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FINITE ELEMENT ANALYSIS OF VERTICAL EXCAVATIONS

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ABSTRACT

The critical height of vertical excavations in both plane strain and axisymmetry has been considered using finite element analysis incorporating plasticity theory. The numerical solutions obtained are found to compare favourably with values from centrifuge tests and limit analyses. The results are presented in a form which shows the variation of the stability number at failure with the height/width ratio of the excavation. Graphical output is also included to emphasise the changing mechanisms as this ratio varies.

INTRODUCTION

The stability of slopes in soft homogeneous clay in undrained shear was one of the earliest soil mechanics problems. Taylor[1] presented charts for estimating the stability of slopes in plane strain expressed through a stability number N, where -

$$N = \frac{\gamma H}{C_u} \qquad (1)$$

Referring to figure 1a, γ and C_u represent the saturated unit weight and undrained shear strength of the soil. The stability number represents the combination of material properties and slope geometry to just cause failure. The stability of a vertical excavation in plane strain may also be obtained from Taylor's chart by considering a slope inclined at 90^o to the horizontal.

Limit analyses, such as those of Pastor[2] and Heyman[3] are in close agreement with Taylor and show that the stability number of a vertical excavation in plane strain must lie in the range -

$$3.635 \leqslant N \leqslant 3.835 \qquad (2)$$

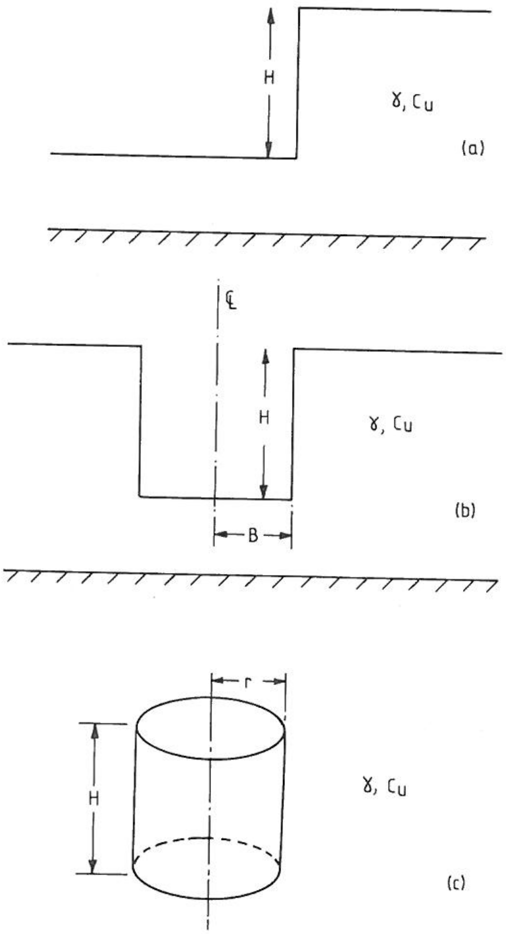


Fig.1 Geometry of excavations

For trench excavations, such as that in figure 1b, the width of the trench is not normally thought to have a large effect on the stability number in plane strain. For the circular excavation of figure 1c, however, the stability number has been shown to rise[4] as the H/r ratio increases.

The intention of this paper is to report on some numerical studies performed on vertical cuts in both plane strain and axisymmetry. Particularly of interest is the effect that the ratios H/B and H/r (figs. 1b and 1c respectively) have on the stability number.

Solutions to similar problems obtained by other authors using limit analyses, numerical analyses and centrifuge tests are referred to for comparison.

NUMERICAL SOLUTION TECHNIQUE

Finite element discretisation of the vertical cut problem used 8-noded quadrilateral elements with "reduced" integration although as a check some of the axisymmetric cases were repeated using 15-noded triangles.

A gravity turn-on procedure was used which resulted in a set of "consistent" nodal forces obtained from the expression -

$$\underline{F} = \sum \gamma \int \underline{N}^T \, dV \qquad \text{for all elements} \qquad (3)$$

Elastic, perfectly plastic material behaviour was assumed with a Tresca failure criterion. The non-linear material behaviour was introduced numerically using a viscoplastic algorithm[5]. This technique has been shown to be an efficient and reliable method[6] for solving plasticity problems in geomechanics. In this case, viscoplasticity has the added advantage that the gravity loads can be applied in one increment whereas other techniques, such as initial stress methods, require the loads to be applied in several steps.

