Undrained Bearing Capacity of Two-Strip Footings on Spatially Random Soil

D. V. Griffiths[1]; Gordon A. Fenton[2]; and N. Manoharan[3]

Abstract: A probabilistic study on the interference of two parallel rough rigid strip footings on a weightless soil with a randomly distributed undrained shear strength performed. The problem is studied using the random finite element method, where nonlinear finite element analysis is merged with random field theory within a Monte Carlo framework. The variability of undrained shear strength is characterized by a lognormal distribution and an exponentially decaying spatial correlation length. The estimated bearing capacity statistics of isolated and two footings cases are compared and the effect of footing interference discussed. Although interference between footings on frictionless materials is not very great, the effect is shown to be increased by soil variability and spatial correlation length.

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Introduction

In general, most geotechnical analyses are treated deterministically, in which the soil medium is considered as a single homogeneous layer or a layered medium with uniform material properties in each layer based on "average" values of soil parameters. However, in nature, soil parameters generally show significant spatial variation in both vertical and horizontal directions, and the results of deterministic analyses are only expected values which may vary from actual performance of constructed facilities. Probabilistic studies to assess the influence of various sources of variability on the estimated performance of geotechnical structures are becoming increasingly popular among engineers, and a wide range of geotechnical applications have been reported in the literature (see e.g., Mostyn and Li 1993; Phoon et al. 2000; Li and Lo 1993; Lemaire et al. 1995; Shackelford et al. 1996; Pula 2000). More recently, probabilistic studies on bearing capacity of an isolated footing have been reported by Griffiths and Fenton (2000, 2001), Nobahar and Popescu (2001) for $\phi_u=0$ soils, and Fenton and Griffiths (2000, 2003) for $c'-\phi'$ soils.

On some occasions, it may be necessary to place footings quite close together, to accommodate structural details or to limit footing loads. In such cases, the interference of failure zones could alter the bearing capacity and load-settlement behavior of footings from the isolated footing condition. Studies on the interference of footings have been reported previously by various authors. In frictional soils, experimental studies have shown that when the spacing between the footings is reduced, interference of failure zones occurs and the bearing capacity of the footings increase. (e.g., Stuart 1962; Mandel 1965; West and Stuart 1965; Saran and Agarwal 1974; Deshmukh 1978; Dembicki et al. 1981; Patankar and Khadilkar 1981; Das and Larbi-Cherif 1983; Selvadurai and Rabbaa 1983; Graham et al. 1984) For footings on the surface of fine grained undrained soils however, interference due to footing proximity is much less pronounced, and any increase in bearing capacity is usually not considered in design.

Probabilistic studies on multifootings are so far limited to the estimation of differential settlements (e.g., Beacher and Ingra 1981; Fenton and Griffiths 2002). When examining bearing capacity, it might be expected that an important parameter would be the spatial correlation length of the soil in relation to the footing width and spacing.

This paper reports the results of parametric studies relating to soil variability and spatial correlation, on the bearing capacity of two parallel rough rigid strip footings on a weightless soil with randomly varying undrained shear strength. For this probabilistic study, a plane strain, nonlinear elastic-perfectly plastic (Tresca) finite element analysis is combined with random field theory using Monte Carlo simulation.

Brief Description of the Random Finite-Element Method

The undrained shear strength is obtained through the transformation

$$c_{u_i}=\exp\{\mu_{\ln c_u}+\sigma_{\ln c_u}g_i\} \quad (1)$$

in which c_{u_i}=undrained shear strength assigned to the ith element; g_i=local average of a standard Gaussian random field, g, over the domain of the ith element; and $\mu_{\ln c_u}$ and $\sigma_{\ln c_u}$=mean and standard deviation of the logarithm of c_u (obtained from the "point" mean and standard deviation μ_{c_u} and σ_{c_u} after local averaging).

The LAS technique (Fenton 1994; Fenton and Vanmarcke 1990) generates realizations of the local averages g_i which are derived from the random field g having zero mean, unit variance,

