$$\bar{A} = AB \tag{7}$$

By combining elastic theory with the Mohr–Coulomb criterion for a similar material an analogous expression can be obtained[5] for the triaxial case given by

$$D_f = \frac{\sigma_3(K_p - 1)(3\beta_t + 1)}{(K_p + 2)\beta_t + 1} \tag{8}$$

where

$$\beta_t = \frac{(1 - 2\nu')K_e}{E'} \tag{9}$$

The elastic properties have been combined in the dimensionless number $\beta_t$ and equations (5) and (8) are equivalent if

$$\bar{A}_f = \frac{\beta_t}{3\beta_t + 1} \tag{10}$$

By varying $\beta_t$, $\bar{A}_f$ values can be obtained in the range

$$0 \leqslant \bar{A}_f < \tfrac{1}{3} \tag{11}$$

Equation (8) is shown plotted in Figure 1, and it should be noted that the solution is essentially constant for $\beta_t \geqslant 20$.

By similar arguments, an element loaded axially under plane strain conditions yields the following expressions (Figure 2):

$$D_f = \frac{\sigma_3(K_p - 1)(2\beta_{ps} + 1)}{(K_p + 1)\beta_{ps} + 1} \tag{12}$$

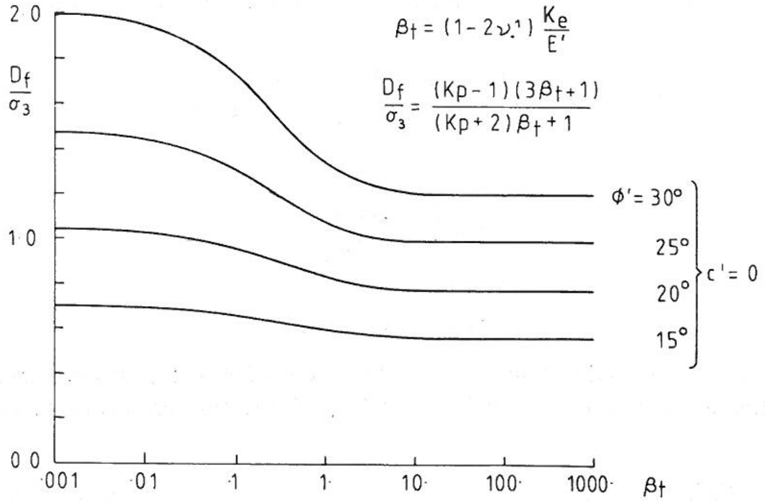


Figure 1. Deviator stress at failure vs. $\beta_t$ (triaxial)

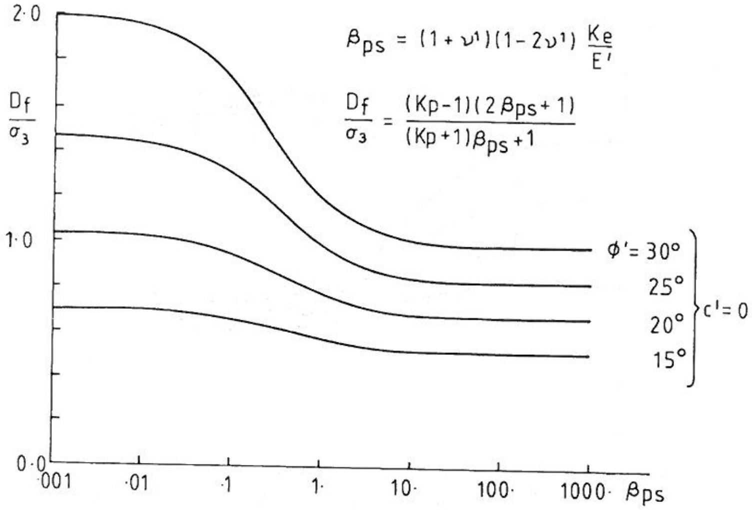


Figure 2. Deviator stress at failure vs. $\beta_{ps}$ (plane strain)

where

$$\beta_{ps} = (1 + v')(1 - 2v')\frac{K_e}{E'} \tag{13}$$

Equivalence this time is given by putting

$$\bar{A}_f = \frac{\beta_{ps}}{2\beta_{ps} + 1} \tag{14}$$

and the corresponding values of $\bar{A}_f$ lie in the range

$$0 \leqslant \bar{A}_f < \tfrac{1}{2} \tag{15}$$

It may be noted that when $\beta = 0$, drained conditions are implied in both cases.