Failure of the system was initiated by gradually reducing the undrained strength C_u until a sudden increase in the magnitude of plastic strains was recorded. This was usually accompanied by an equally sudden rise in the number of iterations required for convergence. It may be noted that due to the dimensionless nature of the stability number, it would be equally valid to keep the undrained shear strength constant and gradually increase the unit weight of the soil, much as would be done in a centrifuge test.

In the present work, it was decided to keep the unit weight and dimensions of the problem constant while reducing the strength. This means that the gravity loads vector needs to be generated once only, and these integrations were conveniently carried out simultaneously with the stiffness matrix formulation.

Having obtained the gravity loads vector, an initial "guess" for the value of C_u was made. This value of C_u was then reduced, at first using fairly crude steps, with a full elasto plastic analysis performed for each. During this process, the number of iterations required for convergence and the magnitude of the plastic displacements were monitered. Once the value of C_u to cause failure was determined to fairly narrow limits the process was repeated within these limits, but using much more refined steps.

The gravity turn-on procedure was checked on a column of material of height H and unit weight γ (figure 2). As the problem was one-dimensional, with no shear stress developing, the surface settlement was expected to be uniform, and given by -

$$\delta_v = m_v \int_0^H \sigma_z \, dz \qquad (4)$$

where $$\sigma_z = \gamma z \quad \text{and} \quad m_v = \frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \qquad (5)$$

Integration of equation (4) gives

$$\delta_v = \frac{(1+\nu)(1-2\nu)\gamma H^2}{2E(1-\nu)} \qquad (6)$$

and this value was closely reproduced by the finite element procedure under both plane and axisymmetric strain conditions.

The majority of the analyses were performed using a Young's modulus of 10^5 kN/m^2 and Poisson's ratio of 0.45, although some brief parametric studies were made to investigate their influence, if any, on failure.

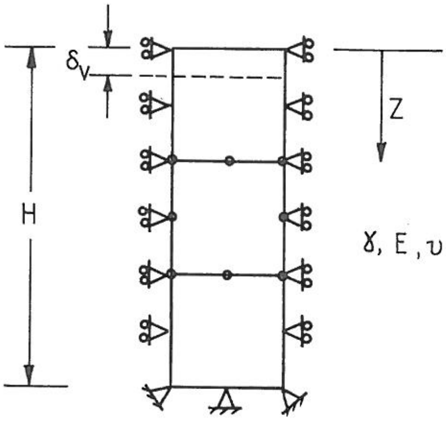


Fig.2 Mesh for checking gravity loads

RESULTS AND DISCUSSION

1. PLANE STRAIN CONDITIONS

The geometry and boundary conditions imposed for the finite element analyses are illustrated in figure 3. The influence of soil thickness beneath the excavation has been considered by Cascini[7]. It was found that

N-values at failure were hardly changed for H/D $\leqslant$ 1 so H/D = 1 has been adopted for the present work. For lateral boundaries, it was found that if they were placed at least 1.5H from the vertical face, then little interference was apparent.

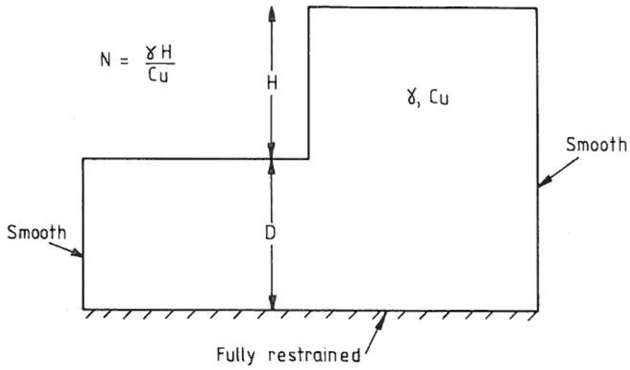


Fig.3 Boundary Conditions

Firstly, the influence of mesh refinement was examined using the meshes of figure 4 with the computed stability number at failure plotted in figure 5. It can be seen that in this case, a four-fold increase in the number of degrees of freedom does not have great effect, but as expected the solution has been improved from the upper-bound side.

Plastic displacement vectors at failure for mesh B are shown in figure 6 and it is apparent that for this particular geometry, the mechanism is confined almost exclusively to the wall of the excavation. The failure mechanism is what Taylor would have called a 'toe failure'.