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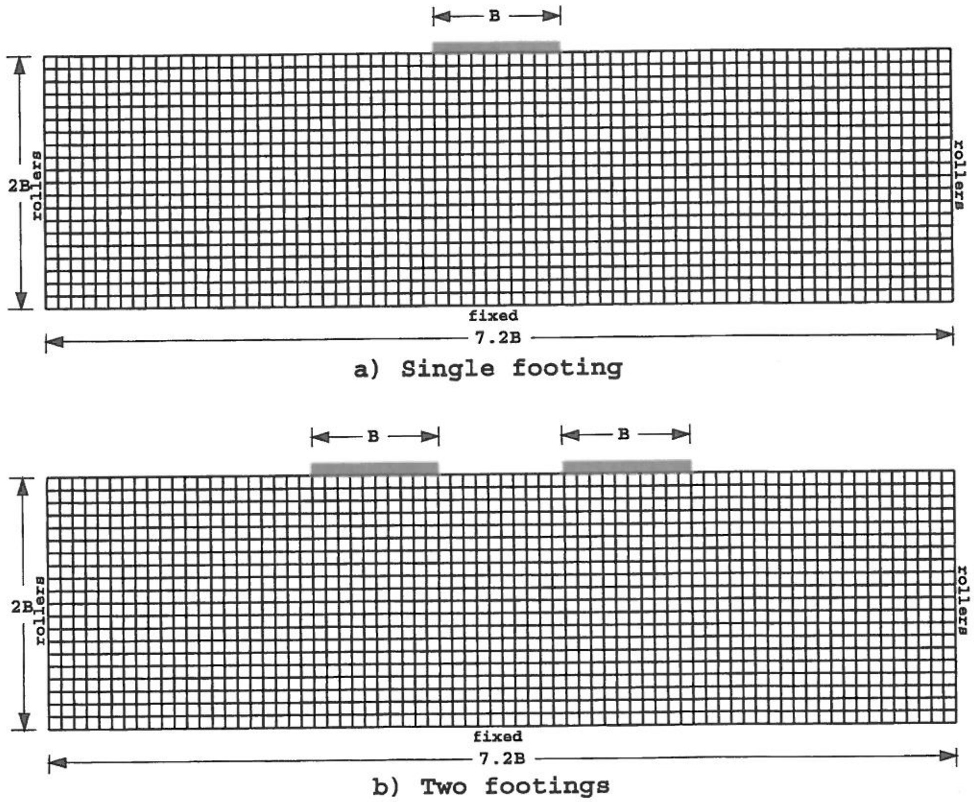

Fig. 1. Mesh used in probabilistic bearing capacity analyses (a) isolated footing; (b) two footings

and a spatial correlation length, $\theta_{\ln c_u}$. As the spatial correlation length tends to infinity, g_i becomes equal to g_j for all elements i and j—that is, the field of shear strengths tends to become uniform for each realization. At the other extreme, as the spatial correlation length tends to zero, g_i and g_j become independent for all $i \neq j$—the soil's undrained shear strength changes rapidly from point to point. In the present study, a Markovian spatial correlation function was used, of the form:

$$\rho(|\tau|) = \exp\left\{-\frac{2}{\theta_{\ln c_u}}|\tau|\right\} \qquad (2)$$

where ρ=correlation coefficient between the logarithm of the undrained strength values at any two points separated by a distance τ in a random field with spatial correlation length $\theta_{\ln c_u}$.

In the two-dimensional analyses presented in this paper, the spatial correlation lengths in the vertical and horizontal directions are taken to be equal (isotropic) for simplicity.

A local averaging process has been included in the formulation to take full account of the level of mesh discretization, and the size of the finite elements onto which the random field is to be mapped. Local averaging preserves the mean, but reduces the standard deviation of the underlying normal field to a "target" value (it could also be noted that local averaging preserves the *median* of the lognormal field). The amount by which the standard deviation is reduced, depends on the size of the elements, the spatial correlation length and the nature of the spatial correlation function governing the field. More specifically, there is a function called the "variance function," which can be derived from the correlation function, which governs the rate at which the standard deviation drops as the averaging domain grows larger (see e.g., Griffiths and Fenton 2004). The interested reader is also referred to Vanmarcke (1977) for a detailed description of local averaging and to Fenton and Griffiths (1993) and Griffiths and Fenton (1993) for early implementations of the random finite-element method (RFEM).

Deterministic Analysis

The bearing capacity analyses are carried out by the finite element method using a viscoplastic algorithm incorporating an elastic-perfectly plastic (Tresca) failure criterion (see e.g., Smith and Griffiths 2004). A typical finite element mesh shown in Fig. 1, consists of 1,440 8-node isoparametric plane strain square elements in 72 columns and 20 rows. The footing width B occupies 10 elements. The footing is placed at the center as shown in Fig. 1(a) for an isolated footing problem. In the two footing problem the footings are placed symmetrically about the centerline as shown in Fig. 1(a). The nodes representing the footing width are incrementally displaced by an equal amount in the vertical direction, simulating a rough rigid footing condition with a uniform vertical settlement and no rotation. The footing load for each increment is the summation of the nodal forces backcomputed from the converged stress field after each increment.

Results for the isolated and two footing cases on a homogeneous soil, are shown in Fig. 2. For an isolated footing, the finite element analysis gave a bearing capacity factor $N_c = q_{f_d}/c_u = 5.42$, which is marginally (5.5%) higher than Prandtl's closed form solution of $N_c = 5.14$. This is due in part to the rough boundary condition beneath the footing, combined with the relatively coarse discretization beneath the footing edge. Under rigid footing conditions, soil in the vicinity of the footing edge experiences stress concentrations and high plastic strains. Use of a coarse mesh in this vicinity leads to somewhat higher values of the back-

