Load and resistance factor design of shallow foundations against bearing failure

Gordon A. Fenton, D.V. Griffiths, and Xianyue Zhang

Abstract: Shallow foundation designs are typically governed either by settlement, a serviceability limit state, or by bearing capacity, an ultimate limit state. While geotechnical engineers have been designing against these limit states for over half a century, it is only recently that they have begun to migrate towards reliability-based designs. At the moment, reliability-based design codes are generally derived through calibration with traditional working stress designs. To take advantage of the full potential of reliability-based design the profession must go beyond calibration and take geotechnical uncertainties into account in a rational fashion. This paper proposes a load and resistance factor design (LRFD) approach for the bearing capacity design of a strip footing, using load factors as specified by structural codes. The resistance factors required to achieve an acceptable failure probability are estimated as a function of the spatial variability of the soil and by the level of "understanding" of the soil properties in the vicinity of the foundation. The analytical results, validated by simulation, are primarily intended to aid in the development of the next generation of reliability-based geotechnical design codes, but can also be used to assess the reliability of current designs.

Key words: bearing capacity, reliability, resistance factors, load and resistance factor design, ultimate limit state, shallow foundation.

Résumé : Les conceptions de fondations superficielles sont typiquement régies soit par le tassement, un état limite de service, ou par la capacité portante, un état limite ultime. Alors que les ingénieurs en géotechnique ont conçu en utillisant ces états limites depuis plus d'un demi-siècle, ce n'est que récemment qu'ils ont commencé à migrer vers les conceptions basées sur la fiabilié. Présentement, les codes de conception basés sur la fiabilité sont généralement dérivés au moyen de calibrage avec les calculs traditionnels de contrainte de travail. Pour prendre avantage du plein potentiel de la conception basée sur la fiabilité, la profession doit aller au delà du calibrage et prendre en compte les incertitudes géotechniques de manière rationnelle. Cet article propose une approche de conception basée sur un facteur de charge et de résistance (« LRFD ») pour le calcul de la capacité portante d'une semelle filante en utilisant des facteurs de charge tels que spécifiés dans les codes de structure. Les facteurs de résistance requis pour obtenir une probabilité de rupture acceptable sont estimés comme une fonction de la variabilité spatiale du sol et par le niveau de "compréhension" des propriétés du sol dans le voisinage de la fondation. Les résultats analytiques, validés par simulation, sont principalement destinés à aider au développement de la prochaine génération de codes de conception géotechnique basé sur la fiabilité, mais peuvent aussi être utilisés pour évaluer la fiabilité des conceptions courantes.

Mots-clés : capacité portante, fiabilité, facteurs de résistance, conception avec les facteurs de force et de résistance, état limite ultime, fondation superficielle.

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Introduction

The design of a shallow footing typically begins with a site investigation aimed at determining the strength and compressibility characteristics of the founding soil or rock. Once this information has been gathered, the geotechnical engineer is in a position to determine the footing dimensions required to avoid entering various limit states. In so doing, it will be assumed here that the geotechnical engineer is in close communication with the structural engineer(s) and is aware of the loads that the footings are being designed to support. The limit states that are usually considered in the footing design are serviceability limit states (typically deformation) and ultimate limit states. The latter is concerned with safety and includes the load-carrying capacity, or bearing capacity, of the footing. Figure 1 illustrates a bearing capacity (ultimate limit state) failure of a strip footing founded on a spatially variable soil. The failure surface passes through the lower strength (lighter) regions of the soil.

This paper develops a load and resistance factor design (LRFD) approach for shallow foundations designed against bearing capacity failure. The design goal is to determine the footing dimensions such that the ultimate geotechnical resistance based on characteristic soil properties, $\widehat{R}_u$, satisfies

$$\phi_g \widehat{R}_u \geq I \sum_i \alpha_i \widehat{L}_i \qquad [1]$$

where ϕ_g is the geotechnical resistance factor, I is an importance factor, α_i is the ith load factor, and $\widehat{L}_i$ is the ith char-

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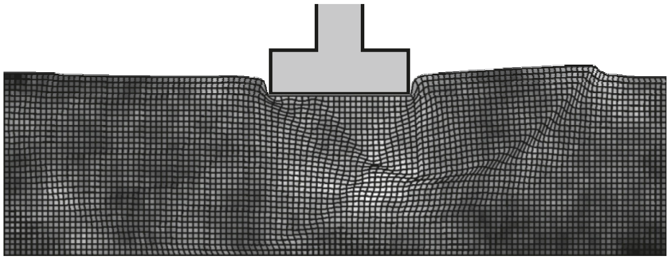
Fig. 1. Bearing failure of a strip footing founded on a spatially variable soil by the random finite element method.



acteristic load effect. The goal of this paper is to determine the relationship between ϕ_g and the probability that the designed footing will experience a bearing capacity failure. The authors recognize that the use of the symbol ϕ_g is somewhat confusing in a geotechnical paper as the friction angle also uses the symbol ϕ; however, the symbol ϕ is commonly used for resistance factor by the *National Building Code of Canada* (NBCC) (NRC 2005) and in other structural codes (see, for example, the use of the symbol ϕ for resistance factor in Commentary K "Foundations" of the User's Guide – NBC 2005 Structural Commentaries, NRC 2006) to represent resistance factor. The authors are also adopting the common notation where the subscript denotes the material that the resistance factor governs. For example, where ϕ_c and ϕ_s are resistance factors governing concrete and steel, the letter g in ϕ_g will be taken to denote "geotechnical" or "ground".

The importance factor, I, reflects the severity of the failure consequences and may be larger than 1.0 for important structures, such as hospitals, whose failure consequences are severe and whose target probabilities of failure are much less than for typical structures. Typical structures usually are designed using I = 1, which will be assumed in this paper. Structures with low failure consequences (minimal risk of loss of life, injury, and (or) economic impact) may have I < 1.

Only one load combination will be considered in this paper, $\alpha_L\widehat{L}_L + \alpha_D\widehat{L}_D$, where $\widehat{L}_L$ is the characteristic live load, $\widehat{L}_D$ is the characteristic dead load, and α_L and α_D are the live and dead load factors, respectively. The load factors used in this paper will be as specified by the NBCC (NRC 2005); α_L = 1.5 and α_D = 1.25. The theory presented here, however, is easily extended to other load combinations and factors, so long as their (possibly time-dependent) distributions are known.

The characteristic loads will be assumed to be defined in terms of the means of the load components in the following fashion:

$$[2a] \qquad \widehat{L}_L = k_{L_e}\mu_{L_e}$$

$$[2b] \qquad \widehat{L}_D = k_D\mu_D$$

where μ_{L_e} and μ_D are the means of the live and dead loads, respectively, and k_{L_e} and k_D are live and dead load bias factors, respectively. The bias factors provide some degree of "comfort" by increasing the loads from the mean value to a value having a lesser chance of being exceeded. Since live loads are time varying, the value of μ_{L_e} is more specifically defined as the mean of the maximum live load experienced over a structure's lifetime (the subscript e denotes extreme). This definition has the following interpretation: if a series of similar structures, all with the same lifespan, is considered and the maximum live load experienced in each throughout its lifespan is recorded, then a histogram of this set of recorded maximum live loads could be plotted. This histogram then becomes an estimate of the distribution of these extreme live loads and the average of the observed set of maximum values is an estimate of μ_{L_e}. As an aside, the distribution of live load is really quite a bit more complicated than what is suggested by this explanation, as it actually depends on both spatial position and time (e.g., regions near walls tend to experience much higher live load than seen near the center of rooms). However, historical estimates of live loads are quite appropriately based on spatial averages both conservatively and for simplicity, as discussed next.

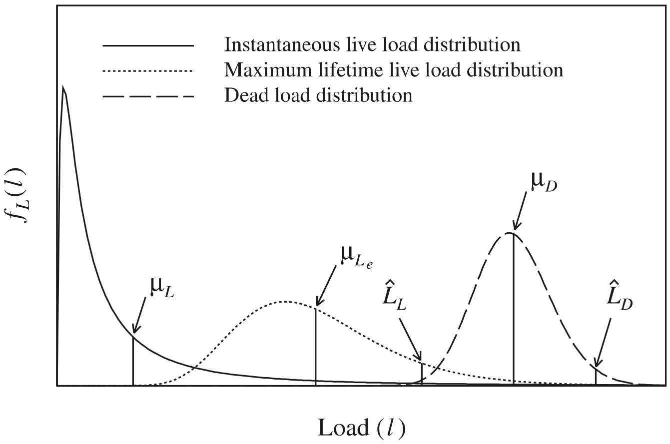
For typical multistory office buildings, Allen (1975) estimates μ_{L_e} to be 1.7 kN/m^2, based on a 30 year lifetime. The corresponding characteristic live load given by the NBCC (NRC 2005) is $\widehat{L}_L = 2.4\,\text{kN/m}^2$, which implies that $k_{L_e} = 2.4/1.7 = 1.41$. Allen further states that the mean live load at any time is approximately equal to the 30 year maximum mean averaged over an infinite area. The NBCC provides for a reduction in live loads with tributary area using the formula $0.3 + \sqrt{9.8/A}$, where A is the tributary area (A > 20 m^2). For $A \rightarrow \infty$, the mean live load at any time is thus approximately μ_L = 0.3(1.7) = 0.51 kN/m^2. The bias factor that translates the instantaneous mean live load, μ_L to the characteristic live load, $\widehat{L}_L$, is thus quite large having value k_L = 2.4/0.51 = 4.7.

Dead load, on the other hand, is largely static, and the time span considered (e.g., lifetime) has little effect on its distribution. Becker (1996*b*) estimates k_D to be 1.18. Figure 2 illustrates typical probability density functions for the three types of loads (instantaneous live, extreme live, and dead) commonly considered and the relative locations of the characteristic load values.

The characteristic ultimate geotechnical resistance, $\widehat{R}_u$, is determined using characteristic soil properties, in this case characteristic values of the soil's cohesion, c, and friction angle, ϕ (note that although the primes are omitted from these quantities it should be recognized that the theoretical developments described in this paper are applicable to either total or effective strength parameters). To obtain the characteristic soil properties, the soil is sampled over a single column somewhere in the vicinity of the footing, for example, a single CPT or SPT sounding near the footing. The sample is assumed to yield a sequence of m observed cohesion values, $c_1^o, c_2^o, \ldots, c_m^o$, and m observed friction angle values, $\phi_1^o, \phi_2^o, \ldots, \phi_m^o$. The superscript o denotes an observation. It is assumed here that the observations are error-free, which is an unconservative assumption. If the actual observations have considerable error, then the resistance factor used in the design should be reduced. This issue is discussed further in the conclusions.

The characteristic value of the cohesion, $\widehat{c}$, is defined in this paper as the median of the sampled observations, c_i^o, which, assuming c is lognormally distributed, can be computed using the geometric average,

Fig. 2. Characteristic and mean values of live and dead loads along with their corresponding distributions.



$$[3] \qquad \widehat{c} = \left[\prod_{i=1}^{m} c_i^{\mathrm{o}}\right]^{1/m} = \exp\left(\frac{1}{m}\sum_{i=1}^{m}\ln c_i^{\mathrm{o}}\right)$$

The geometric average is used here because if c is lognormally distributed, as assumed, then $\widehat{c}$ will also be lognormally distributed.

The characteristic value of the friction angle is computed as an arithmetic average

$$[4] \qquad \widehat{\phi} = \frac{1}{m}\sum_{i=1}^{m}\phi_i^{\mathrm{o}}$$

The arithmetic average is used here because ϕ is assumed to follow a symmetric bounded distribution and the arithmetic average preserves the mean. That is, the mean of $\widehat{\phi}$ is the same as the mean of ϕ.

To determine the characteristic ultimate geotechnical resistance, $\widehat{R}_{\mathrm{u}}$, it will first be assumed that the soil is weightless. This simplifies the calculation of the ultimate bearing stress, q_{u}, to

$$[5] \qquad q_{\mathrm{u}} = cN_{\mathrm{c}}$$

The assumption of weightlessness is conservative as the soil weight contributes to the overall bearing capacity. This assumption also allows the analysis to explicitly concentrate on the role of cN_{c} on ultimate bearing capacity, as this is the only term that includes the effects of spatial variability relating to both shear strength parameters c and ϕ.