## PASSIVE EARTH PRESSURE

In order to extend the previous reasoning to a more interesting problem, the case of a smooth rigid wall translated horizontally into a mass of undrained soil was considered. This problem was chosen because to a close extent it approximates the condition $\Delta\sigma_3 = 0$.

Before movement of the wall commences, it may be assumed that the soil is fully consolidated under its submerged unit weight $\gamma'$.

At failure, the net horizontal thrust exerted by the soil is given by

$$P'_p = \tfrac{1}{2}\gamma' H K_p \sigma'_{vf} \tag{16}$$

where $H$ is the height of the wall and $\sigma'_{vf}$ the vertical effective stress acting on an element of soil at its base.

From the principle of effective stress,

$$\sigma'_{vf} = \gamma' H - \Delta u_f \tag{17}$$

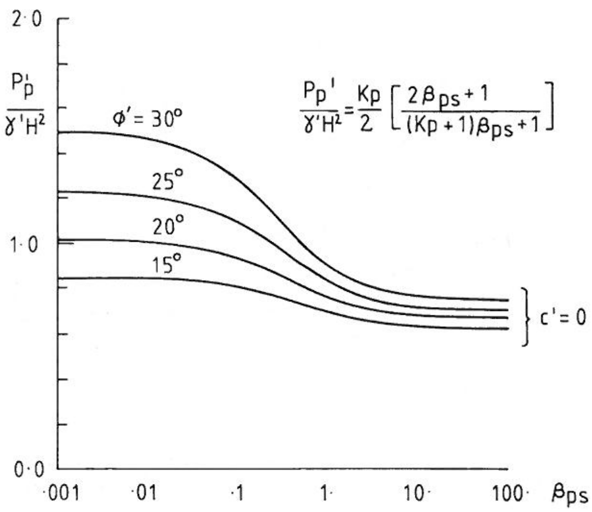


Figure 3. Maximum passive force vs. $\beta_{ps}$

where $\Delta u_f$ is the pore pressure at failure in an undrained plane strain element initially consolidated to a stress of $\gamma' H$.

It can be shown[5] that

$$\Delta u_f = \frac{\gamma' H (K_p - 1)\beta_{ps}}{(K_p + 1)\beta_{ps} + 1} \tag{18}$$

hence from equations (16) and (17),

$$P_p' = \tfrac{1}{2}\gamma' H^2 K_p \left[\frac{2\beta_{ps} + 1}{(K_p + 1)\beta_{ps} + 1}\right] \tag{19}$$

In Figure 3, the maximum passive resistance of the soil is plotted as a function of $\beta_{ps}$ giving the following range:

$$\tfrac{1}{2}\gamma' H^2 K_p \geqslant P_p' \geqslant \tfrac{1}{2}\gamma' H^2 K_p \left[\frac{2}{K_p + 1}\right] \tag{20}$$

The upper bound, corresponding to $\beta_{ps} = 0$, is equivalent to the drained Rankine solution.

## FINITE ELEMENT IMPLEMENTATION

The single 4-noded element shown in Figure 4 was assumed to behave as an elastic–perfectly plastic Mohr–Coulomb material and was assigned the following effective soil parameters:

$$\left.\begin{aligned} \phi' &= 30° \\ c' &= 0 \\ \psi &= 0 \\ E' &= 10^5\,\mathrm{kN/m^2} \\ \nu' &= 0{\cdot}3 \end{aligned}\right\} \tag{21}$$

and the $\beta$-parameter was varied by altering the value of the fluid modulus.

