Probabilistic Analysis of Coupled Soil Consolidation

Jinsong Huang, M.ASCE[1]; D. V. Griffiths, F.ASCE[2]; and Gordon A. Fenton, M.ASCE[3]

Abstract: Coupled Biot consolidation theory was combined with the random finite-element method to investigate the consolidation behavior of soil deposits with spatially variable properties in one-dimensional (1D) and two-dimensional (2D) spaces. The coefficient of volume compressibility (m_v) and the soil permeability (k) are assumed to be lognormally distributed random variables. The random fields of m_v and k are generated by the local average subdivision method which fully takes account of spatial correlation, local averaging, and cross correlations. The generated random variables are mapped onto a finite-element mesh and Monte Carlo finite-element simulations follow. The results of parametric studies are presented, which describe the effect of the standard deviation, spatial correlation length, and cross correlation coefficient on output statistics relating to the overall "equivalent" coefficient of consolidation. It is shown that the average degree of consolidation defined by excess pore pressure and settlement are different in heterogeneous soils. The dimensional effect on the soil consolidation behaviors is also investigated by comparing the 1D and 2D results.

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Introduction

It has long been realized (Rowe 1972) that reliable prediction of consolidation rates in soil deposits is difficult due to variable properties and soil fabric. In recent years, many studies have been made to quantify soil variability and assess the resulting uncertainty for various applications. There have been a few studies that used stochastic approaches to investigate coupled consolidation and settlement problems. Freeze (1977) dealt with one-dimensional (1D) consolidation by taking account of the cross correlation between the coefficient of volume compressibility and the soil permeability, which were chosen randomly from probability density functions. Hwang and Witczak (1984) investigated the dimensional effect on soil consolidation. Chang (1985) investigated the influence of a gamma-distributed coefficient of consolidation (c_v) on 1D layered systems. Hong (1992) also studied 1D consolidation by treating c_v as a random variable. Darrag and Tawil (1993) analyzed a similar Terzaghi-type problem and introduced a variable initial pore pressure. Hong and Shang (1998) analyzed consolidation with prefabricated vertical drains for soil improvement. Most recently, Badaoui et al. (2007) investigated 1D consolidation by the thin layer method combined with Monte Carlo simulations. There are also related works that dealt with foundation problems using probabilistic approaches (see, e.g., Zeitoun and Baker 1992; Brzakala and Puła 1996). Cassiani and Zoccatelli (2000) and Frias et al. (2004) analyzed stochastic subsidence problems by taking account of a reservoir's heterogeneous properties.

Although the previously mentioned works have dealt with probabilistic soil consolidation over a wide range of situations, there are virtually no coupled two-dimensional (2D) studies and there have been no systematic studies of the problem over a range of parametric variations. This paper will fill in this gap and cover the following topics.

1. Both the coefficient of volume compressibility and the soil permeability play important roles in the consolidation of heterogeneous soil but they cannot be embodied into a single coefficient of consolidation (see, e.g., Lee et al. 1992 ; Pyrah 1996). Both m_v and k will be treated as random variables and the coupled Biot consolidation theory (Biot 1941) will be applied;
2. The average degree of consolidation as defined by excess pore pressure is known to be different from that defined by settlement in heterogeneous soil (see, e.g., Lee et al. 1992). These differences will be fully investigated in a probabilistic framework to emphasize that using the average degree of consolidation defined by excess pore pressure to predict settlement rates will give misleading results;
3. Spatial correlation of random variables will be taken into account systematically using random field theory (see, e.g., Griffiths and Fenton 1993; Fenton and Griffiths 2008) and will be shown to have a considerable influence on soil consolidation behavior;
4. The influence of cross correlation between random variables m_v and k on the soil consolidation will be investigated by parametric studies; and
5. Both 1D and 2D random consolidation analyses will be performed and results contrasted.