Bearing capacity predictions, involving specification of the N_{c} factor in this case, are generally based on plasticity theories (see, e.g., Prandtl 1921; Terzaghi 1943; Sokolovski 1965) in which a rigid base is punched into a softer material. These theories assume that the soil underlying the footing has properties that are spatially constant (everywhere the same). This type of ideal soil will be referred to as a uniform soil henceforth. Under this assumption, most bearing capacity theories (e.g., Prandtl 1921; Meyerhof 1951, 1964) assume that the failure slip surface takes on a logarithmic spiral shape to give

$$[6] \qquad N_{\mathrm{c}} = \frac{\mathrm{e}^{\pi\tan\phi}\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) - 1}{\tan\phi}$$

See Griffiths and Fenton (2001) for a probabilistic analysis of a footing founded on a purely cohesive (ϕ = 0) soil. The current paper considers the more general case of a c–ϕ soil. One can always set ϕ = 0 in the following theory to perform a total stress analysis on an ‘‘undrained clay’’ soil.

Consistent with the theoretical results presented by Fenton et al. (2007), this paper will also concentrate on the design of a strip footing, as illustrated in Fig. 1. In this case, the characteristic ultimate geotechnical resistance, $\widehat{R}_{\mathrm{u}}$, becomes

$$[7] \qquad \widehat{R}_{\mathrm{u}} = B\widehat{q}_{\mathrm{u}}$$

where B is the footing width and $\widehat{R}_{\mathrm{u}}$ has units of load per unit length out-of-plane, that is, in the direction of the strip foot. The characteristic ultimate bearing stress, $\widehat{q}_{\mathrm{u}}$, is defined by

$$[8] \qquad \widehat{q}_{\mathrm{u}} = \widehat{c}\widehat{N}_{\mathrm{c}}$$

where the characteristic N_{c} factor is determined using the characteristic friction angle in eq. [6],

$$[9] \qquad \widehat{N}_{\mathrm{c}} = \frac{\mathrm{e}^{\pi\tan\widehat{\phi}}\tan^2\left(\frac{\pi}{4}+\frac{\widehat{\phi}}{2}\right) - 1}{\tan\widehat{\phi}}$$

For the strip footing and just the dead and live load combination, the LRFD equation becomes

$$[10] \qquad \phi_{\mathrm{g}}B\widehat{q}_{\mathrm{u}} = I(\alpha_{\mathrm{L}}\widehat{L}_{\mathrm{L}} + \alpha_{\mathrm{D}}\widehat{L}_{\mathrm{D}}) \Rightarrow B = \frac{I(\alpha_{\mathrm{L}}\widehat{L}_{\mathrm{L}} + \alpha_{\mathrm{D}}\widehat{L}_{\mathrm{D}})}{\phi_{\mathrm{g}}\widehat{q}_{\mathrm{u}}}$$

To determine the resistance factor, ϕ_{g}, required to achieve a certain acceptable reliability of the constructed footing, it is necessary to estimate the probability of bearing capacity failure of a footing designed using eq. [10]. This paper will use the theoretical results presented by Fenton et al. (2007) for a strip footing, which will be summarized in the section entitled ‘‘Analytical approximation to the probability of failure’’. Once the probability of failure, p_{f}, for a certain design using a specific value for ϕ_{g} is known, this probability can be compared to the maximum acceptable failure probability, p_{m}. If p_{f} exceeds p_{m}, then the resistance factor must be reduced and the footing redesigned. Similarly, if p_{f} is less than p_{m}, then the design is overconservative and the value of ϕ_{g} can be increased. A specific relationship between p_{m} and ϕ_{g} will be given here. Design curves will also be presented from which the value of ϕ_{g} required to achieve a maximum acceptable failure probability can be determined.

As previously suggested, the determination of the required resistance factor, ϕ_{g}, involves deciding on a maximum acceptable failure probability, p_{m}. The choice of p_{m} derives from a consideration of acceptable risk and directly influences the size of ϕ_{g}. Different levels of p_{m} may be considered to reflect the ‘‘importance’’ of the supported structure — p_{m} may be much smaller for a hospital than for an uninhabited storage warehouse.

The choice of a maximum failure probability, p_{m}, should consider the margin of safety implicit in current foundation designs and the levels of reliability for geotechnical design as reported in the literature. The values of p_{m} for foundation designs are nearly the same or somewhat less than those for concrete and steel structures because of the difficulties and high expense of foundation repairs. A literature review of the suggested acceptable probability of failure for foundations is listed in Table 1.

Meyerhof (1995) was quite specific about acceptable risks: "The order of magnitude of lifetime probabilities of stability failure is about 10^{-2} for offshore foundation, about 10^{-3} for earthworks and earth retaining structures, and about 10^{-4} for foundations on land".

In this paper three maximum lifetime failure probabilities, 10^{-2}, 10^{-3}, and 10^{-4} will be considered. In general, and without regard to the aforementioned structural categorizations made by Meyerhof, these probabilities are deemed by the authors to be appropriate for designs involving low, medium, and high failure consequence structures, respectively. Resistance factors required to achieve these target probabilities will be recommended for the specific c–ϕ soil considered. These resistance factors are smaller than those that the theory suggests for an undrained soil, because a $\phi = 0$ soil has only one source of uncertainty. In other words, the resistance factors based on a generalized c–ϕ soil are considered to be reasonably conservative.

It is also noted that the effect of structural importance should actually be reflected in the importance factor, I, of eq. [1] and not in the resistance factor. The resistance factor should be aimed at a medium, or common, structural importance level, and the importance factor should be varied above and below 1.0 to account for more and less important structures, respectively. However, because acceptable failure probabilities may not be simply connected to structural importance, $I = 1$ is assumed in the following. For code provisions, the factors recommended here should be considered to be the ratio ϕ_g/I.

Table 1. Literature review of lifetime probabilities of failure of foundations.

Source	p_m
Meyerhof (1970, 1993, 1995)	$10^{-2} - 10^{-4}$
Simpson et al. (1981)	10^{-3}
NCHRP (1991)	$10^{-2} - 10^{-4}$
Becker (1996*a*)	$10^{-3} - 10^{-4}$

The random soil model

The soil cohesion, c, is assumed to be lognormally distributed with mean μ_c, standard deviation σ_c, and spatial correlation length $\theta_{\ln c}$. A lognormally distributed random field is obtained from a normally distributed random field, $G_{\ln c}(\underset{\sim}{x})$ having zero mean, unit variance, and spatial correlation length $\theta_{\ln c}$ through the transformation

$$[11] \qquad c(\underset{\sim}{x}) = \exp\left[\mu_{\ln c} + \sigma_{\ln c} G_{\ln c}(\underset{\sim}{x})\right]$$

where $\underset{\sim}{x}$ is the spatial position at which c is desired, $\sigma^2_{\ln c} = \ln(1 + v_c^2)$, $\mu_{\ln c} = \ln(\mu_c) - \sigma^2_{\ln c}/2$, and $v_c = \sigma_c/\mu_c$ is the coefficient of variation.

The correlation coefficient between the log-cohesion at a point $\underset{\sim}{x}_1$ and a second point $\underset{\sim}{x}_2$ is specified by a correlation function, $\rho_{\ln c}(\underset{\sim}{\tau})$, where $\underset{\sim}{\tau} = \underset{\sim}{x}_1 - \underset{\sim}{x}_2$ is the vector between the two points. In this paper, a simple exponentially decaying (Markovian) correlation function will be assumed, having the form

$$[12] \qquad \rho_{\ln c}(\underset{\sim}{\tau}) = \exp\left(\frac{-2|\underset{\sim}{\tau}|}{\theta_{\ln c}}\right)$$

where $|\underset{\sim}{\tau}| = \sqrt{\tau_1^2 + \tau_2^2}$ is the length of the vector $\underset{\sim}{\tau}$. The spatial correlation length, $\theta_{\ln c}$, is loosely defined as the separation distance within which two values of ln c are significantly correlated. Mathematically, $\theta_{\ln c}$ is defined as the area under the correlation function, $\rho_{\ln c}(\underset{\sim}{\tau})$ (Vanmarcke 1984).

The spatial correlation function, $\rho_{\ln c}(\underset{\sim}{\tau})$ has a corresponding variance reduction function, $\gamma_{\ln c}(D)$, which specifies how the variance is reduced upon local averaging of ln c over some domain D. In the two-dimensional (2D) analysis considered here, $D = D_1 \times D_2$ is an area and the 2D variance reduction function is defined by

$$[13] \qquad \gamma_{\ln c}(D_1, D_2) = \frac{4}{(D_1 D_2)^2}\int_0^{D_1} \times \int_0^{D_2} (D_1 - \tau_1)(D_2 - \tau_2)\rho(\tau_1, \tau_2)\, \mathrm{d}\tau_1\, \mathrm{d}\tau_2$$

which can be evaluated using Gaussian quadrature (see Fenton and Griffiths 2003; Griffiths and Smith 2006 for more details).

It should be emphasized that the previously selected correlation function acts between values of ln c. This is because ln c is normally distributed and a normally distributed random field is simply defined by its mean and covariance structure. In practice, the correlation length $\theta_{\ln c}$ can be estimated by evaluating spatial statistics of the log-cohesion data directly (see, e.g., Fenton 1999). Unfortunately, such studies are scarce so that little is currently known about the spatial correlation structure of natural soils. For the problem considered here, it turns out that a worst case correlation length exists that can be conservatively assumed in the absence of improved information.

The random field is also assumed here to be statistically isotropic (the same correlation length in any direction through the soil). Although the horizontal correlation length is often greater than the vertical as a result of soil layering, taking this into account was deemed to be a site-specific refinement that does not lead to an increase in the general understanding of the probabilistic behaviour of shallow foundations. The theoretical results presented here, however, apply also to anisotropic soils, so that the results are easily extended to specific sites. The authors have found that when the soil is sampled at some distance from the footing (i.e., not directly under the footing), increasing the correlation length in the horizontal direction to values above the worst-case isotropic correlation length leads to a decreased failure probability, so that the isotropic case is also conservative for low to medium levels of site understanding. When the soil is sampled directly below the footing, the failure probability increases as the horizontal correlation length is increased above the worst case scale, which is unconservative.

The friction angle, ϕ, is assumed to be bounded both above and below, so that neither normal nor lognormal distributions are appropriate. A beta distribution is often used for bounded random variables. Unfortunately, a beta distributed random field has a complex joint distribution and simulation is cumbersome and numerically difficult. To keep things simple, a bounded distribution is selected that resembles a beta distribution but it arises as a simple transformation of a standard normal random field, $G_\phi(\underset{\sim}{x})$, according to

$$[14] \qquad \phi(\underset{\sim}{x}) = \phi_{\min} + \frac{1}{2}(\phi_{\max} - \phi_{\min})\left[1 + \tanh\left(\frac{sG_\phi(\underset{\sim}{x})}{2\pi}\right)\right]$$

where $\phi_{\min}$ and $\phi_{\max}$ are the minimum and maximum friction angles in radians, respectively, and s is a scale factor that governs the friction angle variability between its two bounds (see Fenton and Griffiths 2008, for more details). Figure 3 shows how the distribution of ϕ (normalized to the interval [0,1]) changes as s changes, going from an almost uniform distribution at $s = 5$ to a very normal looking distribution for smaller s. Thus, varying s between about 0.1 and 5.0 leads to a wide range in the stochastic behaviour of ϕ. In all cases, the distribution is assumed to be symmetric so that the midpoint between $\phi_{\min}$ and $\phi_{\max}$ is the mean. Values of s less than 1 lead to narrower distributions, however those with s greater than about 5 lead to U-shaped distributions (higher at the boundaries), which are unrealistic.