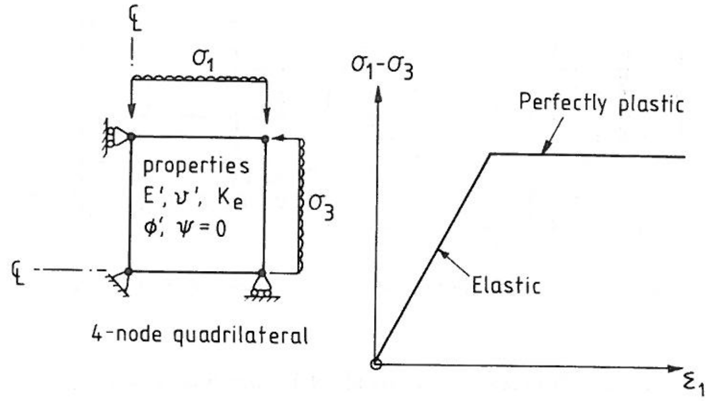


Figure 4. Single element tests on undrained elastic–perfectly plastic soil

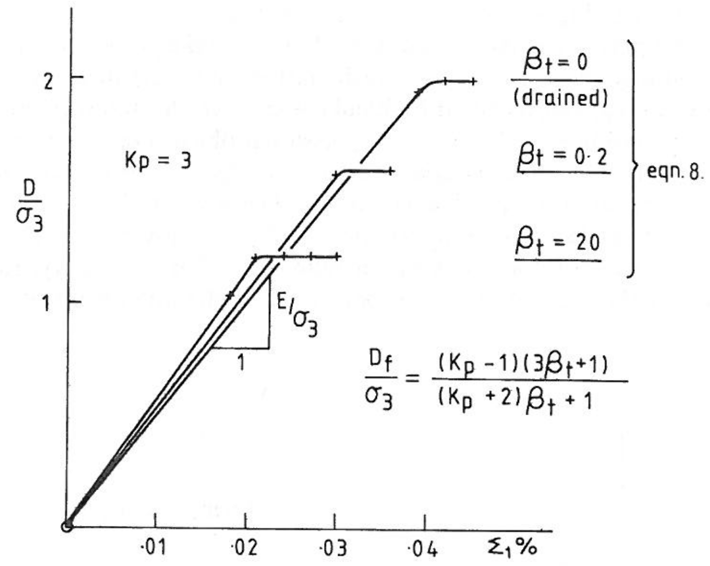


Figure 5. Computed stress–strain curves in triaxial case

For the triaxial case, an axisymmetric strain analysis was performed, and close agreement with equation (8) was observed regarding the deviator stress at failure. It was noted in Figure 5 that as $\beta_t$ was increased, the soil exhibited a stiffer initial response with respect to total parameters before failing at a reduced value. This was expected from equation (3), which gives that

$$E \geqslant E' \quad \text{for all } K_e \tag{22}$$

The three cases shown in Figure 5 with $\beta_t$ varying between zero and twenty were stable numerically, but for much larger $\beta_t$ values, numerical stability and convergence became increasingly hard to achieve. This was due to the total Poisson's ratio becoming very close to $\frac{1}{2}$, as indicated by equation (4). Fortunately, the need for large $\beta_t$ values never arises because as indicated in Figure 1, the collapse load has essentially levelled out for $\beta_t \geqslant 20$.

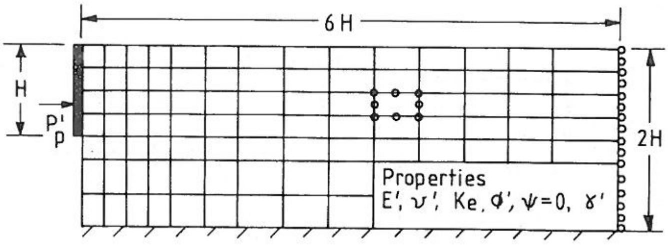


Figure 6. Passive resistance of an undrained elastic–perfectly plastic soil

For the case of passive earth pressure, the mesh of Figure 6 was employed using the same soil properties, and close agreement with equation (19) was obtained as indicated in Figure 7. As before, by putting $\beta_{ps} = 20$ stable results were obtained and the value of $P'_p$ computed was essentially the minimum value as given by the range of equation (20).