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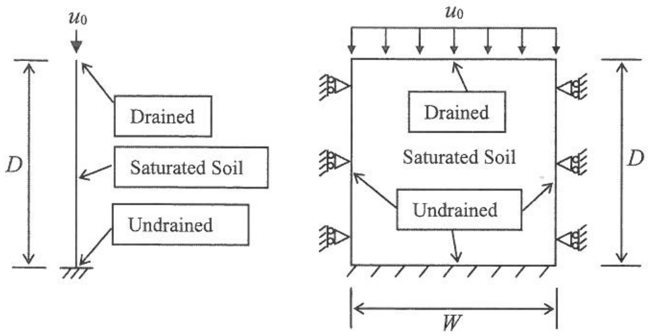

Fig. 1. Geometry of analyzed 1D and 2D consolidation test problems

Review of Terzaghi's 1D and Biot Consolidation Theories

The dimensions and boundary conditions of the 1D and 2D test problems that will be analyzed in this paper are shown in Fig. 1. The soil skeleton is treated as a porous elastic solid in which the pore fluid is coupled to the solid by the conditions of equilibrium and continuity. The permeability and compressibility are related to changes in void ratio and effective stresses. The changes in permeability and compressibility can significantly affect the generation and dissipation of excess pore-water pressure and the rate of settlement. Comprehensive formulations that account for both material nonlinearity and finite strains have been developed (e.g., Gibson et al. 1967; Schiffman et al. 1984). Although solutions that deal with the fully nonlinear problem are available (e.g., Cornetti and Battaglio 1994; Schiffman et al. 1996; Lamcellotta and Preziosi 1997; Morris 2002), these aspects are not considered in the current paper. The remainder of this section uses the same notation and is a condensed version of the review paper by Griffiths (1994).

First, for 2D equilibrium in the absence of body forces, the gradient of effective stress must be combined with gradients of the fluid pressure as follows:

$$\frac{\partial \sigma_x'}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial u_w}{\partial x} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y'}{\partial y} + \frac{\partial u_w}{\partial y} = 0 \qquad (1)$$

where $\sigma_x' = \sigma_x - u_w$ and $\sigma_y' = \sigma_y - u_w$=effective stresses; τ_{xy}=shear stress; and u_w=excess pore pressure.

Assuming plane strain condition and small strains, and following the usual sequence of operations for a displacement method, the stress terms in Eq. (1) can be presented in terms of displacements to give

$$\frac{E'(1-\nu')}{(1+\nu')(1-2\nu')}\left[\frac{\partial^2 u_x}{\partial x^2} + \frac{(1-2\nu')}{2(1-\nu')}\frac{\partial^2 u_x}{\partial y^2} + \frac{1}{2(1-\nu')}\frac{\partial^2 u_y}{\partial x\,\partial y}\right] + \frac{\partial u_w}{\partial x} = 0$$

$$\frac{E'(1-\nu')}{(1+\nu')(1-2\nu')}\left[\frac{1}{2(1-\nu')}\frac{\partial^2 u_x}{\partial x\,\partial y} + \frac{\partial^2 u_y}{\partial y^2} + \frac{(1-2\nu')}{2(1-\nu')}\frac{\partial^2 u_y}{\partial x^2}\right] + \frac{\partial u_w}{\partial y} = 0 \qquad (2)$$

where E' and ν'=effective elastic parameters and u_x and u_y =displacements in the x- and y-directions, respectively.

Second, from continuity, and assuming fluid incompressibility, the net flow rate equals the rate of change of volume of the element of soil, such that

$$\frac{\partial}{\partial t}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) + \frac{k_x}{\gamma_w}\frac{\partial^2 u_w}{\partial x^2} + \frac{k_y}{\gamma_w}\frac{\partial^2 u_w}{\partial y^2} = 0 \qquad (3)$$

where γ_w=unit weight of water and k_x and k_y=soil permeabilities in the x- and y-directions, respectively.

Eqs. (2) and (3) represent the two "Biot" equations for a 2D poroelastic material. A solution to these equations will enable the displacements u_x and u_y and excess pore pressure u_w to be estimated at spatial location (x,y) at a given time t.