The following relationship between s and the variance of ϕ derives from a third-order Taylor series approximation to tanh and a first-order approximation to the final expectation,

$$[15] \qquad \begin{aligned} \sigma_\phi^2 &= (0.5)^2(\phi_{\max} - \phi_{\min})^2 \mathrm{E}\left[\tanh^2\left(\frac{sG_\phi}{2\pi}\right)\right] \\ &\simeq (0.5)^2(\phi_{\max} - \phi_{\min})^2 \mathrm{E}\left[\frac{(sG_\phi/2\pi)^2}{1 + (sG_\phi/2\pi)^2}\right] \\ &\simeq (0.5)^2(\phi_{\max} - \phi_{\min})^2 \frac{s^2}{4\pi^2 + s^2} \end{aligned}$$

where $\mathrm{E}[\cdot]$ is the expectation operator and $\mathrm{E}[G_\phi^2] = 1$ because G_ϕ is a standard normal random variable. Equation [15] slightly overestimates the true standard deviation of ϕ, from 0% when $s = 0$ to 11% when $s = 5$. A closer approximation over the entire range $0 \le s \le 5$ is obtained by slightly decreasing the 0.5 factor to 0.46 (this is an empirical adjustment)

$$[16] \qquad \sigma_\phi \simeq \frac{0.46(\phi_{\max} - \phi_{\min})s}{\sqrt{4\pi^2 + s^2}}$$

The close agreement is illustrated in Fig. 4.

Equation [15] can be generalized to yield the covariance between $\phi(\underset{\sim}{x}_i)$ and $\phi(\underset{\sim}{x}_j)$, for any two spatial points $\underset{\sim}{x}_i$ and $\underset{\sim}{x}_1$ as follows,

$$[17] \qquad \begin{aligned} \mathrm{Cov}[\phi(\underset{\sim}{x}_i), \phi(\underset{\sim}{x}_j)] &= (0.5)^2(\phi_{\max} - \phi_{\min})^2 \mathrm{E}\left[\tanh\left(\frac{sG_\phi(\underset{\sim}{x}_i)}{2\pi}\right)\tanh\left(\frac{sG_\phi(\underset{\sim}{x}_i)}{2\pi}\right)\right] \\ &\simeq (0.5)^2(\phi_{\max} - \phi_{\min})^2 \mathrm{E}\left\{\frac{[sG_\phi(\underset{\sim}{x}_i)/2\pi][sG_\phi(\underset{\sim}{x}_j)/2\pi]}{1 + \frac{1}{2}\left[(sG_\phi(\underset{\sim}{x}_i)/2\pi)^2 + (sG_\phi(\underset{\sim}{x}_j)/2\pi)^2\right]}\right\} \\ &\simeq (0.46)^2(\phi_{\max} - \phi_{\min})^2 \frac{s^2\rho_\phi(\underset{\sim}{x}_i - \underset{\sim}{x}_j)}{4\pi^2 + s^2} \\ &= \sigma_\phi^2\rho_\phi(\underset{\sim}{x}_i - \underset{\sim}{x}_j) \end{aligned}$$

where the empirical correction found in eq. [16] was introduced in the second to last step.

It seems reasonable to assume that if the spatial correlation structure of a soil is caused by changes in the constitutive nature of the soil over space, then both cohesion and friction angle would have similar correlation lengths. Thus, θ_ϕ is taken to be equal to $\theta_{\ln c}$ in this study, and ϕ is assumed to have the same correlation structure as c (eq. [12]), that is, $\rho_\phi(\underset{\sim}{\tau}) = \rho_{\ln c}(\underset{\sim}{\tau})$. Both correlation lengths will be referred to generically from now on simply as θ, and both correlation functions as $\rho(\underset{\sim}{\tau})$, remembering that this length and correlation function reflects correlation between points in the underlying normally distributed random fields, $G_{\ln c}(\underset{\sim}{x})$ and $G_\phi(\underset{\sim}{x})$, and not directly between points in the cohesion and friction fields (although the correlation lengths in the different spaces are quite similar). The correlation lengths can be estimated by statistically analyzing data generated by inverting eqs. [11] and [14]. Because both fields have the same correlation function, $\rho(\underset{\sim}{\tau})$, they will also have the same variance reduction function, that is, $\gamma_{\ln c}(D) = \gamma_\phi(D) = \gamma(D)$, as defined by eq. [13].

The two random fields, c and ϕ, are assumed to be independent. Nonzero correlations between c and ϕ were found by Fenton and Griffiths (2003) to have only a minor influence on the estimated probabilities of bearing capacity failure. Because the general consensus is that c and ϕ are negatively correlated (Wolff 1985; Cherubini 2000) and the mean bearing capacity for independent c and ϕ was slightly lower than for the negatively correlated case (Fenton and Griffiths 2003), the assumption of independence between c and ϕ is slightly conservative.

Analytical approximation to the probability of failure

In this section, an analytical approximation to the probability of bearing capacity failure of a strip footing is summarized. Equation [5] was developed assuming an ideal soil whose shear strength is the same everywhere (i.e., a uniform soil). When soil properties are spatially variable, as they are in reality, then the hypothesis made in this study is that eq. [5] can be replaced by

Fig. 3. Bounded distribution of friction angle normalized to the interval [0, 1].

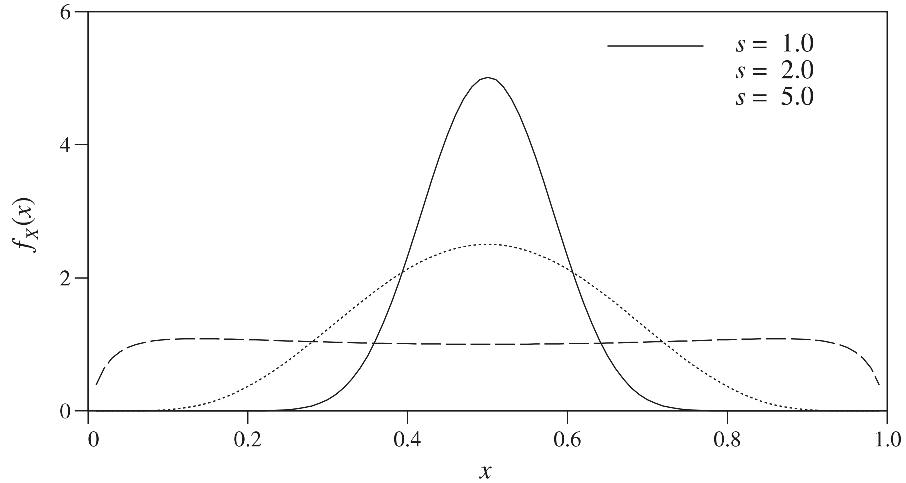
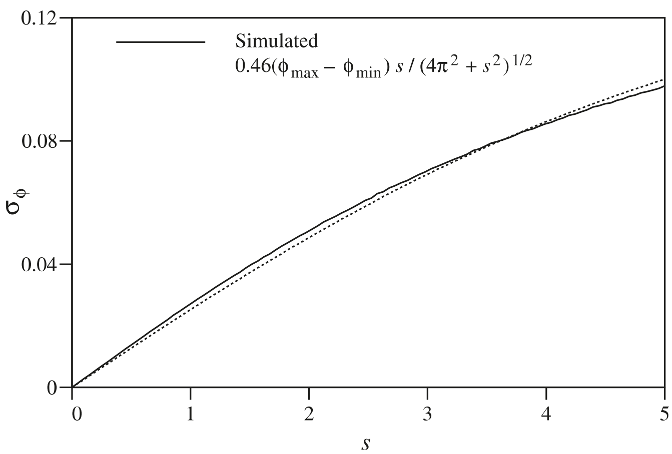


Fig. 4. Relationship between σ_ϕ and s derived from simulation (100 000 realizations for each s) and the Taylor's series derived approximation given by eq. [16]. The vertical scale corresponds to $\phi_{max} - \phi_{min}$ = 0.349 radians (20°).



$$[18]\qquad q_u = \bar{c}\bar{N}_c$$

where $\bar{c}$ and $\bar{N}_c$ are the equivalent cohesion and equivalent N_c factor, defined as those uniform soil parameters that lead to the same bearing capacity as observed in the real, spatially varying, soil. In other words, it is proposed that equivalent soil properties, $\bar{c}$ and $\bar{\phi}$, exist such that a uniform soil having these properties will have the same bearing capacity as the actual spatially variable soil. The value of $\bar{N}_c$ is obtained by using the equivalent friction angle, $\bar{\phi}$, in eq. [6],

$$[19]\qquad \bar{N}_c = \frac{e^{\pi \tan \bar{\phi}} \tan^2\left(\frac{\pi}{4} + \frac{\bar{\phi}}{2}\right) - 1}{\tan \bar{\phi}}$$

In the design process, eq. [18] is replaced by eq. [8] and the design footing width, B, is obtained using eq. [10], which, in terms of the characteristic design values becomes

$$[20]\qquad B = \frac{I(\alpha_L \hat{L}_L + \alpha_D \hat{L}_D)}{\phi_g \hat{c} \hat{N}_c}$$

The design philosophy proceeds as follows: find the required footing width B such that the probability that the actual load, L, exceeds the actual resistance, $q_u B$, is less than some small acceptable failure probability, p_m. If p_f is the actual failure probability, then

$$[21]\qquad p_f = \mathbf{P}[L > q_u B] = \mathbf{P}[L > \bar{c}\bar{N}_c B]$$

and a successful design methodology will have $p_f \le p_m$. Substituting eq. [20] into eq. [21] and collecting random terms to the left of the inequality leads to

$$[22]\qquad p_f = \mathbf{P}\left[L\frac{\hat{c}\hat{N}_c}{\bar{c}\bar{N}_c} > \frac{I(\alpha_L \hat{L}_L + \alpha_D \hat{L}_D)}{\phi_g}\right]$$

Letting

$$[23]\qquad Y = L\frac{\hat{c}\hat{N}_c}{\bar{c}\bar{N}_c}$$

means that

$$[24]\qquad p_f = \mathbf{P}\left[Y > \frac{I(\alpha_L \hat{L}_L + \alpha_D \hat{L}_D)}{\phi_g}\right]$$

and the task is to find the distribution of Y. Assuming that Y is lognormally distributed (an assumption found to be reasonable by Fenton et al. 2007, which is also supported to some extent by the central limit theorem), then

$$[25]\qquad \ln Y = \ln L + \ln \hat{c} + \ln \hat{N}_c - \ln \bar{c} - \ln \bar{N}_c$$

is normally distributed and p_f can be found once the mean and variance of ln Y are determined. The mean of ln Y is

$$[26]\qquad \mu_{\ln Y} = \mu_{\ln L} + \mu_{\ln \hat{c}} + \mu_{\ln \hat{N}_c} - \mu_{\ln \bar{c}} - \mu_{\ln \bar{N}_c}$$

and the variance of ln Y is

$$[27]\qquad \sigma^2_{\ln Y} = \sigma^2_{\ln L} + \sigma^2_{\ln \hat{c}} + \sigma^2_{\ln \bar{c}} + \sigma^2_{\ln \hat{N}_c} + \sigma^2_{\ln \bar{N}_c} - 2\mathrm{Cov}[\ln \bar{c}, \ln \hat{c}] - 2\mathrm{Cov}[\ln \bar{N}_c, \ln \hat{N}_c]$$

where the load, L, and soil properties, c and ϕ have been assumed to be mutually independent.

To find the parameters in eqs. [26] and [27], the following two assumptions are made:

(1) The equivalent cohesion, $\bar{c}$, is the geometric average of the cohesion field over some zone of influence, D, under the footing,

$$[28] \qquad \bar{c} = \exp\left[\frac{1}{D}\int_D \ln c(\underset{\sim}{x})\, \mathrm{d}\underset{\sim}{x}\right]$$

Note that in this 2D analysis, D is an area and the above is a 2D integration. If $c(\underset{\sim}{x})$ is lognormally distributed as assumed, then $\bar{c}$ is also lognormally distributed. This relationship also preserves the mean, i.e. $\mu_{\bar{\phi}} = \mu_\phi$.