As the cohesion is reduced there is, at first, a gradual increase in deformation but, eventually, a lower bound on the value of C_u is reached where the displacements increase considerably. Figure 7 shows the variation of plastic displacements with cohesion for mesh B. When using a large cohesion of 67.5 kN/m^2, the mesh remained essentially elastic but subsequent reduction of the shear strength resulted in failure at C_u = 35.9 kN/m^2.

Further analyses of plane strain stability were performed on the more confined meshes of figure 8 in order to observe the effect of the H/B ratio. Practically no variation in the stability number was observed as this ratio was increased. As shown in figure 9, the mechanism of failure was still confined to the wall of the excavation, with little heave of the base.

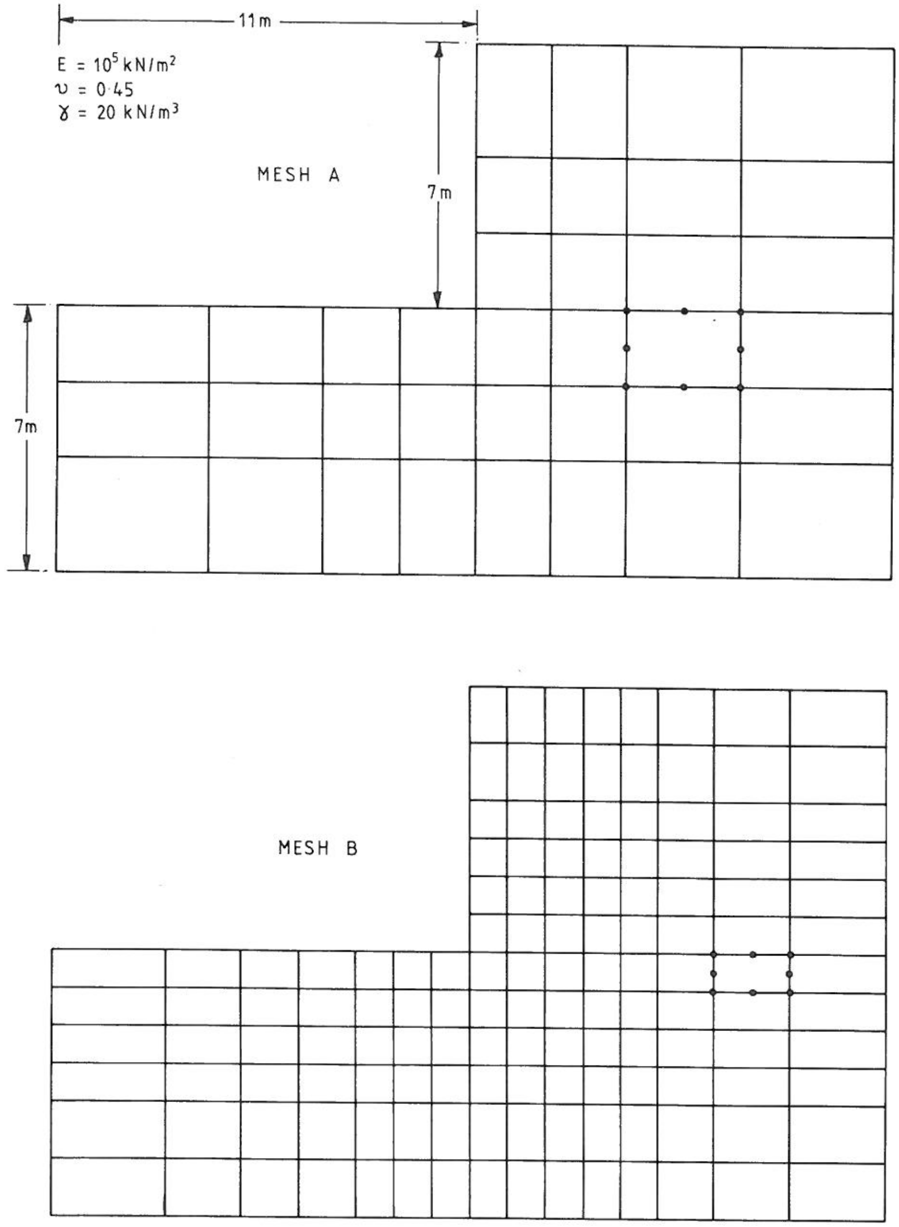


Fig.4 Two meshes of difference coarseness

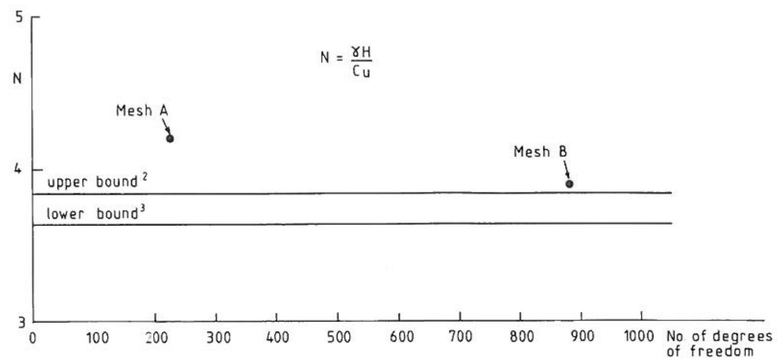


Fig.5 Effect of mesh coarseness on computed stability number

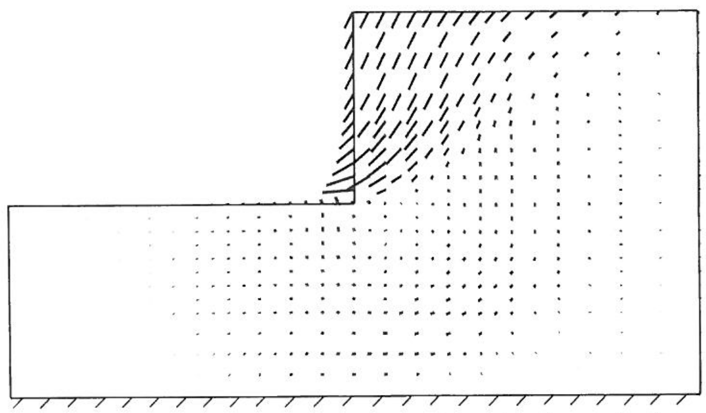


Fig.6 Failure displacement vectors in plane strain

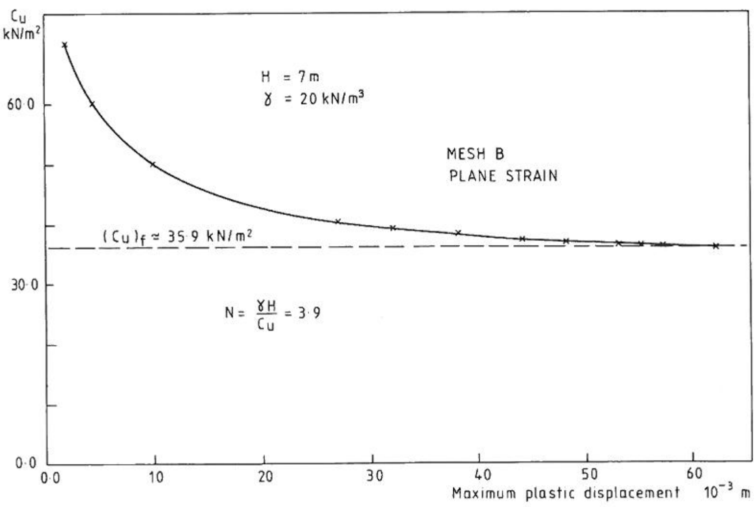
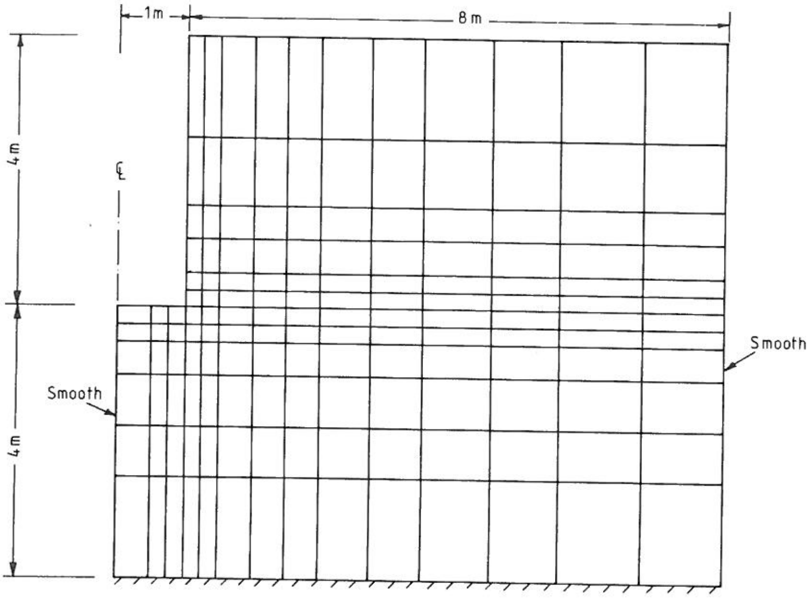