In a 1D system, if m_v and k/γ_w are constant throughout the soil layers, the Terzaghi's 1D consolidation equation is

$$c_v\frac{\partial^2 u_w}{\partial z^2} = \frac{\partial u_w}{\partial t} \qquad (4)$$

where c_v=coefficient of consolidation as defined by

$$c_v = \frac{k}{m_v \gamma_w} \qquad (5)$$

where m_v=coefficient of volume compressibility. As mentioned in the Introduction, however, the uncoupled Eq. (4) is not applicable to 1D layered systems.

Average Degree of Consolidation of Heterogeneous Soil

The average degree of consolidation of a coupled system can be expressed in terms of excess pore pressure or settlement. In the 1D case, if the initial (uniform) excess pore pressure is given by u_0 and the maximum drainage path by D, the average degree of consolidation defined by excess pore pressure is

$$U_{avp} = 1 - \frac{1}{D}\int_0^D \frac{u}{u_0}dz \qquad (6)$$

The average degree of consolidation defined by settlement is

$$U_{avs} = \frac{s_t}{s_u} \qquad (7)$$

where s_u=long term (ultimate) settlement and s_t=settlement at time t.

In the 2D heterogeneous cases, the settlement will be different in different places so average values of s_t and s_u will be used to quantify the average degree of consolidation U_{avs}. The U_{avp} in 2D cases is defined as

$$U_{avp} = 1 - \frac{1}{DW}\int_0^D \int_0^W \frac{u}{u_0}dxdy \qquad (8)$$

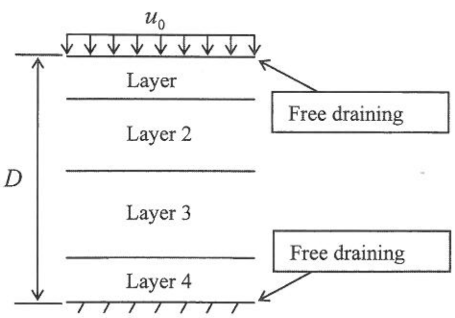

Fig. 2. Four-layer system [used by Schiffman and Stein (1970)]

Validation of the Coupled 1D Computer Program

2D coupled consolidation Program 9.3 in the text by Smith and Griffiths (2004) which was well tested will be used directly for the 2D analyses. A 1D coupled consolidation finite-element program was developed using two-node "rod" elements in the style of Program 9.3 and will be validated in this section by an example used by Schiffman and Stein (1970).

The soil profile shown in Fig. 2 consists of four compressible layers with free drainage permitted at the top and bottom boundaries. The geotechnical data of the system are shown in Table 1 where h_i is the thickness of the ith layer. A unit load u_0 is applied to the top surface at time $t=0$ and maintained at that value.

The calculated excess pore pressures at different times after loading are shown in Fig. 3. It can be seen that the coupled results are essentially the same as those presented by Schiffman and Stein (1970). Also plotted in Fig. 3 are the uncoupled results which were obtained by Terzaghi's 1D consolidation [Eq. (4)] and the c_{vi} values listed in Table 1. 1D uncoupled 8.1 in the text by Smith and Griffiths (2004) was used. The uncoupled results are quite different from the coupled ones.

Fig. 4 plots the average degree of consolidation as a function of time. It can be seen that the coupled approach predicts faster consolidation with respect both to settlement and excess pore pressure in comparison to the uncoupled (Terzaghi) approach. Comparing the coupled results, it can be seen from Fig. 4 that U_{avp} is always smaller than U_{avs}. For the example presented above, if coupled U_{avp} is used to predict settlement, a conservative result (smaller settlement) will be obtained. It should be mentioned that even when the four layers have same value of c_{vi}, but different compressibility and permeability characteristics, U_{avp} and U_{avs} will not only be different from each other, but different from the value predicted by the Terzaghi approach.

Random Finite-Element Method

Soil is one of the most inherently variable as it exists in its natural state. The most realistic interpretation that can be placed on measured values from a site exploration programs is in terms of the probability density functions for each properties, together with the correlation relationships between the properties and the possible spatial trends or autocorrelation structures that may be present. The previous section shows that coupled theory must be adopted for heterogeneous soil consolidation. This section will show the procedure that RFEM (Fenton and Griffiths 2008) is used to statistically predict consolidations of soils.