(2) The equivalent friction angle, $\bar{\phi}$, is the arithmetic average of the friction angle over the zone of influence, D

$$[29] \qquad \bar{\phi} = 1/D \int_{\mathrm{D}} \phi(\underset{\sim}{x})\, \mathrm{d}\underset{\sim}{x}$$

Probably the greatest source of uncertainty in this analysis involves the choice of the domain, D, over which the equivalent soil properties are averaged under the footing. The averaging domain was found by trial and error to be best approximated by $D = W \times W$, centered directly under the footing (see Fig. 5). In this study, W is taken as 80% of the average mean depth of the wedge zone directly beneath the footing, as given by the classical Prandtl failure mechanism,

$$[30] \qquad W = \frac{0.8}{2}\hat{\mu}_{\mathrm{B}} \tan\left(\frac{\pi}{4} + \frac{\mu_\phi}{2}\right)$$

and where μ_ϕ is the mean friction angle (in radians), within the zone of influence of the footing, and $\hat{\mu}_{\mathrm{B}}$ is an estimate of the mean footing width obtained by using mean soil properties (μ_{c} and μ_ϕ) in eq. [10],

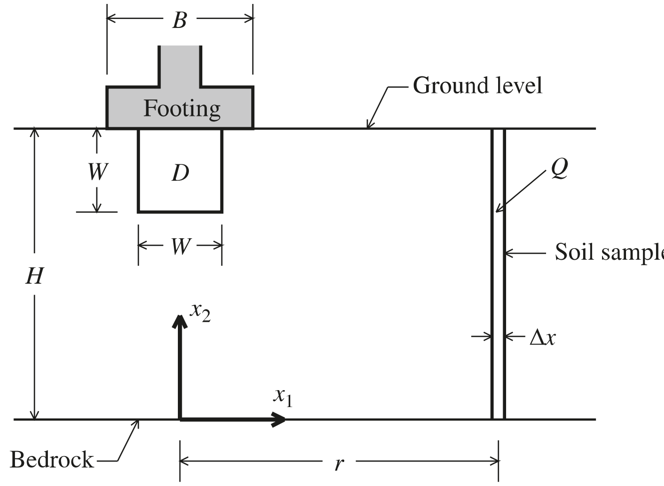
$$[31] \qquad \hat{\mu}_{\mathrm{B}} = \frac{I(\alpha_{\mathrm{L}}\hat{L}_{\mathrm{L}} + \alpha_{\mathrm{D}}\hat{L}_{\mathrm{D}})}{\phi_{\mathrm{g}}\mu_{\mathrm{c}}\mu_{N_{\mathrm{c}}}}$$

The footing shown in Fig. 5 is just one possible realization since the footing width, B, is actually a random variable. The averaging area D with dimension W suggested by eq. [30] is significantly smaller than that suggested by Fenton and Griffiths (2003). The 2003 study assumed that the footing width was known, rather than designed, and recognized that the larger averaging region did not well represent the mean bearing capacity, which of course is the most important value in probability calculations. The smaller averaging region used in this study may be reasonable if one considers the actual quantity of soils involved in resisting the bearing failure along the failure surfaces. That is, D would be the area of soil that deforms during failure. As this area will change, sometimes dramatically, from realization to realization, this can only be considered to be a rough empirical approximation. The problem of deciding on an appropriate averaging region needs further study. In the simulations performed to validate the theory presented here, the soil depth is taken to be H = 4.8 m and Δx = 0.15 m, where Δx is the width of the columns of finite elements used in the simulations (see Fig. 1).

To first order, the mean of N_{c} is,

$$[32] \qquad \mu_{N_{\mathrm{c}}} \simeq \frac{\mathrm{e}^{\pi \tan \mu_\phi} \tan^2\left(\frac{\pi}{4} + \frac{\mu_\phi}{2}\right) - 1}{\tan \mu_\phi}$$

Fig. 5. Averaging regions used to predict probability of bearing capacity failure.



Armed with the aformentioned information and assumptions, the components of eqs. [26] and [27] can be computed as follows (given the basic statistical parameters of the loads, c, ϕ, the number and locations of the soil samples, and the averaging domain size D):

(1) Assuming that the total load L is equal to the sum of the maximum live load, $L_{L_{\mathrm{e}}}$, acting over the lifetime of the structure and the static dead load, L_{D}, that is, $L = L_{L_{\mathrm{e}}} + L_{\mathrm{D}}$, both of which are random, then

$$[33a] \qquad \mu_{\ln L} = \ln(\mu_{\mathrm{L}}) - \frac{1}{2}\ln(1 + v_{\mathrm{L}}^2)$$

$$[33b] \qquad \sigma^2_{\ln L} = \ln(1 + v_{\mathrm{L}}^2)$$

where $\mu_{\mathrm{L}} = \mu_{L_{\mathrm{e}}} + \mu_{\mathrm{D}}$ is the sum of the mean (max lifetime) live and (static) dead loads, and v_{L} is the coefficient of variation of the total load defined by

$$[34] \qquad v_{\mathrm{L}}^2 = \frac{\sigma^2_{L_{\mathrm{e}}} + \sigma^2_D}{(\mu_{L_{\mathrm{e}}} + \mu_{\mathrm{D}})^2}$$

(2) With reference to eq. [3],

$$[35] \qquad \mu_{\ln \hat{c}} = \mathrm{E}\left[\frac{1}{m}\sum_{i=1}^{m} \ln c_i^{\mathrm{o}}\right] = \mu_{\ln c}$$

$$[36] \qquad \sigma^2_{\ln \hat{c}} \simeq \frac{\sigma^2_{\ln c}}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\rho(\underset{\sim}{x}_i^{\mathrm{o}} - \underset{\sim}{x}_j^{\mathrm{o}})$$

where $\underset{\sim}{x}_i^{\mathrm{o}}$ is the spatial location of the center of the ith soil sample (i = 1,2,…m) and ρ is the correlation function defined by eq. [12]. The approximation in the variance arises because correlation coefficients between the local averages associated with observations (in that all tests are performed on samples of some finite volume) are approximated by correlation coefficients between the local average centers. Assuming that $\ln \hat{c}$ actually represents a local average of $\ln c$ over a domain of size $\Delta x \times H$, where Δx is the horizontal dimension of the soil sample, which, for example, can be thought of as the horizontal zone of influence of a CPT or SPT sounding, and H is the depth over which the samples are taken, then $\sigma^2_{\ln \hat{c}}$ is probably more accurately computed as

$$\sigma^2_{\ln \hat{c}} = \sigma^2_{\ln c}\gamma(\Delta x, H) \tag{37}$$

(3) With reference to eq. [28],

$$\mu_{\ln \bar{c}} = \mathrm{E}\left[\frac{1}{D}\int_{\mathrm{D}} \ln c(\underset{\sim}{x})\, \mathrm{d}\underset{\sim}{x}\right] = \mu_{\ln c} \tag{38}$$

$$\sigma^2_{\ln \bar{c}} = \sigma^2_{\ln c}\gamma(D) \tag{39}$$

where $\gamma(D) = \gamma(W,W)$, as discussed previously, is defined by eq. [13].

(4) Because $\mu_{\hat{\phi}} = \mu_{\phi}$ (see eq. [4]), the mean and variance of $\widehat{N}_{\mathrm{c}}$ can be obtained using first order approximations to expectations of eq. [9] (Fenton and Griffiths 2003), as follows:

$$\mu_{\ln \widehat{N}_{\mathrm{c}}} = \mu_{\ln N_{\mathrm{c}}} \simeq \ln \frac{\mathrm{e}^{\pi \tan \mu_{\phi}} \tan^2\left(\frac{\pi}{4} + \frac{\mu_{\phi}}{2}\right) - 1}{\tan \mu_{\phi}} \tag{40}$$

$$\begin{aligned} \sigma^2_{\ln \widehat{N}_{\mathrm{c}}} &\simeq \sigma^2_{\hat{\phi}}\left(\frac{d \ln \widehat{N}_{\mathrm{c}}}{d\hat{\phi}}\bigg|_{\mu_{\phi}}\right)^2 \\ &= \sigma^2_{\hat{\phi}}\left\{\frac{bd}{bd^2 - 1}[\pi(1 + a^2)d + 1 + d^2] - \frac{1 + a^2}{a}\right\}^2 \end{aligned} \tag{41}$$

where $a = \tan(\mu_{\phi})$, $b = \mathrm{e}^{\pi a}$, $d = \tan[(\pi/4) + (\mu_{\phi}/2)]$. The variance of $\hat{\phi}$ can be obtained by making use of eq. [17],

$$\begin{aligned} \sigma^2_{\hat{\phi}} &\simeq \frac{\sigma^2_{\phi}}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\rho(\underset{\sim}{x}_i^{\mathrm{o}} - \underset{\sim}{x}_j^{\mathrm{o}}) \\ &= \sigma^2_{\phi}\gamma(\Delta x, H) \end{aligned} \tag{42}$$

where $\underset{\sim}{x}_i^{\mathrm{o}}$ is the spatial location of the center of the ith soil observation ($i = 1,2,\ldots,m$). See eq. [16] for the definition of σ_{ϕ}.

(5) Because $\mu_{\bar{\phi}} = \mu_{\phi}$ (see eq. [29]), the mean and variance of $\overline{N}_{\mathrm{c}}$ can be obtained in the same fashion as for $\widehat{N}_{\mathrm{c}}$ (in fact, they only differ as a result of different local averaging in the variance calculation). With reference to eqs. [19] and [32]

$$\mu_{\ln \overline{N}_{\mathrm{c}}} = \mu_{\ln \widehat{N}_{\mathrm{c}}} = \mu_{\ln N_{\mathrm{c}}} \tag{43}$$

$$\begin{aligned} \sigma^2_{\ln \overline{N}_{\mathrm{c}}} &\simeq \sigma^2_{\bar{\phi}}\left(\frac{d \ln \overline{N}_{\mathrm{c}}}{d\bar{\phi}}\bigg|_{\mu_{\phi}}\right)^2 \\ &= \sigma^2_{\bar{\phi}}\left\{\frac{bd}{bd^2 - 1}[\pi(1 + a^2)d + 1 + d^2] - \frac{1 + a^2}{a}\right\}^2 \end{aligned} \tag{44}$$

$$\sigma^2_{\bar{\phi}} = \sigma^2_{\phi}\gamma(D) = \sigma^2_{\phi}\gamma(W, W) \tag{45}$$

See previous item for definitions of a, b, and d. The variance reduction function, $\gamma(W,W)$ is defined for 2D by eq. [13] and eq. [16] defines σ_{ϕ}.

(6) The covariance between the observed cohesion values and the equivalent cohesion beneath the footing is obtained as follows for $D = W \times W$ and $Q = \Delta x \times H$:

$$\begin{aligned} \mathrm{Cov}[\ln \bar{c}, \ln \hat{c}] &\simeq \frac{\sigma^2_{\ln c}}{D^2Q^2}\int_D \\ &\quad \times \int_Q \rho(\underset{\sim}{x}_1 - \underset{\sim}{x}_2)\, \mathrm{d}\underset{\sim}{x}_1\, \mathrm{d}\underset{\sim}{x}_2 = \sigma^2_{\ln c}\gamma_{\mathrm{DQ}} \end{aligned} \tag{46}$$

where γ_{DQ} is the average correlation coefficient between the two areas D and Q. The area D denotes the averaging region below the footing over which equivalent properties are defined, and the area Q denotes the region over which soil samples are gathered. These areas are illustrated in Fig. 5. In detail, γ_{DQ} is defined by

$$\begin{aligned} \gamma_{\mathrm{DQ}} &= \frac{1}{(W^2\Delta xH)^2}\int_{-W/2}^{W/2}\int_{H-W}^{H}\int_{r-\Delta x/2}^{r+\Delta x/2} \\ &\quad \times \int_0^H \rho(\xi_1 - x_1, \xi_2 - x_2)\, \mathrm{d}\xi_2\, \mathrm{d}\xi_1\, \mathrm{d}x_2\, \mathrm{d}x_1 \end{aligned} \tag{47}$$

where r is the horizontal distance between the footing centerline and the centerline of the soil sample column. Equation [47] can be evaluated by Gaussian quadrature.