Fig.7 Effect of cohesion on plastic displacements



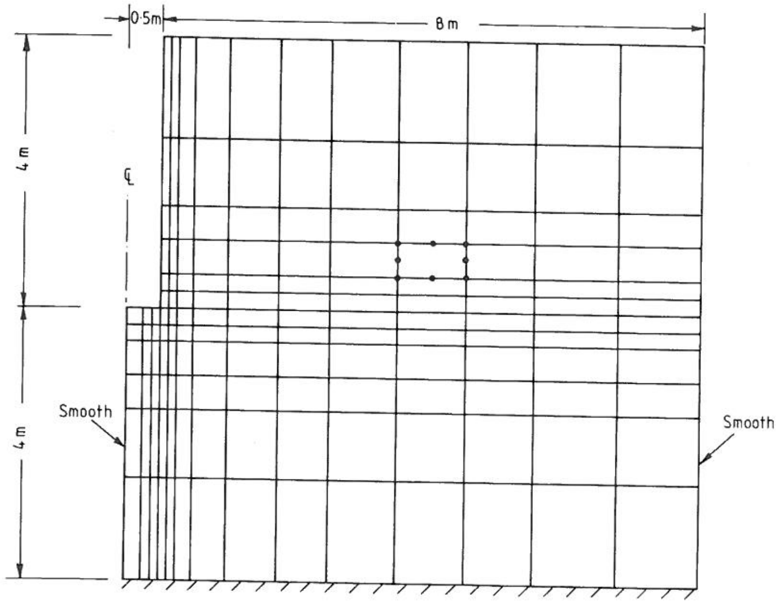


Fig.8 More confined meshes for analysis of excavations

In general, problems of plane strain slope stability are highly amenable to elasto-plastic finite element analyses of the type described herein. Reports of similar analyses for frictional soils[6,8] have also given results in close correspondence with more traditional slope stability methods.

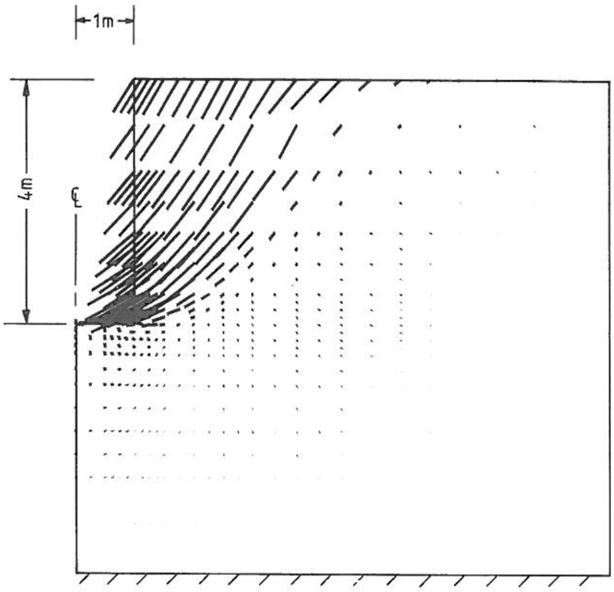


Fig.9 Failure displacement vectors in plane strain (H/B = 4)