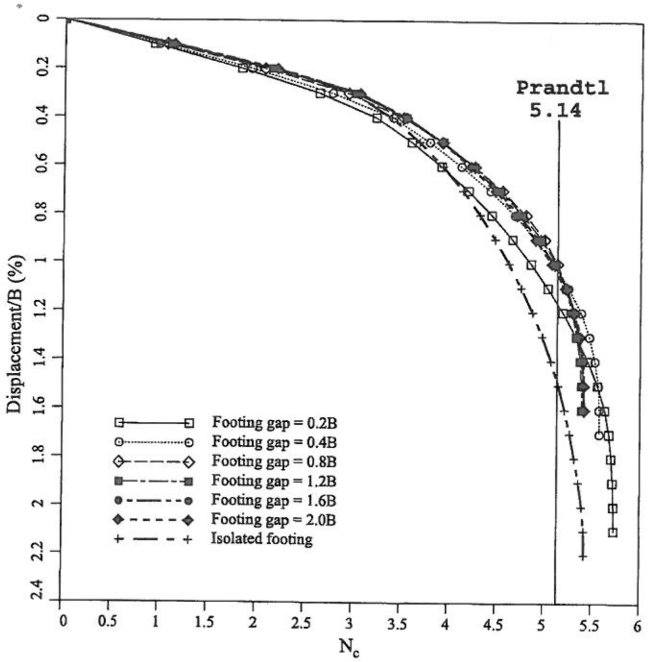

Fig. 2. Analysis of footing interference on a homogeneous soil

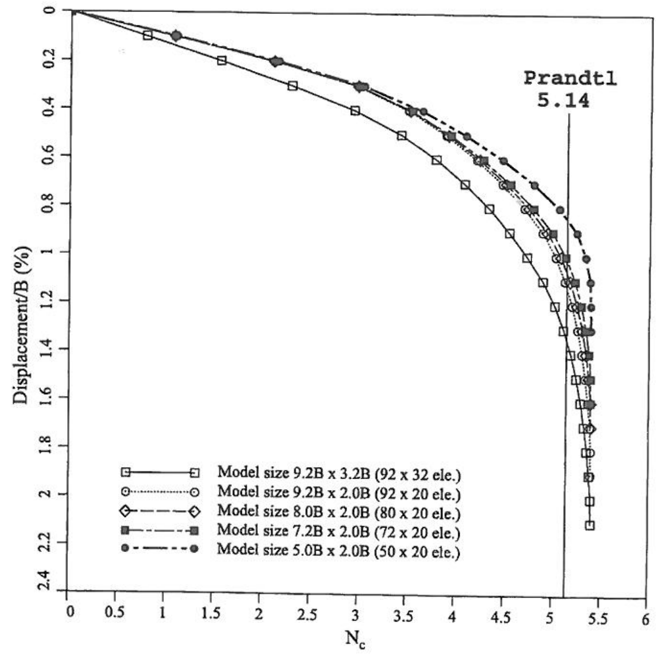

Fig. 3. Influence of boundary proximity on computed bearing capacity on a homogeneous soil

computed loads. Better agreement could be obtained using a finer mesh near the footing edge. In the present study however, a uniform finite element mesh consisting of square elements is used in order to facilitate and simplify the random field generation and mapping.

For the two footing case, the gap between the footings was varied from $0.2B$ to $2B$. In spite of some difference in the load-settlement behaviors, the estimated bearing capacities are very similar in all cases. These results confirm that for a homogeneous fine grained undrained soil, the effect of interference on the bearing capacity of two closely spaced footings is insignificant. It may be noted that the total stress analyses presented in this paper take no account of pore pressures generated by the footing and any influence this might have on bearing capacity. The writers have developed RFEM programs that perform effective stress analysis with shear dilatancy coupling, however these studies are left for future work.

The boundary effect of the considered finite element model on the bearing capacity of a two footing problem was also considered by running the analysis with model widths ranging from $5B$ to $9.2B$ and two different depths of $2B$ and $3B$ while maintaining the footing spacing at B. The results in Fig. 3 show that the bearing capacity is essentially the same for all the considered model sizes. The only variation that was observed occurred in the load-settlement behavior.

The results from Figs. 2 and 3 suggest that the finite element model shown in Fig. 1, is adequate to capture the bearing capacity of two footings with reasonable accuracy.

A displacement vector plot at bearing failure of two footings in the homogeneous case is shown in Fig. 4. Due to the interference of failure mechanisms between the footings, the soil mass is sheared mostly outward.

Probabilistic Study

Analyses have been performed for isolated and two footings cases with the meshes of Figs. 1(a and b), respectively, and with the input parameters taking the following values:

$$\Theta_{c_u} = 0.125, 0.5, 1, 2, 4, 8$$

$$\mathrm{COV}_{c_u} = 0.25, 0.5, 1, 2, 8$$

where $\mathrm{COV}_{c_u} = \sigma_{c_u}/\mu_{c_u}$ and $\Theta_{c_u} = \theta_{\ln c_u}/B$.

Although typical maximum values of $\mathrm{COV}_{c_u} \approx 0.5$ are quoted by some researchers (e.g., Lee et al. 1983), the current investigation covers a much wider range of values going up to $\mathrm{COV}_{c_u} = 8$. The justification for this lies in the need to study the trends in the results for highly variable materials. From a practical viewpoint, it also seems likely that the actual variability of soil may be considerably higher than "typical values" which are often based on rather limited data bases and small sample sizes.