(7) The covariance between $\overline{N}_{\mathrm{c}}$ and $\widehat{N}_{\mathrm{c}}$ is similarly approximated by

$$\mathrm{Cov}[\ln \overline{N}_{\mathrm{c}}, \ln \widehat{N}_{\mathrm{c}}] \simeq \sigma^2_{\ln N_{\mathrm{c}}}\gamma_{\mathrm{DQ}} \tag{48}$$

$$\begin{aligned} \sigma^2_{\ln N_{\mathrm{c}}} &\simeq \sigma^2_{\phi}\left(\frac{d \ln N_{\mathrm{c}}}{d\phi}\Big|_{\mu_{\phi}}\right)^2 \\ &= \sigma^2_{\phi}\left\{\frac{bd}{bd^2 - 1}[\pi(1 + a^2)d + 1 + d^2] - \frac{1 + a^2}{a}\right\}^2 \end{aligned} \tag{49}$$

Substituting these results into eqs. [26] and [27] gives

$$\mu_{\ln Y} = \mu_{\ln L} \tag{50}$$

$$\begin{aligned} \sigma^2_{\ln Y} &= \sigma^2_{\ln L} \\ &\quad + (\sigma^2_{\ln c} + \sigma^2_{\ln N_{\mathrm{c}}})[\gamma(\Delta x, H) + \gamma(W, W) - 2\gamma_{\mathrm{DQ}}] \end{aligned} \tag{51}$$

which can now be used in eq. [24] to produce estimates of p_{f}. If the friction angle is nonrandom, as in a purely cohesive soil where $\phi = 0$, then $\sigma^2_{\ln N_{\mathrm{c}}} = 0$ and eq. [51] simplifies slightly. The primary impact of taking $\phi = 0$ is that the variability of Y is reduced as only one soil parameter is now random. As will be shown shortly, this means that if only cohesion is random, a larger resistance factor can be used, as expected. In other words, the resistance factors presented for the more general c–ϕ case are conservative. Letting

$$q = I(\alpha_{\mathrm{L}}\widehat{L}_{\mathrm{L}} + \alpha_{\mathrm{D}}\widehat{L}_{\mathrm{D}}) \tag{52}$$

allows the probability of failure to be expressed as

$$\begin{aligned} p_{\mathrm{f}} &= \mathbf{P}[Y > q/\phi_{\mathrm{g}}] \\ &= \mathbf{P}[\ln Y > \ln (q/\phi_{\mathrm{g}})] \\ &= 1 - \Phi\left[\frac{\ln (q/\phi_{\mathrm{g}}) - \mu_{\ln Y}}{\sigma_{\ln Y}}\right] \end{aligned} \tag{53}$$

where Φ is the standard normal cumulative distribution function.

Figure 6 illustrates the best and worst agreement between failure probabilities estimated via simulation and those computed using eq. [53]. The failure probabilities are slightly underestimated at the worst case correlation lengths when the sample location is not directly below the footing. Given all the approximations made in the theory, the agreement is very good (within a 10% relative error), allowing the resistance factors to be computed with confidence even at probability levels that the simulation cannot estimate; the simulation involved only 2000 realizations and so cannot properly resolve probabilities much less than 0.001.

Required resistance factor

Equation [53] can be inverted to find a relationship between the acceptable probability of failure, $p_f = p_m$, and the resistance factor, ϕ_g, required to achieve that acceptable failure probability,

$$[54] \qquad \phi_g = \frac{I(\alpha_L \widehat{L}_L + \alpha_D \widehat{L}_D)}{\exp(\mu_{\ln Y} + \sigma_{\ln Y}\beta)}$$

where β is the desired reliability index corresponding to p_m. That is $\Phi(\beta) = 1 - p_m$. For example, if $p_m = 0.001$, then $\beta = 3.09$.

The computation of $\sigma_{\ln Y}$ in eq. [54] involves knowing the size of the averaging domain, D, under the footing. In turn, D depends on the average mean wedge zone depth (by assumption) under the footing, which depends on the mean footing width, $\widehat{\mu}_B$. Unfortunately, the mean footing width given by eq. [31] depends on ϕ_g, so solving eq. [54] for ϕ_g is not entirely straightforward. One possibility is to iterate eq. [54] until a stable solution is obtained. However, the authors have found that eq. [54] is quite insensitive to the initial size of D and using an "average" value of ϕ_g in eq. [31] of 0.7 is quite sufficient. In other words, approximating

$$[55] \qquad \widehat{\mu}_B = \frac{I(\alpha_L \widehat{L}_L + \alpha_D \widehat{L}_D)}{0.7\mu_c \mu_{N_c}}$$

allows $\sigma_{\ln Y}$ to be suitably estimated for use in eq. [54].

In the following, the value of ϕ_g required to achieve three target lifetime failure probability levels (10^{-2}, 10^{-3}, and 10^{-4}) for a specific case (a strip footing founded on a soil with specific statistic parameters) will be investigated. The results are to be viewed relatively. It is well known that the true probability of failure for any design will only be known once an infinite number of replications of that particular design have been observed over infinite time (and thus exposed to all possible loadings). One of the great advantages of probabilistic models is that it is possible to make probabilistic statements immediately, so long as we are willing to accept the fact that the probability estimates are only approximate. In that past history provides a wealth of designs that have been deemed by society to be acceptably reliable (or not, as the case may be), the results presented here need to be viewed relative to past designs so that the acceptable risk levels based on the past many years of experience are incorporated. In other words, the results presented in the following, although rational and based on rigorous research, need to be moderated and adjusted by past experience.

The following parameters will be varied in the simulation study to investigate their effects on the resistance factor required to achieve a target lifetime failure probability p_m:

(1) Three values of p_m are considered, 0.01, 0.001, and 0.0001, corresponding to reliability indices of approximately 2.3, 3.1, and 3.7, respectively.
(2) The correlation length, θ is varied from 0.0 to 50.0 m.
(3) The mean cohesion was set to $\mu_c = 100$ kN/m^2. Four coefficients of variation for cohesion are considered, $v_c = 0.1$, 0.2, 0.3, and 0.5. The s factor for the friction angle distribution (see Fig. 3) is set correspondingly to $s = 1$, 2, 3, and 5. That is, when $v_c = 0.2$, s is set to 2.0, and so on. The friction angle distribution is assumed to range from $\phi_{min} = 0.1745$ radians (10°) to $\phi_{max} = 0.5236$ radians (30°). The corresponding coefficients of variation for friction angle are $v_\phi = 0.07$, 0.14, 0.20, and 0.29.
(4) Three sampling locations are considered: $r = 0$, 4.5, and 9.0 m from the footing centerline (see Fig. 5 for the definition of r).

The design problem considered involves a strip footing supporting loads having means and standard deviations

$$[56a] \qquad \mu_{L_e} = 200\ \text{N/m} \quad \sigma_{L_e} = 60\ \text{kN/m}$$

$$[56b] \qquad \mu_D = 600\ \text{N/m} \quad \sigma_D = 90\ \text{kN/m}$$

Assuming bias factors $k_D = 1.18$ (Becker 1996*b*) and $k_{L_e} = 1.41$ (Allen 1975) gives the characteristic loads

$$[57a] \qquad \widehat{L}_L = 1.41(200) = 282\ \text{kN/m}$$

$$[57b] \qquad \widehat{L}_D = 1.18(600) = 708\ \text{N/m}$$

and the total factored design load (assuming $I = 1$) is

$$[58] \qquad \begin{aligned} q &= I(\alpha_L \widehat{L}_L + \alpha_D \widehat{L}_D) \\ &= 1.5(282) + 1.25(708) \\ &= 1308\ \text{kN/m} \end{aligned}$$

So long as the ratio of dead to live load (assumed to be 3.0 in this study), the coefficients of variation of the load (assumed to be $v_{L_e} = 0.3$ and $v_D = 0.15$), and the characteristic bias factors, k_{L_e} and k_D, are unchanged, the results presented here are independent of the load applied to the strip footing. Minor changes in load ratios, coefficients of variation, and bias factors should not result in significant changes to the resistance factor.

Considering the slightly unconservative underestimation of the probability of failure in some cases (see Fig. 6*b*), it is worthwhile investigating how sensitive eq. [54] is to changes in p_m of the same order as the errors in estimation of p_f. If p_m is replaced by $p_m/1.5$, then this corresponds to underestimating the failure probability by a factor of 1.5, which was well above the maximum difference seen between theory and simulation. It can be seen from Fig. 7, which illustrates the effect of errors in the estimation of the failure probability, that the effect on ϕ_g is minor, especially

Fig. 6. Comparison of failure probabilities estimated from simulation based on 2000 realizations and theoretical estimates using eq. [53]. Plot (*a*) shows probabilities when the soil has been sampled directly under the footing, while (*b*) shows the probabilities when the soil has been sampled 9 m from the footing centerline. Note the change in the vertical scales; the probability of failure is much lower when samples are taken directly under the proposed footing.

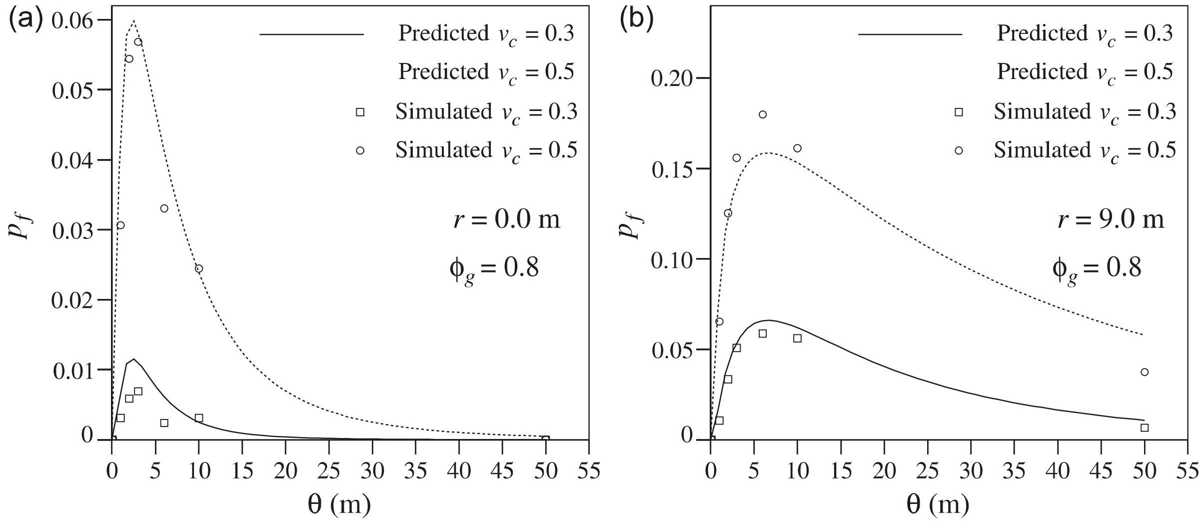
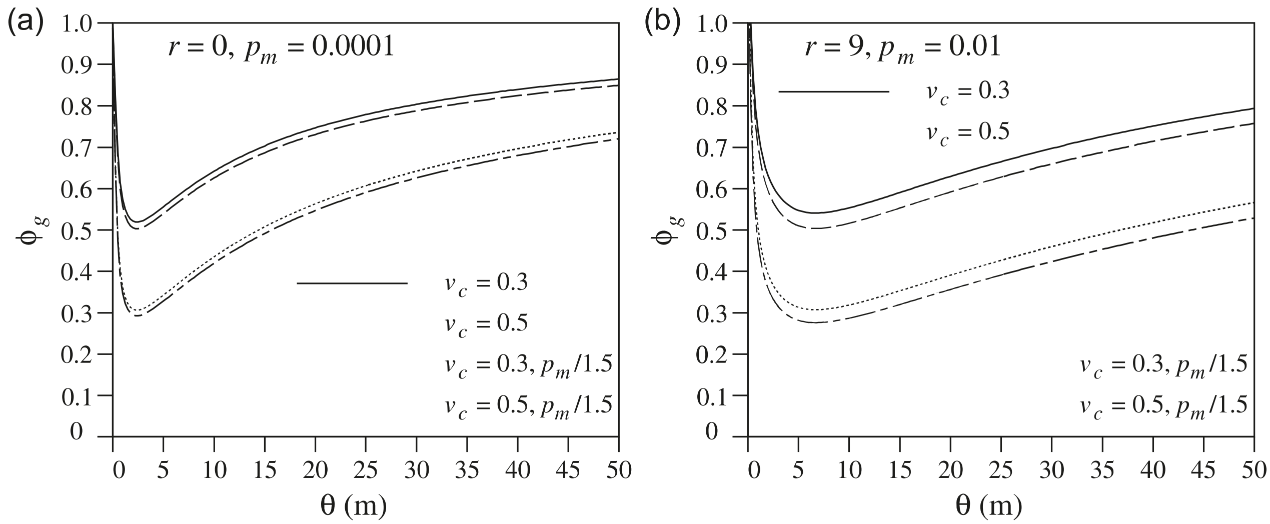


Fig. 7. Effect of failure probability underestimation on the resistance factor required by eq. [54].



considering all other sources of error in the analysis. Of the cases considered in this study, the ϕ_g values least affected by an underestimation of the probability occur when the soil is sampled under the footing (r = 0) and for small p_m, as seen in Fig. 7*a*. The worst case is shown in Fig. 7*b* and all other results (not shown) were seen to lie between these two plots. Even in the worst case of Fig. 7*b*, the change in ϕ_g due to errors in probability estimation are relatively small and will be ignored.