2. AXISYMMETRIC CONDITIONS

The greater confinement imposed by axisymmetric conditions and the corresponding restraint this has on plastic flow, results in generally higher stability numbers than are observed in plane strain. The effect of the H/r ratio on stability of circular excavations has been discussed by Britto and Kusakabe[4] on the evidence of centrifuge tests. For small values of the ratio, the problem approximates to plane strain conditions, but as the ratio is increased so does the stability number. The observed variation was found to be related to the form of the failure mechanism. For $H/r \leqslant 2$ a similar wall mechanism to that observed in plane strain was obtained. For $2 < H/R \leqslant 7$, the mechanism was still confined mainly to the wall, but with more curvature and activity towards the base. For higher H/r values, the mechanism extended fully into the base of the excavation. The base mechanism proposed, however, seemed to overestimate the stability of the excavation and this was thought to be due to the need for the mechanism to extend fully from the base to ground level. Field studies by Bjerrum and Eide[9] have suggested a more localised failure mechanism which could be approximated by Skempton's[10] bearing capacity factor for deep circular foundations. A summary of the proposed upper bound as a function of H/r is given in figure 10. Some other solutions are also included in this figure. For example, for H/r = 1, Pastor and Turgeman[11] obtained an upper bound of N = 5.298 and for the same geometry, Pastor[12] presented a lower bound of N = 3.464. Using finite element analysis, Sloan[13] obtained a stability number of 7.7 for H/r = 4, and Britto and Kusakabe[14] reported values of 6.6, 8.12 and 8.12 for H/r equal to 2, 7 and 9 respectively.

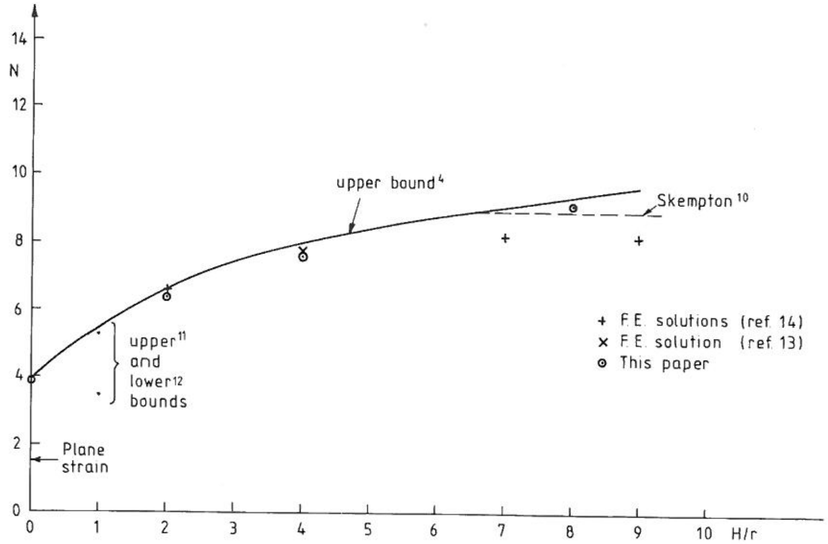


Fig.10 Upper bound and F.E. solutions for the circular excavation

Using the meshes of figure 8 in an axisymmetric elasto-plastic analysis, a range of H/r ratios were considered. As before, the shear strength C_u was gradually reduced until a sudden increase in plastic deformation was observed and these results are also given in figure 10.
All the values lie close to the upper-bound solution proposed by Britto and Kusakabe[4]. A typical plot of plastic displacement at failure shear for H/r = 4 is given in figure 11. The constrast with figure 9 for the plane strain case is striking and reflected in a stability number some 90% higher.

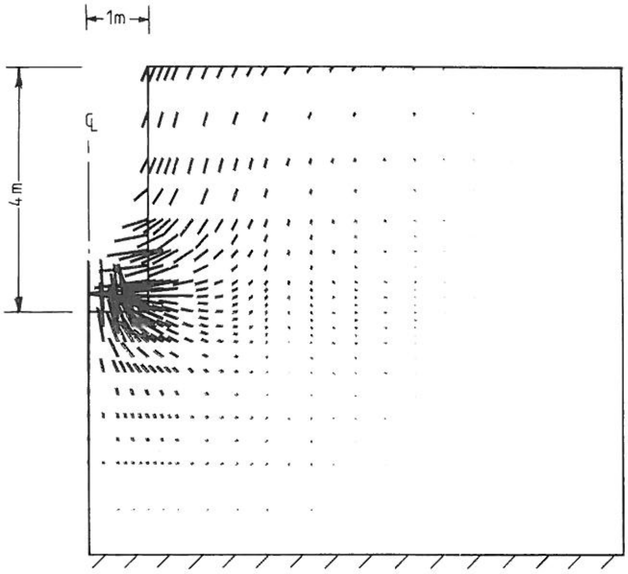


Fig.11 Failure displacement vectors in axisymmetry (H/r = 4)

Finally, a study was made of the effects of element type on the results achieved in axisymmetry. An element that has been recommended for problems of incompressible plastic flow in axisymmetry is the 15-noded triangle[15] with 16 integration points. A typical mesh is shown in figure 12a. The analyses proceeded exactly as before except due to the larger bandwidths an 'out of core' solver was required for the equilibrium equations[16]. Very similar stability numbers to those computed previously were obtained. For comparison, the displacement vector plot at failure for the mesh with H/r = 4 is shown in figure 12b.