The gap between the footings is kept constant at B for this parametric study. For each set of assumed statistical properties given by COV_{c_u}, and Θ_{c_u}, Monte Carlo simulations were performed. These involved the shear strength random field generation and the subsequent deterministic finite element analysis of bearing capacity for each realization. It was observed that 500 realizations was generally adequate to achieve stable bearing capacity statistics within the considered range of parameters. For further discussion on the optimal number of realizations the reader is referred to Griffiths and Fenton (2001). Each realization, although having the same underlying statistics, will lead to a quite different spatial pattern of shear strength values beneath the foot-

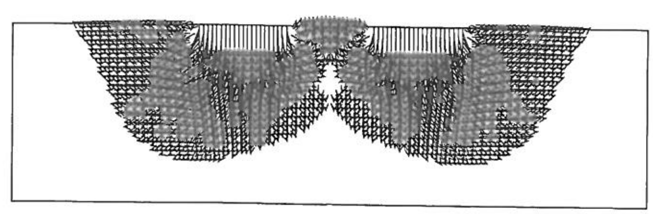

Fig. 4. Displacement vectors at bearing failure in a two-footing analysis on a homogeneous soil

ing and hence a different bearing capacity. Unlike the homogeneous analysis, the load-settlement behavior and bearing capacity of the left and right footings in the same realization were generally different. Following the Monte Carlo process, the mean and standard deviation of the resulting 500 bearing capacities for the left and right footings were computed.

In a two footings case, it is quite possible that both footings may support a single structure (e.g., a portal frame supported by two footings) or two separate, but close structures. When two different structures are supported, failure of one footing might not affect the performance of the other, so the finite element analysis in each realization is carried out until both footings reach failure. In this case, the statistical analysis for the mean and standard deviation is carried out separately for the left and right footings. When both the footings support a single structure, failure of any one footing may be considered as failure of the entire structure, since this may cause excessive damage, irrespective of the safe loading condition of the other footing. For this case, only the minimum value among the left and right footing capacities is recorded at each realization, and subsequently used for the statistical analysis.

In the following discussions, and in order to take account of the discretization error mentioned previously, the mean bearing capacity (μ_{q_f}) is normalized by the finite element deterministic value (q_{f_d}).

Fig. 5 shows typical deformed meshes at failure under two footings and the corresponding displacement vector plots. The deformed mesh is superimposed on a grayscale, in which lighter regions indicate stronger soil and darker regions indicate weaker soil. As would be expected for a heterogeneous soil, the failure mechanisms no longer exhibit symmetry.

Griffiths and Fenton (2001) have reported for an isolated footing, that the bearing capacity of a footing on a heterogeneous will generally be less than the deterministic bearing capacity computed with the mean shear strength μ_{c_u}. For low values of COV_{c_u}, the mean bearing capacity, μ_{q_f} tends to the deterministic value, but for higher values the mean bearing capacity falls quite steeply.

Figs. 6(a and b) compare the normalized mean bearing capacity following Monte Carlo simulations, for the isolated and two footing cases. The two footing results are computed separately for the left and right footings, and also using the minimum value from either footing. As the trend was generally similar for all Θ_{c_u} values considered, only results corresponding to the smallest and largest values of Θ_{c_u} are shown here.

In the deterministic analysis for a footing gap of B, there is virtually no increase in the bearing capacity due to footing interference. In the probabilistic study, it can be observed from Fig. 6 that the mean bearing capacities of the footings (either left or right) tends to be higher than for an isolated footing. On the other hand, the mean bearing capacity based on the assumption of any footing failure, tends to be lower than for an isolated footing. In both cases, the difference becomes more pronounced as Θ_{c_u} is increased. The left and right footing mean bearing capacities are very similar in both cases as might be expected.

This effect is shown in Fig. 7, where the soil variability is fixed at $\text{COV}_{c_u}=1$, and the spatial correlation length Θ_{c_u} is varied. The maximum differences from the isolated footing case in both directions is of the order of 10% when $\Theta_{c_u}=8$. Fig. 7 also demonstrates the minimum or "worst case" bearing capacity corresponding to $\Theta_{c_u}\approx 1$ or $\theta_{\ln c_u}\approx B$.

Fig. 8 compares the coefficient of variation of the bearing capacities (COV_{q_f}) against the coefficient of variation of the input undrained shear strength (COV_{c_u}) for the case of $\Theta_{c_u}=1$. The differences in COV_{q_f} are not particularly significant at low values of COV_{c_u}, but increase at higher values.