Figures 8, 9, and 10 show the resistance factors required for the cases where the soil is sampled directly under the footing, at a distance of 4.5 m and at a distance of 9.0 m from the footing centerline, respectively, to achieve the three target failure probabilities. The worst case correlation length is clearly between about 1 and 5 m, as evidenced by the fact that in all plots the lowest resistance factor occurs when $1 < \theta < 5$ m. This worst case correlation length is of the same magnitude as the mean footing width ($\widehat{\mu}_B$ = 1.26 m), which can be explained as follows: if the random soil fields are stationary then soil samples yield perfect information, regardless of their location, if the correlation length is either zero (assuming soil sampling involves some local averaging) or infinity. When the information is perfect the probability of a bearing capacity failure goes to zero and $\phi_g \rightarrow 1.0$ (or possibly greater than 1.0 to compensate for the load bias factors). When the correlation length is zero, the soil sample will consist of an infinite number of independent "observations" whose average is equal to the true mean (or true median, if the average is a geometric average). Because the footing also averages the soil properties, the footing "sees" the same true mean (or true median) value predicted by the soil sample. When the correlation length goes to infinity, the soil becomes uniform, having the same value everywhere. In this case, any soil sample also perfectly predicts conditions under the footing.

At intermediate correlation lengths soil samples become

Fig. 8. Resistance factors required to achieve acceptable failure probability, p_m, when soil is sampled directly under the footing (r = 0).

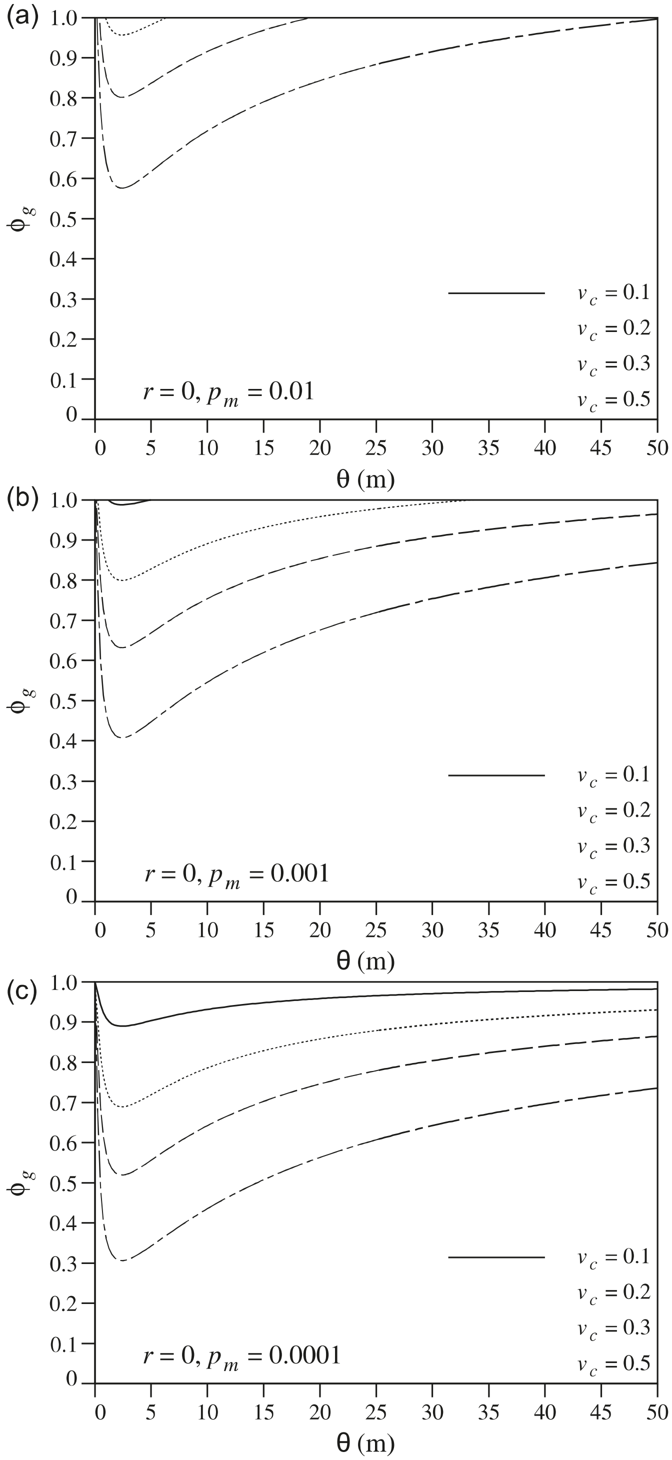
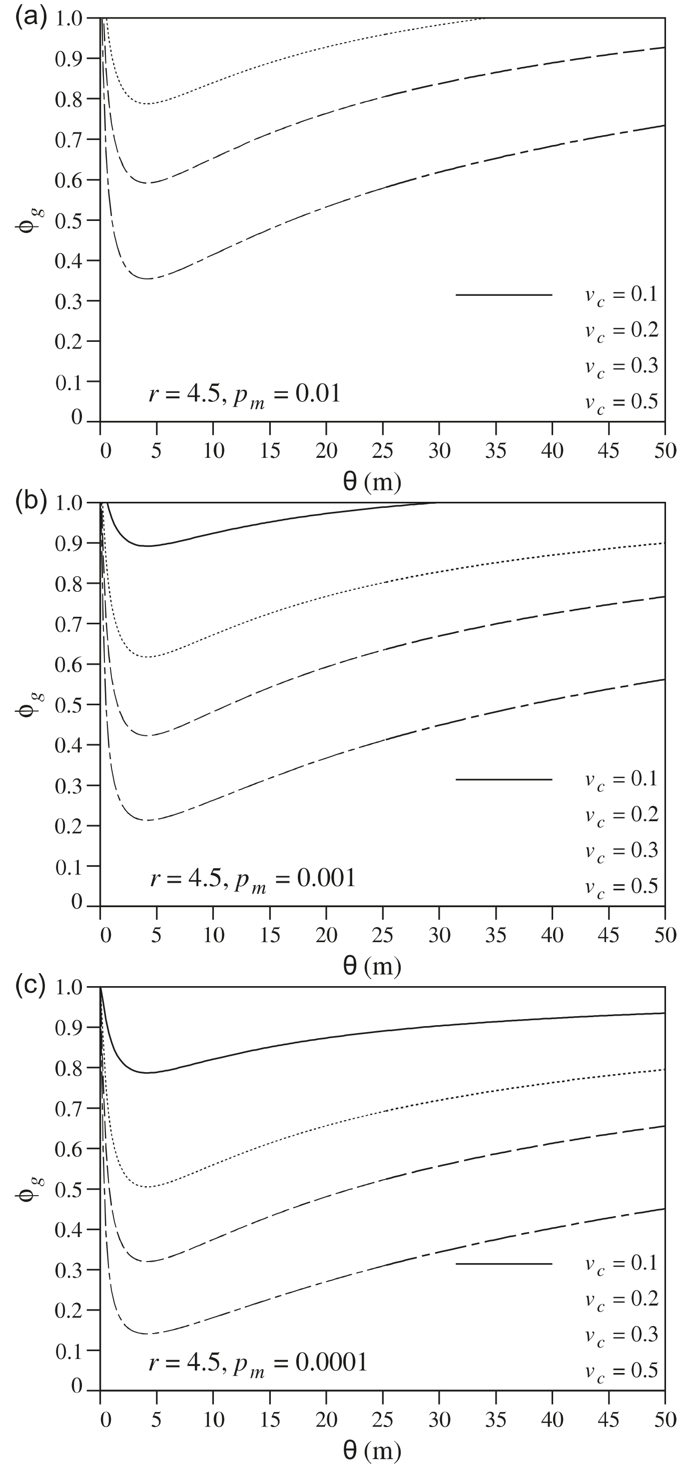


Fig. 9. Resistance factors required to achieve acceptable failure probability, p_m, when soil is sampled at r = 4.5 m from the footing centerline.



imperfect estimators of conditions under the footing, and so the probability of bearing capacity failure increases, or conversely, the required resistance factor decreases. Thus, the minimum required resistance factor will occur at some correlation length between 0 and infinity. The precise value depends on the geometric characteristics of the problem under consideration, such as the footing width, depth to bedrock, length of soil sample, and (or) the distance to the sample point. Notice in Figs. 8, 9, and 10 that the worst case point does show some increase as the distance to the sample location, r, increases.

As expected, the smallest resistance factors correspond with the smallest acceptable failure probability considered, p_m = 0.0001, and with the poorest understanding of the soil

Fig. 10. Resistance factors required to achieve acceptable failure probability, p_m, when soil is sampled at r = 9.0 m from the footing centerline.

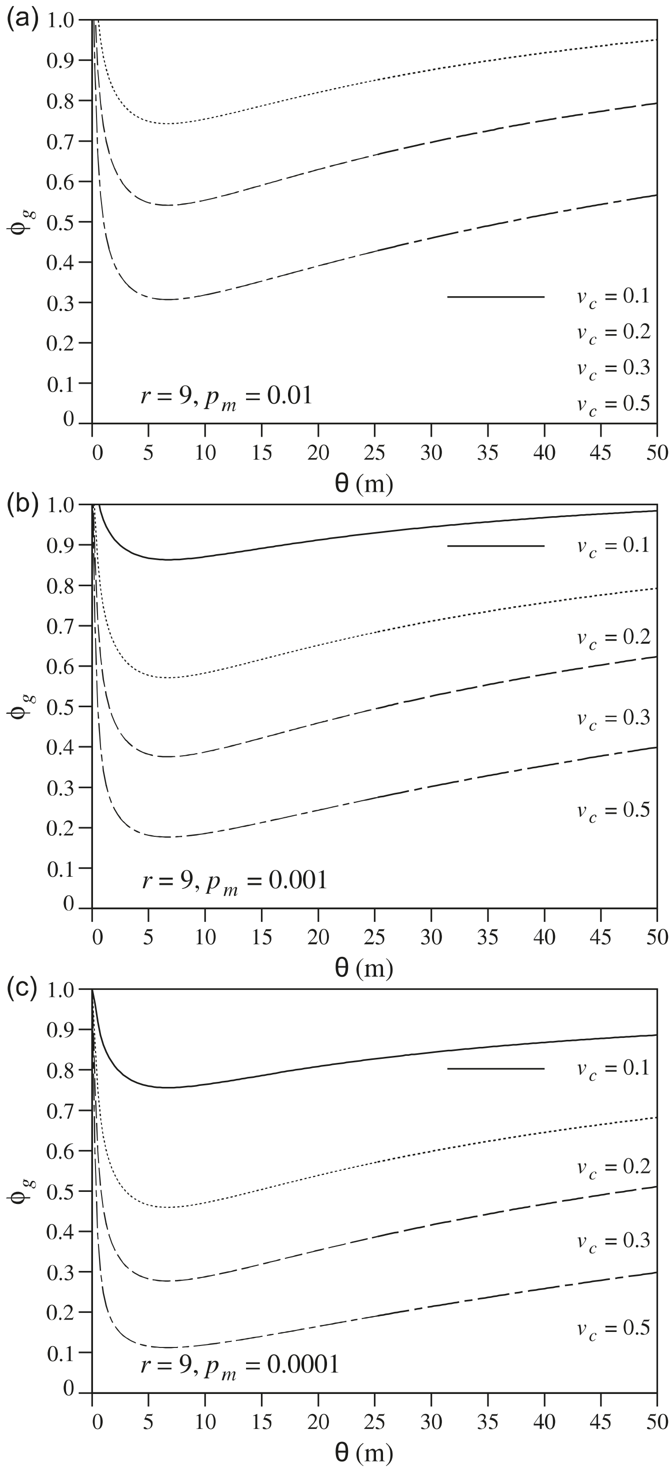
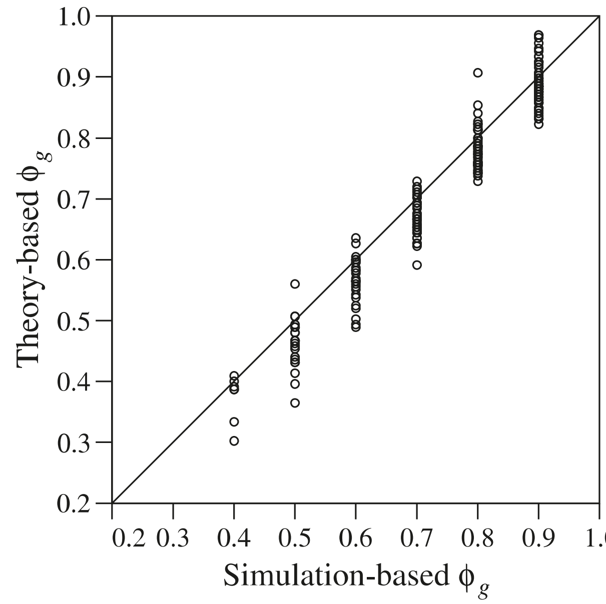