The RFEM involves the generation and mapping of a random field of properties onto a finite-element mesh. Full account is taken of local averaging and variance reduction (Fenton and Vanmarcke 1990) over each element, and an exponentially decaying (Markov) spatial correlation function is incorporated. The random field is initially generated and properties assigned to the elements. The coupled consolidation analysis follows. The analysis is repeated numerous times using Monte Carlo simulations. Each simulation of the Monte Carlo process involves the same mean, standard deviation, and spatial correlation length of soil proper-

Table 1. Data Used in the Four-Layer System (Schiffman and Stein 1970)

Layer number	h_i (m)	k_i (cm/s)	m_{vi} (kPa^{-1})	c_{vi} (cm^2/s)
1	3.05	2.78×10^{-9}	6.41×10^{-5}	4.42×10^{-4}
2	6.10	8.25×10^{-9}	4.08×10^{-5}	2.06×10^{-3}
3	9.14	1.17×10^{-9}	2.04×10^{-5}	5.85×10^{-4}
4	6.10	2.94×10^{-9}	4.08×10^{-5}	7.35×10^{-4}

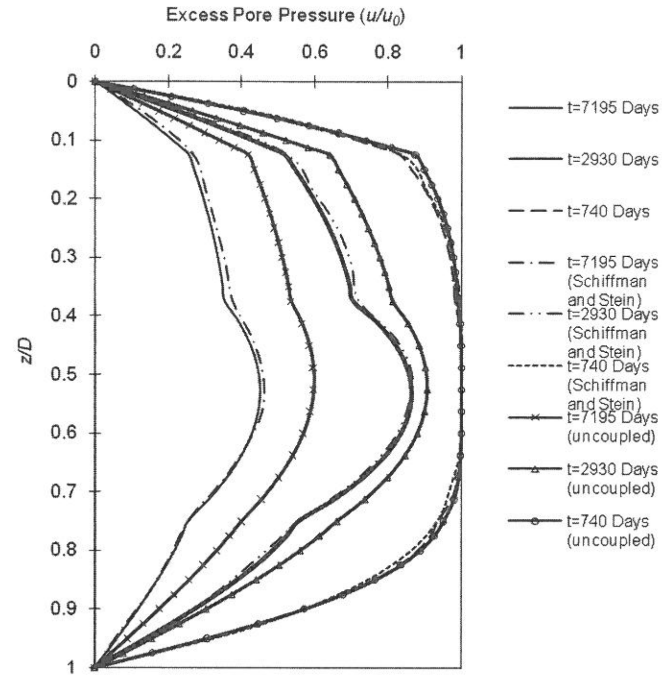

Fig. 3. Excess pore pressure isochrones

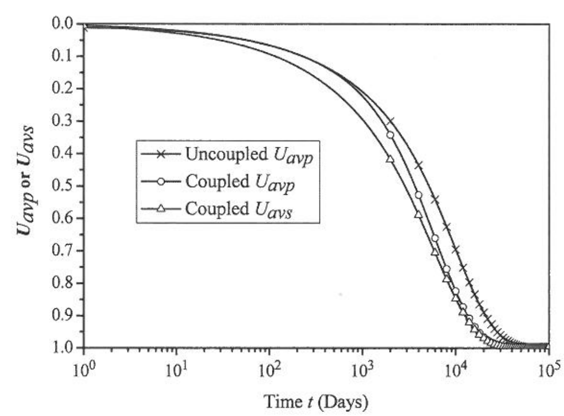

Fig. 4. Coupled and uncoupled results of the four-layer system [used by Schiffman and Stein (1970)]

ties; however the spatial distribution of properties varies from one simulation to the next. Following a "sufficient" number of simulations, output quantities of interest, such as the settlement and the average degree of consolidation, can be assimilated and statistically analyzed to produce estimates of probability density functions and ultimately probabilities of events such as, for example, excessive settlement. The analysis has the option of including cross correlation between properties and anisotropic spatial correlation lengths (e.g., the spatial correlation length in a naturally occurring stratum of soil is often higher in the horizontal direction). Further details of RFEM can be found in Fenton and Griffiths (2008).