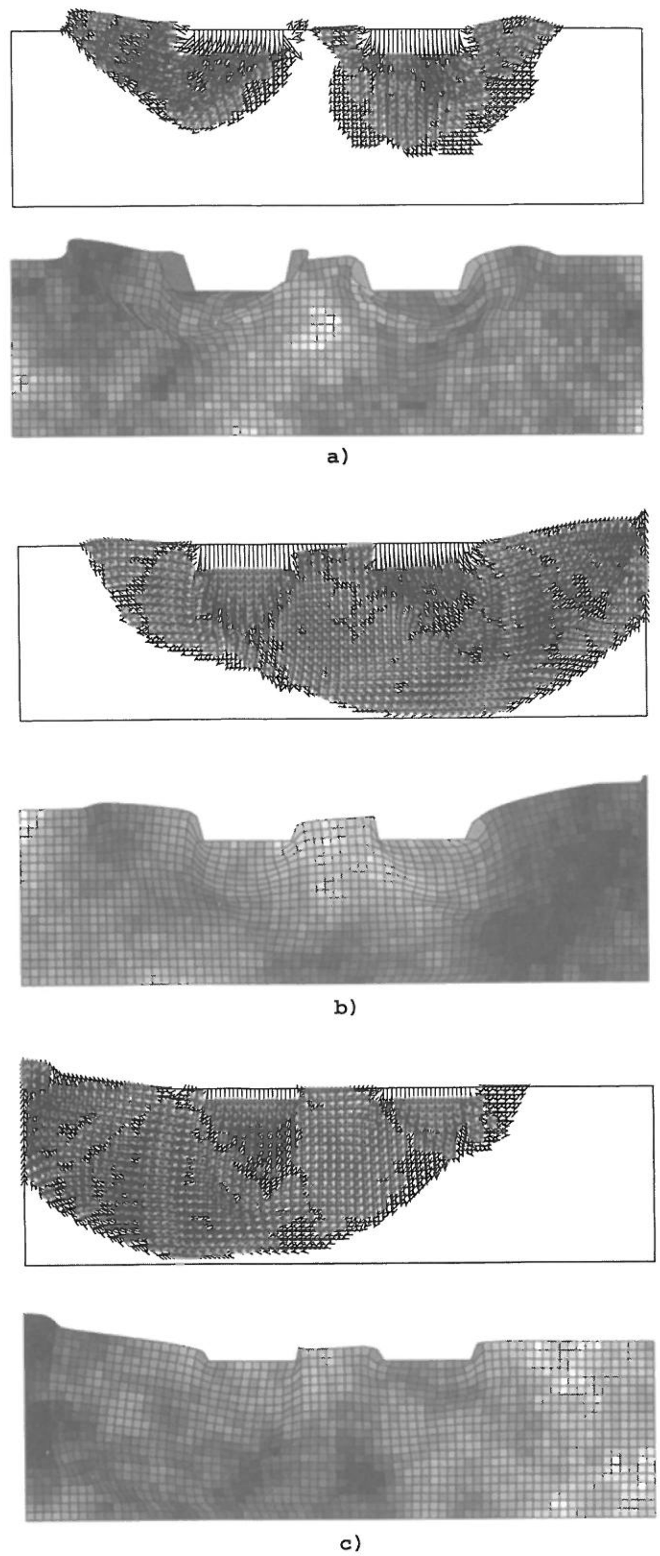

Fig. 5. Typical displacement vectors and deformed meshes at bearing failure in a two-footing analysis on a stochastic soil

Keeping the footing spacing at B, Fig. 9 shows how the mean value of the absolute *difference* between the left and right footing bearing capacities, normalized by the deterministic bearing capacity value, varies with Θ_{c_u} and COV_{c_u}. For small values of Θ_{c_u}, the