Fig. 11. Required resistance factors, ϕ_g, based on simulation versus those based on eq. [54]. For perfect agreement, the points would all lie on the diagonal.



properties under the footing (i.e., when the soil is sampled 9 m away from the footing centerline). When the cohesion coefficient of variation is relatively large, v_c = 0.5, with corresponding $v_\phi \simeq 0.29$, the worst case values of ϕ_g dip almost down to 0.1 in order to achieve p_m = 0.0001. In other words, there will be a significant construction cost penalty if a high reliability footing is designed using a site investigation that is insufficient to reduce the residual variability to less than v_c = 0.5.

The simulation results can also be used to verify the theoretically determined resistance factors. This is done by using the simulation-based failure probabilities as values of p_m in the theory and comparing the resistance factor, ϕ_g, used in the simulation to that predicted by eq. [54]. The comparison is shown in Fig. 11. For perfect agreement between theory and simulation, the points should align along the diagonal. The agreement is deemed to be very good and much of the discrepancy is due to failure probability estimator error, as discussed next. In general, however, the theory-based estimates of ϕ_g are seen to be conservative, that is somewhat less than seen in the simulations on average.

Those simulations having less than 2 failures out of the 2000 realizations were omitted from the comparison in Fig. 11, as the estimator error for such low probabilities is as large, or larger, than the probability being estimated. In fact, for those simulations having 2 failures out of 2000 (included in Fig. 11), the estimated probability of failure is 0.001, which has standard error $\sqrt{0.001(0.999)/2000} = 0.0007$. This error is almost as large as the probability being estimated, having a coefficient of variation of 70%. In fact most of the discrepancies in Fig. 11 are easily attributable to estimator error in the simulation. The coefficient of variation of the estimator at the 0.01 probability level is 20%, which is still larger than most of the relative errors seen in Fig. 11 (the maximum relative error in Fig. 11 is 0.28 at ϕ_g = 0.5).

The "worst case" resistance factors required to achieve the indicated maximum acceptable failure probabilities, as seen in Figs. 8–10, are summarized in Table 2. In the absence of better knowledge about the actual correlation length at the site in question, these factors are the largest values that should be used in the LRFD bearing capacity design of a strip footing founded on a c–ϕ soil.

Table 2. Worst case resistance factors for various coefficients of variation, v_c, distance to sampling location, r, and acceptable failure probabilities, p_m.

	r = 0.0 m			r = 4.5 m			r = 9.0 m		
v_c	p_m = 0.01	p_m = 0.001	p_m = 0.0001	p_m = 0.01	p_m = 0.001	p_m = 0.0001	p_m = 0.01	p_m = 0.001	p_m = 0.0001
0.1	1.00	0.99	0.89	1.00	0.89	0.79	1.00	0.86	0.76
0.2	0.96	0.80	0.69	0.79	0.62	0.51	0.74	0.57	0.46
0.3	0.80	0.63	0.52	0.59	0.42	0.32	0.54	0.38	0.28
0.5	0.58	0.41	0.31	0.35	0.21	0.14	0.31	0.18	0.11

Table 3. Comparison of resistance factors recommended in this study to those recommended by three other sources.

	Load factors				
	$R_{D/L}$	α_L	α_D	ϕ_g	ϕ_g/α
Current study, r = 0 m	3	1.5	1.25	0.63	0.48
Current study, r = 4.5 m	3	1.5	1.25	0.42	0.32
Current study, r = 9.0 m	3	1.5	1.25	0.38	0.29
Foye et al. (2006)	4	1.6	1.20	0.70	0.54
CGS (2006)	3	1.5	1.25	0.50	0.38
Australian Standard (2004)	3	1.8	1.20	0.45	0.33

It is noted, however, that the factors listed in Table 2 are sometimes quite conservative. For example, when v_c = 0.3, r = 4.5 m, and p_m = 0.001, Table 2 suggests that ϕ_g = 0.42 for the c–ϕ soil considered here. However, if the soil is undrained, with ϕ = 0 (all else being the same), then the only source of variability in the shear strength is the cohesion. In this case the above theory predicts a resistance factor of ϕ_g = 0.60, which is considerably larger than that suggested by Table 2.

To compare the resistance factors recommended in Table 2 to resistance factors recommended in the literature and to current geotechnical LRFD codes, changes in the load factors from code to code need to be taken into account. It will be assumed that all other sources define μ_{L_e}, μ_D, k_{L_e}, and k_D in the same way, which is unfortunately by no means certain. The easiest way to compare resistance factors is to compare the ratio of the resistance factor, ϕ_g, to the total load factor, α. The total load factor, defined for fixed dead to live load ratio, is the single load factor that yields the same result as the individual live and dead load factors, that is, $\alpha(\widehat{L}_L + \widehat{L}_D) = \alpha_L\widehat{L}_L + \alpha_D\widehat{L}_D$. For mean dead to live load ratio $R_{D/L} = \mu_D/\mu_{L_e}$ and characteristic bias factors k_D and k_L,

[59] $$\alpha = \frac{\alpha_L\widehat{L}_L + \alpha_D\widehat{L}_D}{\widehat{L}_L + \widehat{L}_D} = \frac{\alpha_L k_L \mu_{L_e} + \alpha_D k_D \mu_D}{k_L\mu_{L_e} + k_D\mu_D} = \frac{\alpha_L k_L + \alpha_D k_D R_{D/L}}{k_L + k_D R_{D/L}}$$

which, for $R_{D/L}$ = 3, k_L = 1.41, k_D = 1.18, gives α = 1.32. Table 3 compares the ratio of the resistance factors recommended in this study to total load factor ratio with three other sources. The individual "current study" values correspond to the moderate case where v_c = 0.3 and acceptable failure probability p = 0.001. The resistance factor derived from the Australian Standard (2004) on bridge foundations assumes a dead to live load ratio of 3.0 (not stated in the code) and that the site investigation is based on CPT tests.

Apparently the resistance factor recommended by Foye et al. (2006) assumes very good site understanding – they specify that the design assumes a CPT investigation that is presumably directly under the footing. Foye's recommended resistance factor is based on a reliability index of β = 3, which corresponds to p_m = 0.0013, which is very close to that used in Table 3 (p_m = 0.001). The small difference between the "current study" r = 0 result and Foye's may be due to differences in load bias factors – these are not specified by Foye et al.

The resistance factor specified by the *Canadian Foundation Engineering Manual* (CFEM) (CGS 2006) is somewhere between that predicted here for the r = 0 and r = 4.5 m results. The CFEM resistance factor apparently presumes a reasonable, but not significant, understanding of the soil properties under the footing (e.g., $r \simeq 3$ m rather than r = 0 m). The corroboration of the rigorous theory proposed here by an experience-based code provision is, however, very encouraging. The authors also note that the CFEM is the only source listed in Table 3 for which the live and dead load bias factors used in this study can be reasonably assumed to also apply.

The AS 5100.3 standard (Australian Standard 2004) resistance factor ratio is very close to that predicted here using r = 4.5 m. It is probably reasonable to assume that the Australian Standard recommendations correspond to a moderate level of site understanding (e.g., r = 4.5 m) and an acceptable failure probability of about 0.001.

Summary

One of the main impediments to the practical use of this paper is that it depends on an a-priori knowledge of the variance, and, to a lesser extent as "worst case" results are presented herein, the correlation structure of the soil properties. However, assuming that at least one CPT or SPT sounding (or equivalent) is taken in the vicinity of the footing, it is probably reasonable to assume that the residual variability is reduced to a coefficient of variation of no more than about 0.3, and often considerably less (the results collected by other investigators, e.g., Phoon and Kulhawy 1999, suggest that this may be the case for "typical" site investigations). If this is so, the resistance factors recommended in Table 2 for v_c = 0.3 are probably reasonable for the load and bias factors assumed in this study.

The resistance factors recommended in Table 2 are conservative in (at least) the following ways.

(1) It is unlikely that the correlation length of the residual random process at a site (after removal of any mean or mean trend estimated from the site investigation, assuming there is one) will actually equal the "worst case" correlation length.
(2) The soil is assumed to be weightless in this study. The addition of soil weight will reduce the failure probability and so result in higher resistance factors for fixed acceptable failure probability.
(3) Sometimes more than one CPT or SPT sounding is taken at the site in the footing region, so that the site understanding may exceed even the r = 0 m case considered here if trends and layering are carefully accounted for.
(4) Both c and ϕ are assumed to be independent, rather than negatively correlated, which leads to a somewhat higher probability of failure and correspondingly lower resistance factor, and therefore to somewhat conservative results. Because the effect of positive or negative correlation of c and ϕ was found by Fenton and Griffiths (2003) to be quite minor, this is not a major source of conservatism.

On the other hand, the resistance factors recommended in Table 2 are unconservative in (at least) the following ways.

(1) Measurement and model errors are not considered in this study. The statistics of measurement errors are very difficult to determine, as the true values need to be known. Similarly, model errors, which relate both the errors associated with translating measured values (e.g., CPT or SPT measurements to friction angle values) and the errors associated with predicting bearing capacity by an equation such as eq. [5], are quite difficult to estimate simply because the true bearing capacity along with the true soil properties are rarely, if ever, known. In the authors' opinions this is the major source of unconservatism in the presented theory. When confidence in the measured soil properties or in the model used is low, the results presented here can still be employed by assuming that the soil samples were taken further away from the footing location than they actually were (e.g., if low-quality soil samples are taken directly under the footing, r = 0, the resistance factor corresponding to a larger value of r, say r = 4.5 m should be used).
(2) The failure probabilities given by the aforementioned theory are slightly underpredicted when soil samples are taken at some distance from the footing. The effect of this underestimation on the recommended resistance factor has been shown to be relatively minor but nevertheless is slightly unconservative.

To some extent the conservative and unconservative factors listed previously cancel one another out. Figure 11 suggests that the theory is generally conservative if measurement errors are assumed to be insignificant. The comparison of resistance factors presented in Table 3 demonstrates that the "worst case" theoretical results presented in Table 2 agree quite well with current literature and LRFD code recommendations, assuming moderate variability and site understanding, suggesting that the theory is reasonably accurate. In any case, the theory provides an analytical basis to extend code provisions beyond mere calibration with the past.

The results presented in this paper are for a c–ϕ soil in which both cohesion and friction contribute to the bearing capacity, and thus to the variability of the strength. If it is known that the soil is purely cohesive (e.g., "undrained clay"), then the strength variability comes from one source only. In this case, not only does eq. [51] simplify as $\sigma^2_{\ln N_c} = 0$, but because of the loss of one source of variability, the resistance factors increase significantly. The net result is that the resistance factors presented in this paper are conservative when ϕ = 0. Additional research is needed to investigate how the resistance factors should generally be increased for "undrained clays".