Although barelling of quadratic elements with reduced integration can occur in crude finite element meshes, evidence produced in this paper and elsewhere[17] suggests that this has little impact on failure loads. Although the eight-node element with reduced integration axisymmetry only satisfies plastic incompressibility at its Gauss-points, any reasonably refined mesh will have sufficient Gauss-points to provide an adequate discretisation.

Finally, some brief analyses were performed on the non-homogeneous case in which the shear strength of the soil increased linearly with depth. This

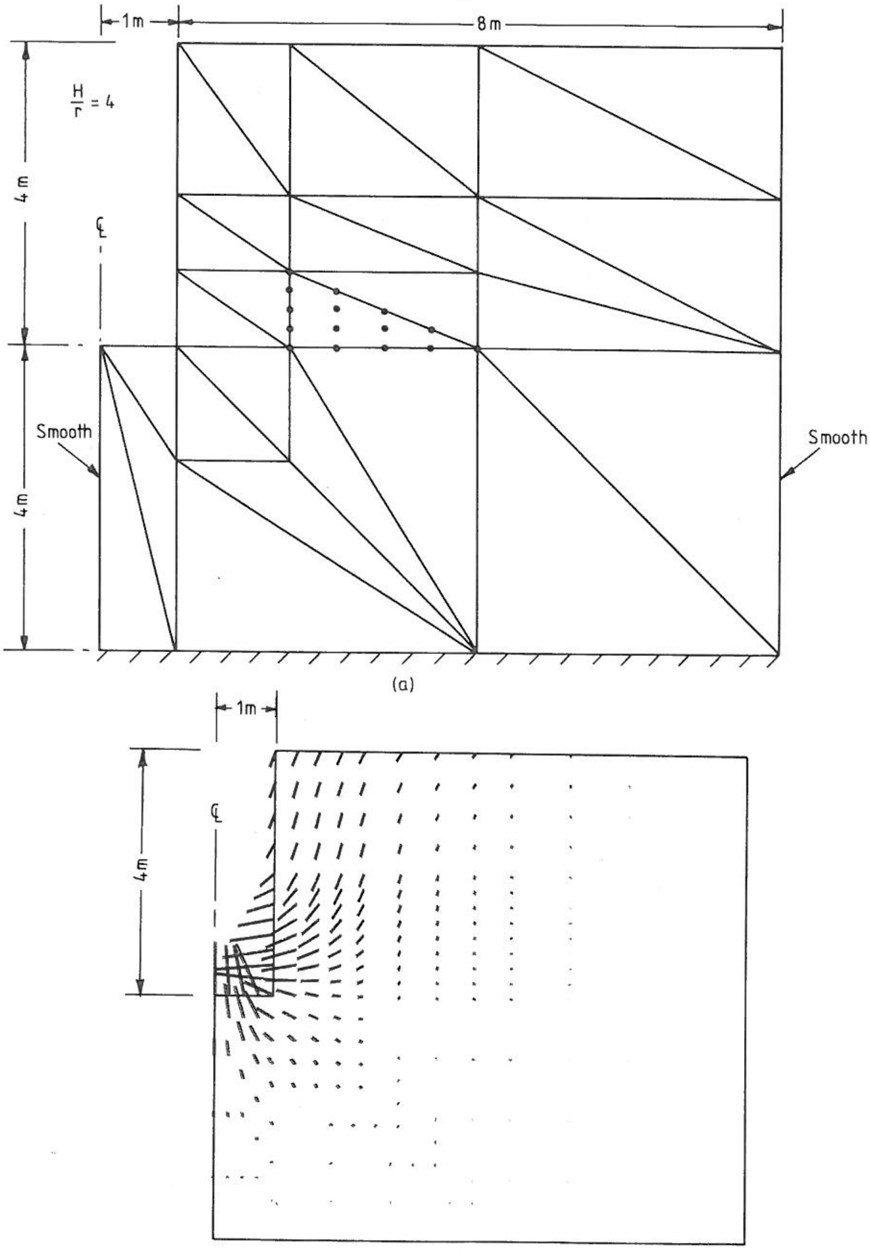


Fig.12 Mesh & displacements at failure using higher order triangular elements

problem has been considered analytically by Davis and Booker[18] and physically modelled in the centrifuge by Britto and Kusakabe[14]. The notation of the latter authors is preserved here.