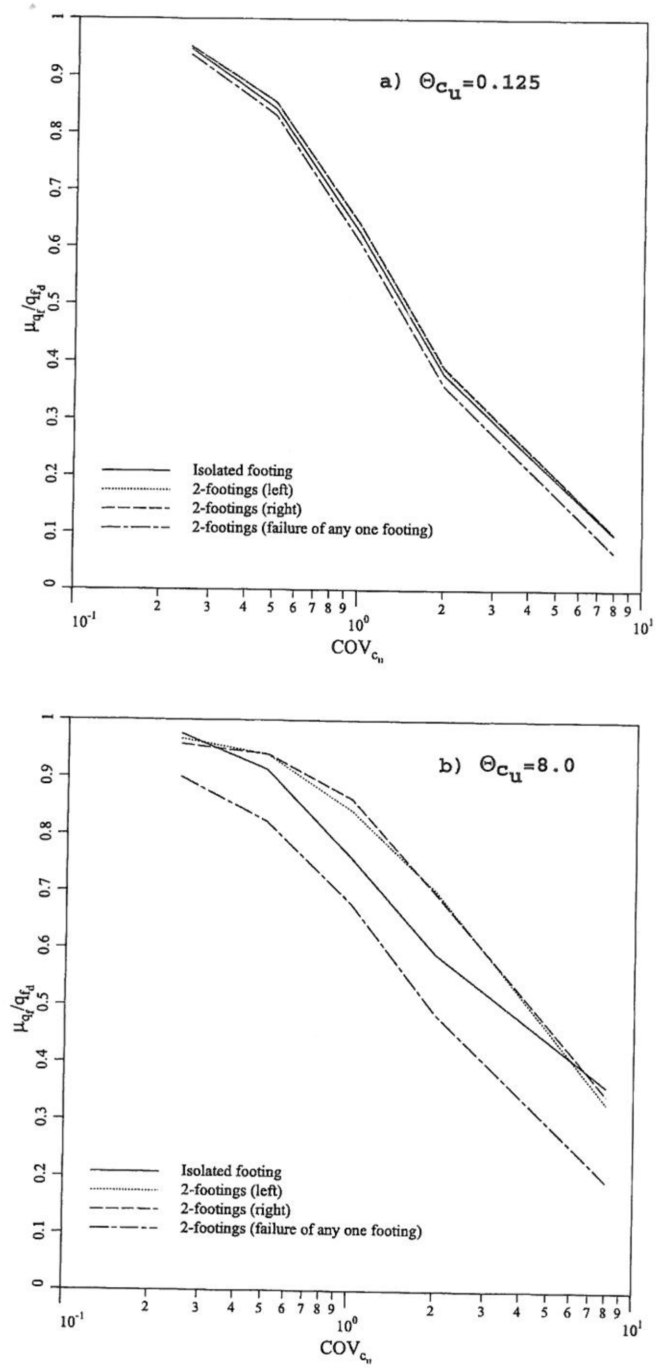

Fig. 6. Normalized mean bearing capacity as a function of COV_{c_u} (a) $\Theta_{c_u}=0.125$; (b) $\Theta_{c_u}=8$

shear strength changes rapidly from element to element, and the local averaging effect tends to give a similar bearing capacity values for both the footings. As the value of Θ_{c_u} increases however, the local averaging effect becomes less pronounced, and the shear strengths field beneath each footing can vary more significantly leading to greater differences between their respective bearing capacity values. For a range of $COV_{c_u}=0.25$–1, the maximum difference in the bearing capacities was observed to reach a peak at about $\Theta_{c_u}=4$.

In the limit of $\Theta_{c_u}\to\infty$, the difference between the bearing capacity of the two footings would tend to zero due to the random field becoming more homogeneous within each realization.

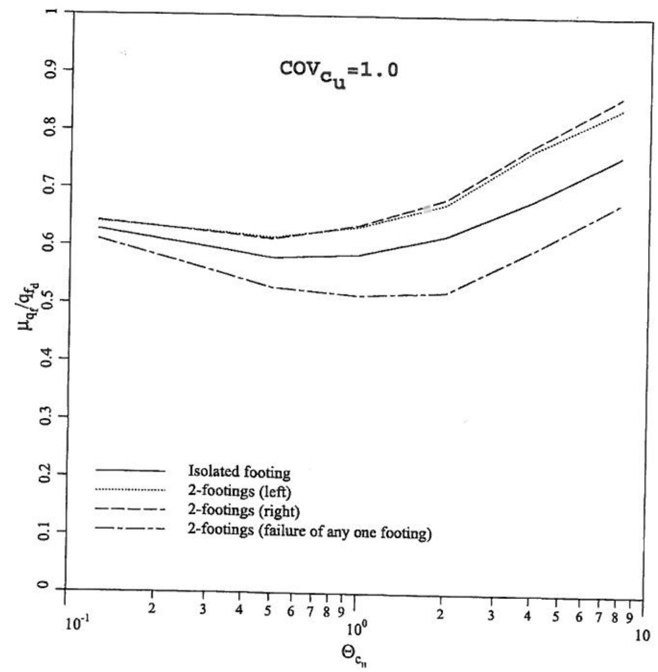

Fig. 7. Influence of Θ_{c_u} on normalized mean bearing capacity for $COV_{c_u}=1$

Concluding Remarks

The paper has described results from probabilistic analyses on interference effects involving the bearing capacity of two parallel rough rigid strip footings on fine grained undrained ($\phi_u=0$) soil. In this mainly qualitative study, the gap between the footings was held constant at the footing width B, and parametric studies performed to study the influence of COV_{c_u} and Θ_{c_u} on the bearing capacity statistics.