The effect of anisotropy on the correlation lengths has not been carefully considered in this study. It is known, however, that increasing the horizontal correlation length above the worst case length is conservative when the soil is not sampled directly below the footing. When the soil is sampled directly under the footing, weak spatially extended horizontal layers below the footing will obviously have to be explicitly handled by suitably adjusting the characteristic soil properties used in the design. If this is done, then the resistance factors suggested here should still be conservative. The theory presented in this paper easily accomodates the anisotropic case.

One of the major advantages to a table such as Table 2 is that it provides geotechnical engineers with evidence that increased site investigation will lead to reduced construction costs and (or) increased reliability. In other words, Table 2 is further evidence that you pay for a site investigation whether you have one or not (Institution of Civil Engineers 1991).

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References

Allen, D.E. 1975. Limit states design—A probabilistic study. Canadian Journal of Civil Engineering, **2**(1): 36–49. doi:10.1139/l75-004.

Australian Standard. 2004. Bridge design, Part 3: Foundations and soil-supporting structures. AS 5100.3-2004. Australian Standard, Sydney, Australia.

Becker, D.E. 1996*a*. Eighteenth Canadian Geotechnical Colloquium: Limit states design for foundations. Part 1. An overview of the foundation design process. Canadian Geotechnical Journal, **33**(6): 956–983. doi:10.1139/t96-124.

Becker, D.E. 1996*b*. Eighteenth Canadian Geotechnical Colloquium: Limit states design for foundations. Part II. Development for the National Building Code of Canada. Canadian Geotechnical Journal, **33**(6): 984–1007. doi:10.1139/t96-125.

CGS. 2006. Canadian foundation engineering manual. 4th ed. Canadian Geotechnical Society, Montreal, Que.

Cherubini, C. 2000. Reliability evaluation of shallow foundation bearing capacity on c', ϕ' soils. Canadian Geotechnical Journal, **37**: 264–269. doi:10.1139/cgj-37-1-264.

Fenton, G.A. 1999. Estimation for stochastic soil models. Journal of Geotechnical and Geoenvironmental Engineering, **125**: 470–485. doi:10.1061/(ASCE)1090-0241(1999)125:6(470).

Fenton, G.A., and Griffiths, D.V. 2003. Bearing capacity prediction of spatially random c–ϕ soils. Canadian Geotechnical Journal, **40**: 54–65. doi:10.1139/t02-086.

Fenton, G.A., and Griffiths, D.V. 2008. Risk assessment in geotechnical engineering. John Wiley & Sons, New York.

Fenton, G.A., Zhang, X.Y., and Griffiths, D.V. 2007. Reliability of shallow foundations designed against bearing failure using LRFD. Georisk, **1**: 202–215.

Foye, K.C., Salgado, R., and Scott, B. 2006. Resistance factors for use in shallow foundation LRFD. Journal of Geotechnical and Geoenvironmental Engineering, **132**: 1208–1218. doi:10.1061/(ASCE)1090-0241(2006)132:9(1208).

Griffiths, D.V., and Fenton, G.A. 2001. Bearing capacity of spatially random soil: The undrained clay Prandtl problem revisited. Géotechnique, **54**: 351–359.

Griffiths, D.V., and Smith, I.M. 2006. Numerical methods for engineers. 2nd ed. Chapman & Hall/CRC Press Inc., Boca Raton, Fla.

Institution of Civil Engineers. 1991. Inadequate site investigation. Thomas Telford, London.

Meyerhof, G.G. 1951. The ultimate bearing capacity of foundations. Géotechnique, **2**: 301–332.

Meyerhof, G.G. 1964. Some recent research on the bearing capacity of foundations. Canadian Geotechnical Journal, **1**: 16–26. doi:10.1139/t63-003.

Meyerhof, G.G. 1970. Safety factors in soil mechanics. Canadian Geotechnical Journal, **7**: 349–355. doi:10.1139/t70-047.

Meyerhof, G.G. 1993. Development of geotechnical limit state design. *In* Proceedings of the International Symposium on Limit State Design in Geotechnical Engineering. Danish Geotechnical Society, Copenhagen, Denmark, 26–28 May 1993.. pp. 1–12.

Meyerhof, G.G. 1995. Development of geotechnical limit state design. Canadian Geotechnical Journal, **32**: 128–136. doi:10.1139/t95-010.

NCHRP. 1991. Manuals for the design of bridge foundations. Report 343. National Cooperative Highway Research Program, Transportation Research Board, NRC, Washington, DC.

NRC. 2005. National Building Code of Canada. National Research Council of Canada, Ottawa, Ont.

NRC. 2006. User's Guide- NBC 2005 Structural Commentaries (Part 4 of Division B). 2nd ed. National Research Council of Canada, Ottawa, Ont.

Phoon, K.-K., and Kulhawy, F.H. 1999. Characterization of geotechnical variability. Canadian Geotechnical Journal, **36**: 612–624. doi:10.1139/cgj-36-4-612.

Prandtl, L. 1921. Uber die Eindringungsfestigkeit (Harte) plastischer Baustoffe und die Festigkeit von Schneiden. Zeitschrift für Angewandte Mathematik und Mechanik, **1**: 15–20.

Simpson, B., Pappin, J.W., and Croft, D.D. 1981. An approach to limit state calculations in geotechnics. Ground Engineering, **14**: 21–28.

Sokolovski, V.V. 1965. Statics of granular media. Pergamon Press, London, UK. 270 pp.

Terzaghi, K. 1943. Theoretical soil mechanics. John Wiley & Sons, New York.

Vanmarcke, E.H. 1984. Random fields: Analysis and synthesis. The MIT Press, Cambridge, Mass.

Wolff, T.H. 1985. Analysis and design of embankment dam slopes: A probabilistic approach. Ph.D. thesis, Purdue University, Lafayette, Ind.

List of symbols

a $\tan \mu_\phi$
A load tributary area
b $e^{\pi a}$
B strip footing width
c cohesion
$\bar{c}$ geometric average of cohesion field over domain D
$\hat{c}$ geometric average of observed (sampled) cohesion values
c_i^o observed (sampled) cohesion value
d $\tan\left(\frac{\pi}{4}+\frac{\mu_\phi}{2}\right)$
D effective soil property averaging domain centered under footing $W \times W$
D_1 x_1 dimension of the averaging domain D
D_2 x_2 dimension of the averaging domain D
$\mathrm{E}[\cdot]$ expectation operator
f_L probability density function of load
$G_{\ln c}$ standard normal random field (log-cohesion)
G_ϕ standard normal random field (underlying friction angle)
H depth to bedrock and depth of assumed soil sample
I importance factor
k_{L_e} extreme lifetime live load bias factor
k_D dead load bias factor
L total true (random) footing load, kN/m
$\hat{L}_i$ ith characteristic load effect
L_D true (random) dead load, kN/m
$\hat{L}_D$ characteristic dead load (= $k_D\mu_D$), kN/m
L_{L_e} true (random) maximum live load over design life, kN/m
$\hat{L}_L$ characteristic live load (= $k_{L_e}\mu_{L_e}$), kN/m
m number of soil observations
N_c N-factor associated with cohesion, which is a function of ϕ
$\bar{N}_c$ effective N-factor associated with cohesion, which is based on an arithmetic average of the friction angle over domain D
$\hat{N}_c$ characteristic N-factor associated with cohesion, which is based on an arithmetic average of the observed friction angles over domain Q (m soil sample observations)
p_f probability of bearing capacity failure
p_m maximum acceptable probability of bearing capacity failure
q_u ultimate bearing stress
$\hat{q}_u$ ultimate bearing stress estimated from characteristic soil properties
q factored design load $(= I(\alpha_L\hat{L}_L + \alpha_D\hat{L}_D))$
Q characteristic soil property averaging domain (= $\Delta x \times H$)
r distance between soil sample and footing center, m
$R_{D/L}$ ratio of mean dead load and mean extreme lifetime live load
$\hat{R}_u$ ultimate resistance based on characteristic soil properties
R_u true ultimate resistance (random)
s scale factor used in distribution of ϕ
v_c coefficient of variation of cohesion
v_D coefficient of variation of dead load
v_L coefficient of variation of total load
v_{L_e} coefficient of variation of extreme lifetime load
v_ϕ coefficient of variation of friction angle
W side dimension of effective averaging domain D
$\underset{\sim}{x}$ spatial coordinate, (x_1,x_2) in 2D
$\underset{\sim}{x}_i^o$ spatial coordinate of the center of the ith soil sample
x_i spatial direction (x_1 and x_2)
Y true load times ratio of estimated to effective bearing capacity
α total load factor
α_i load factor corresponding to the ith load effect
α_L live load factor
α_D dead load factor
β reliability index corresponding to acceptable failure probability, p_m

Δx horizontal dimension of soil samples
ϕ friction angle (radians unless otherwise stated)
ϕ_g resistance factor
$\overline{\phi}$ arithmetic average of ϕ over domain D
$\widehat{\phi}$ arithmetic average of the m observed friction angles
ϕ_{min} minimum friction angle
ϕ_{max} maximum friction angle
ϕ_i^o ith observed friction angle
Φ standard normal cumulative distribution function
$\gamma(D)$ common variance function giving variance reduction due to averaging over domain D
$\gamma_{\ln c}(D)$ variance function giving variance reduction due to averaging log-cohesion over domain D
$\gamma_{\phi}(D)$ variance function giving variance reduction due to averaging G_ϕ over domain D
γ_{DQ} average correlation coefficient between domains D and Q
μ_c cohesion mean
$\mu_{\ln c}$ log-cohesion mean
$\mu_{\ln \widehat{c}}$ mean of the estimate of log-cohesion based on a geometric average of cohesion observations
$\mu_{\ln \overline{c}}$ mean of the effective log-cohesion based on a geometric average of cohesion over domain D
μ_{N_c} mean of N_c
$\mu_{\ln \widehat{N}_c}$ mean of $\ln \widehat{N}_c$
$\mu_{\ln \overline{N}_c}$ mean of $\ln \overline{N}_c$
μ_D mean dead load
μ_L mean total load on strip footing, kN/m
μ_{L_e} mean extreme live load over design life
$\mu_{\ln L}$ mean total log-load on strip footing
μ_ϕ mean friction angle
$\mu_{\widehat{\phi}}$ mean of estimated friction angle
$\mu_{\overline{\phi}}$ mean of effective friction angle in zone of influence under footing
$\mu_{\ln Y}$ mean of $\ln Y$
$\widehat{\mu}_B$ estimated mean footing width
ξ_1, ξ_2 dummy variables used in eq. [47]
θ correlation length of the random fields
$\theta_{\ln c}$ correlation length of the log-cohesion field
θ_ϕ correlation length of the G_ϕ field
$\rho(\underset{\sim}{\tau})$ common correlation function
$\rho_{\ln c}(\underset{\sim}{\tau})$ correlation function giving correlation between two points in the log-cohesion field
$\rho_\phi(\underset{\sim}{\tau})$ correlation function giving correlation between two points in the G_ϕ field
σ_c cohesion standard deviation
σ_D dead load standard deviation
σ_{L_e} standard deviation of extreme lifetime live load
$\sigma_{\ln L}$ standard deviation of total log-load
$\sigma_{\ln c}$ log-cohesion standard deviation
$\sigma_{\ln \overline{c}}$ standard deviation of $\ln \overline{c}$
$\sigma_{\ln \widehat{c}}$ standard deviation of $\ln \widehat{c}$
σ_ϕ standard deviation of ϕ
$\sigma_{\overline{\phi}}$ standard deviation of $\overline{\phi}$
$\sigma_{\ln N_c}$ standard deviation of $\ln N_c$
$\sigma_{\ln \overline{N}_c}$ standard deviation of $\ln \overline{N}_c$
$\sigma_{\ln \widehat{N}_c}$ standard deviation of $\ln \widehat{N}_c$
$\sigma_{\ln Y}$ standard deviation of $\ln Y$
$\underset{\sim}{\tau}$ vector between two points in the soil domain
τ_1 horizontal component of the distance between two points in the soil domain
τ_2 vertical component of the distance between two points in the soil domain