Using the axisymmetric meshes of figure 8 the shear strength at each Gauss-point was made proportional to its distance below ground level. Two different rates of increase of shear strength were considered as shown in figure 13 and failure was achieved by keeping k constant and gradually reducing C_{uo}. Excellent agreement with the centrifuge solutions was recorded.

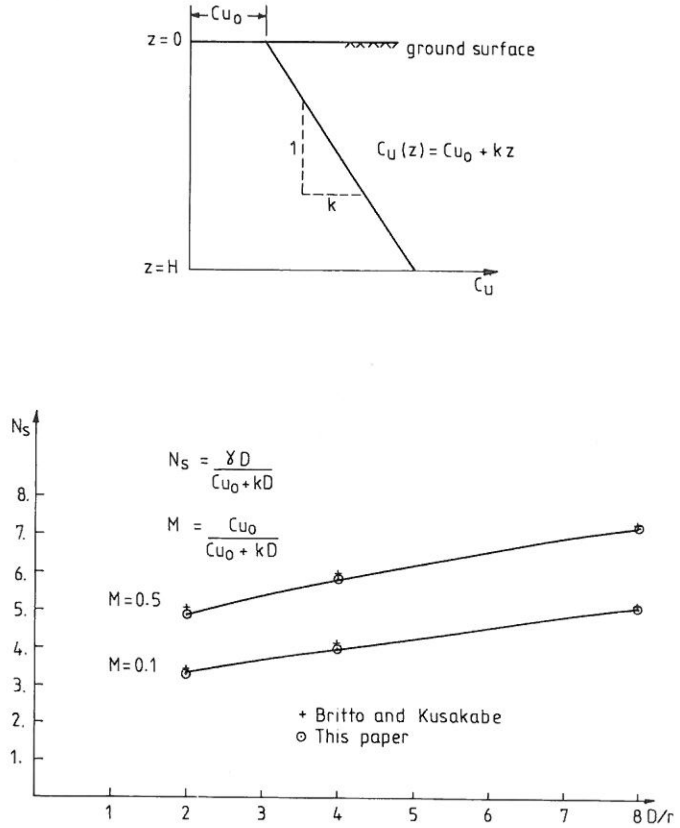


Fig.13. Notation and solutions for non-homogenous soil

CONCLUSIONS

Elasto-plastic finite element methods have been used to analyse the short term stability of vertical excavations in undrained clay. Using a gravity turn-on procedure, failure was induced by keeping the geometry and soil unit-weight constant while gradually reducing the shear strength. Using this approach, both the stability number at failure and the mechanism were indicated. In plane strain the stability number was in close agreement with the classical solutions of Taylor with the mechanism confined mainly to the wall of the excavation. It was also found that stability was hardly influenced by the excavation width. In axisymmetry, however, the stability number tended to rise with reduced excavation width and thus was in agreement with other solutions obtained by limit analyses, finite elements and centrifuge tests. The stabilising effect of reducing the excavation width was due to the mechanism extending fully into the base. The displacement vector plots showed this clearly, with movements at failure in axisymmetry being more localised around the base of the excavation than was observed in plane strain.

The influence of the elastic properties of the soil on the computed stability number at failure was found to be minimal. Young's modulus had no effect although as Poisson's ratio approached the incompressibility limit of 0.5, a stiffening of the soil response prior to failure was apparent.

The finite element procedures described here have been shown to give reliable estimates of the stability number at failure for both plane strain and axisymmetric configurations. The results have shown that the method can reproduce, with sufficient accuracy, known solutions for homogeneous soils. The particular power of the method, however, lies in the ease with which non-homogeneous soil strata can be analysed with no 'a priori' assumption necessary regarding the eventual failure mechanism.

The finite element method therefore remains a powerful tool for assessing the stability of 'awkward' slopes that are not amenable to analysis by simple methods.

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