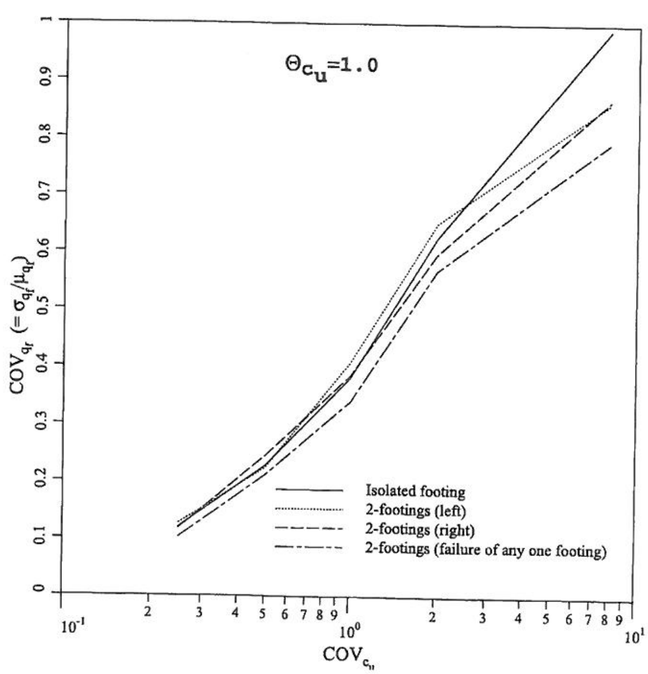

Fig. 8. Influence of COV_{c_u} on COV_{q_f} for $\Theta_{c_u}=1$

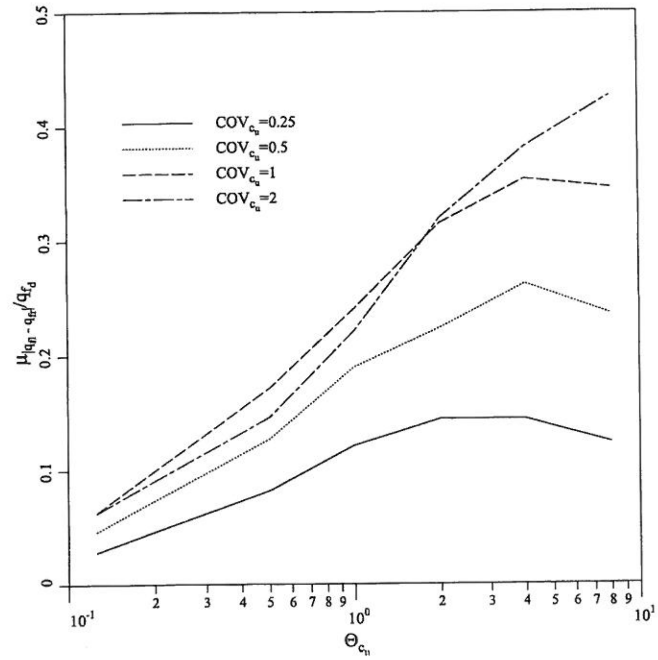

Fig. 9. Influence of Θ_{c_u} on the difference between the bearing capacity of the left and right footings

The probabilistic studies indicated that footing interference generally increased the mean bearing capacity over the isolated footing value, when the footings could be considered to support separate structures. On the other hand, when the footings were considered to be supporting a single structure, in which failure of either footing could be considered failure of the whole, the estimated mean bearing capacity due to interference was lower than that of an isolated footing. In either case the difference from that of an isolated footing on a random soil was no greater than 10% for the range of parameters considered. Small differences such as those observed are of little practical interest in the context of bearing failure of fine grained undrained soils. This is especially true in view of the high factors of safety generally used in bearing capacity calculations.

The difference between the left and right footing bearing capacities was also investigated. The mean absolute difference was influenced by both COV$_{c_u}$ and Θ_{c_u}, however the results indicated a critical value of Θ_{c_u} that led to a maximum differential bearing capacity. For low and high values of Θ_{c_u}, the difference fell due to either local averaging or high correlations leading to a more uniform shear strength field beneath both footings.

In general, footing interference is significantly higher in frictional soils than undrained soils, hence in a probabilistic context, the trends observed in such a study will likely be much more pronounced than those observed in this paper. The influence of property variability in cohesive/frictional soils on geotechnical analysis is an area of on-going research for the writers.

Acknowledgments

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Notation

The following symbols are used in this paper:

B = footing width;
COV$_{c_u}$ = coefficient of variation of undrained shear strength;
COV$_{q_f}$ = estimated coefficient of variation of bearing capacity;
c_u = undrained shear strength;
N_c = bearing capacity factor;
q_f = bearing capacity;
q_{f_d} = deterministic bearing capacity;
Θ_{c_u} = dimensionless spatial correlation of log undrained shear strength;
$\theta_{\ln c_u}$ = spatial correlation length of log undrained shear strength;
μ_{c_u} = mean of undrained shear strength;
μ_{q_f} = estimated mean bearing capacity;
$\mu_{|q_{fl}-q_{fr}|}$ = estimated mean absolute difference in the left and right footing capacities;
σ_{c_u} = standard deviation of undrained shear strength; and
σ_{q_f} = estimated standard deviation of bearing capacity.

References

Beacher, G. B., and Ingra, T. S. (1981). "Stochastic FEM in settlement predictions." *J. Geotech. Engrg. Div.*, 107(4), 449–463.

Das, B. M., and Larbi-Cherif, S. (1983). "Bearing capacity of two closely-spaced shallow foundations on sand." *Soils Found.*, 23(1), 1–7.

Dembicki, E., Odrobinski, W., and Mrozek, W. (1981). "Bearing capacity of subsoil under strip foundations." *Proc., 10th Int. Conf. of Soil Mechanics and Foundation Engineering*, Vol. 2, 91–94.

Deshmukh, A. M. (1978). "Interaction of different types of footings on sand." *Indian Geotechnical J.*, 8(4), 193–204.

Fenton, G. A. (1994). "Error evaluation of three random field generators." *J. Eng. Mech.*, 120(12), 2478–2497.