All the analyses described have assumed that volume changes occurred in the elastic phase of compression only. No plastic volume change was allowed to take place, and this was ensured by putting the dilatation angle ($\psi$) to zero. A positive dilatation angle has a dramatic effect on the shear strength of undrained materials, as noted by Zienkiewicz *et al.*[6] for footings. Figure 8 shows the effects of a positive dilatation angle for the earth pressure problem. Once stresses reach the failure surface, the pore pressures fall as the material tries to dilate. This results in rising mean stresses and apparently limitless resources of passive resistance. The greater the value of $\beta_{ps}$, the more pronounced would be this effect. For drained materials ($\beta_{ps} = 0$), however, the dilatation angle has little influence on collapse loads for relatively unconfined problems. In reality, undrained dilating materials fail due to either cavitation of the pore fluid or the attainment of critical confining pressures.[7]

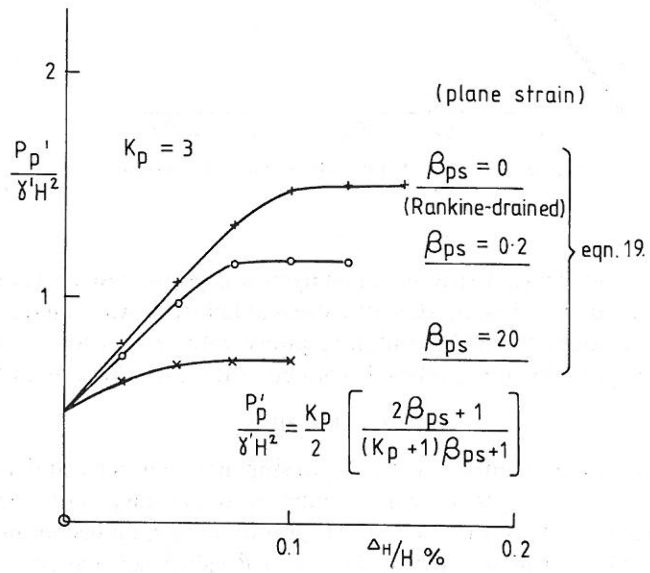


Figure 7. Computed maximum passive pressure

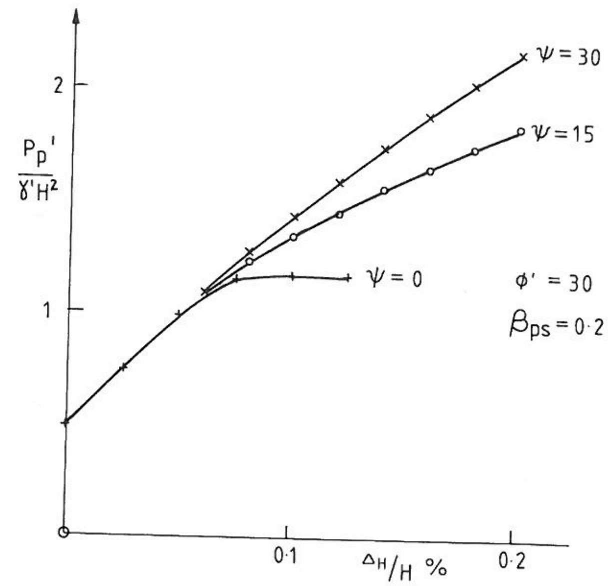


Figure 8. Effect of dilatation angle on undrained passive resistance

## CONCLUSIONS

When modelling undrained phenomena by adding a 'large' fluid modulus into the effective stress–strain matrix, the elastic properties are conveniently expressed in terms of a dimensionless parameter $\beta$. An upper limit on the value of $\beta$ equal to 20 is suggested as suitable for numerical applications.

By reducing $\beta$, a range of values of the pore pressure parameter $\bar{A}_f$ can be modelled for certain problems. The range of $\bar{A}_f$ is limited by the assumed elastic behaviour prior to failure, and the expressions quoted herein are only valid for cases where $\Delta\sigma_3 \simeq 0$.

Although undrained collapse of certain classes of clay may be predicted using the technique described, the use of more sophisticated soil models is essential for undrained dilating materials.

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