Fenton, G. A., and Griffiths, D. V. (1993). "Statistics of block conductivity through a simple bounded stochastic medium." *Water Resour. Res.*, 29(6), 1825–1830.

Fenton, G. A., and Griffiths, D. V. (2000). "Bearing capacity of spatially random soils." *PMC2000 Conf.*, Univ. of Notre Dame, Notre Dame, Ind.

Fenton, G. A., and Griffiths, D. V. (2002). "Probabilistic foundation settlement on spatially random soil." *J. Geotech. Geoenviron. Eng.*, 128(5), 381–390.

Fenton, G. A., and Griffiths, D. V. (2003). "Bearing capacity prediction of spatially random $c-\phi$ soils." *Can. Geotech. J.*, 40(1), 54–65.

Fenton, G. A., and Vanmarcke, E. H. (1990). "Simulation of random fields via local average subdivision." *J. Eng. Mech.*, 116(8), 1733–1749.

Graham, J., Raymond, G. P., and Suppiah, A. (1984). "Bearing capacity of three closely-spaced footings on sand." *Geotechnique*, 34(2), 173–182.

Griffiths, D. V., and Fenton, G. A. (1993), "Seepage beneath water retaining structures founded on spatially random soil." *Geotechnique*, 43(4), 577–587.

Griffiths, D. V., and Fenton, G. A. (2000). Bearing capacity of heterogeneous soils by finite elements." *Memorias Del V Congreso Internacional De Métodos Numéricos En Ingeniería Y Ciencias Aplicadas*, N. Troyani and M. Cerrolaza, eds., Puerto La Cruz, Venezuela, CI27–CI37.

Griffiths, D. V., and Fenton, G. A. (2001). "Bearing capacity of spatially random soil: The undrained clay Prandtl problem revisited." *Geotechnique*, 51(4), 351–359.

Griffiths, D. V., and Fenton, G. A. (2004). "Probabilistic slope stability analysis by finite elements." *J. Geotech. Geoenviron. Eng.*, 130(5), 507–518.

Lee, I. K., White, W., and Ingles, O. G. (1983). *Geotechnical engineering*, Pitman, London.

Lemaire, M., Favre, J.-L., and Mebarki, A., eds. (1995). *Proc., Int. Congress on Applications of Statistics and Probability*, Balkema, Rotterdam, The Netherlands.

Li, K. S., and Lo, S-C. R., eds. (1993). *Probabilistic methods in geotechnical engineering*, Balkema, Rotterdam, The Netherlands.

Mandel, J. (1965). "Interférence plastique de semelles filantes." *Proc., 6th. Int. Conf. Soil Mechanics and Foundation Engineering*, Vol. 2, 127–131.

Mostyn, G. R., and Li, K. S. (1993). "Probabilistic slope analysis: State-of-play." *Probabilistic methods in geotechnical engineering*, K. S. Li and S.-C. R. Lo, eds., Balkema, Rotterdam, The Netherlands, 89–109.

Nobahar, A., and Popescu, P. (2001). "Effects of spatial variability of soil properties on bearing capacity." *Proc., 10th Int. Conf. Computer Methods and Advances in Geomechanics*, C. S. Desai, T. Kundu, S. Harpalani, D. Contractor, and J. Kemeny, eds., Tucson, Ariz., 1479–1484.

Patankar, M. V., and Khadilkar, B. S. (1981). "Nonlinear analysis of interference of three surface strip footings by finite element method." *Indian Geotechnical J.*, 11(4), 327–344.

Phoon, K-K., Kulhawy, F. H., and Grigoriu, M. D. (2000). "Reliability-based design for transmission line structure foundations." *Comput. Geotech.*, 26(3–4), 169–185.

Pula, W., ed. (2000). "Special issue on reliability in geotechnics." *Comput. Geotech.*, 26(3–4), 169–346.

Saran, S., and Agarwal, V. C. (1974). "Interference of surface footings in sands." *Indian Geotechnical J.*, 4(2), 129–139.

Selvadurai, A. P. S., and Rabbaa, S. A. A. (1983). "Some experimental studies concerning the contact stresses beneath interfering rigid strip foundations resting on a granular stratum." *Can. Geotech. J.*, 20(3), 406–415.

Shackelford, C. D., Nelson, P. P., and Roth, M. J. S., eds. (1996). "Uncertainty in the geologic environment: From theory to practice." *Proc., Uncertainty '96*, ASCE, GSP No. 58, Madison, Wis.

Smith, I. M., and Griffiths, D. V. (2004). *Programming the finite element method*, 4th Ed., Wiley, Chichester, N.Y.

Stuart, J. G. (1962). "Interference between foundations, with special reference to surface footings in sand." *Geotechnique*, 12(1), 15–22.

Vanmarcke, E. H. (1977). "Probabilistic modeling of soil profiles." *J. Geotech. Engrg. Div.*, 103(11), 1227–1246.

West, J. M., and Stuart, J. G. (1965). "Oblique loading resulting from interference between surface footings on sand." *Proc., 6th Int. Conf. Soil Mechanics and Foundation Engineering*, Vol. 2, 214–217.