Generation of the Coefficient of Volume Compressibility and the Soil Permeability Values

The coefficients of volume compressibility and the soil permeability are assumed to be characterized statistically by lognormal distributions. The lognormal distribution will be applied at the point level. The lognormal distribution is one of many possible choices (e.g., Fenton and Griffiths 2008); however it offers the advantage of simplicity, in that it is arrived by a simple nonlinear transformation of the classical normal (Gaussian) distribution. Lognormal distributions guarantee that the random variable is always positive. Soil permeability was assumed to follow a lognormal distribution (see, e.g., Freeze 1977; Hoeksema and Kitanidis 1985; Sudicky 1986; Yang et al. 1996; Gui et al. 2000). Volume compressibility was also assumed to be lognormally distributed (see, e.g., Freeze 1975; Freeze 1977). Only the procedure to generate the random soil permeability is summarized here. The same approach was used to generate coefficient of volume compressibility values.

The lognormally distributed soil permeability is characterized by its mean μ_k, its standard deviation σ_k, and its correlation length $\theta_{\ln k}$. The mean and standard deviation can conveniently be expressed in terms of the dimensionless coefficient of variation defined as

$$v_k = \frac{\sigma_k}{\mu_k} \tag{9}$$

Other useful relationships relating to the lognormal function include the standard deviation and mean of the underlying normal distribution (of ln k) as follows:

$$\sigma_{\ln k} = \sqrt{\ln\{1 + v_k^2\}} \tag{10}$$

$$\mu_{\ln k} = \ln \mu_k - \frac{1}{2}\sigma_{\ln k}^2 \tag{11}$$

The soil permeability field is obtained through the transformation

$$k_i = \exp\{\mu_{\ln k} + \sigma_{\ln k} g_{\ln k,i}\} \tag{12}$$

in which k_i=permeability of the ith element; $g_{\ln k,i}$=local average of a standard Gaussian random field, $g_{\ln k}$, over the domain of the ith element; and $\mu_{\ln k}$ and $\sigma_{\ln k}$=mean and standard deviation of the logarithm of k_i (obtained from the "target" mean and standard deviation μ_k and σ_k).

The local average subdivision method (Fenton and Vanmarcke 1990) renders simulations of the local averages $g_{\ln k,i}$, which are derived from the random field $g_{\ln k}$ having zero mean, unit variance, and a spatial correlation controlled by the correlation length. Random number generator RAN2 (Press et al. 1992) is used. It should be mentioned that when there are more than one random fields (e.g., $g_{\ln k}$ and $g_{\ln m_v}$) and the cross correlations need to be considered, $g_{\ln k}$ and $g_{\ln m_v}$ will be adjusted according to the cross correlation (see, e.g., Fenton and Griffiths 2008). The spatial correlation length is measured with respect to this underlying field, that is, with respect to ln k. In particular, the spatial correlation length ($\theta_{\ln k}$) describes the distance over which the spatially random values will tend to be significantly correlated in the underlying Gaussian field. Thus, a large value of $\theta_{\ln k}$ will imply a smoothly varying field, while a small value will imply a ragged field. A dimensionless correlation length Θ is defined as

$$\Theta = \frac{\theta_{\ln k}}{D} \tag{13}$$

where $\theta_{\ln k}$ and D=spatial correlation length and maximum drainage path, respectively.

In the 2D analyses presented in this paper, the correlation lengths in the vertical and horizontal directions are taken to be equal (isotropic) for simplicity. Although beyond the scope of this paper, it should be noted that for a layered soil mass the horizontal scale of fluctuation is generally larger than the vertical scale due to the natural stratification of many soil deposits. The 2D model used herein implies that the out-of-plane scale of fluctuation is infinite—soil properties are constant in this direction.

Probabilistic Analyses

The 1D layered system shown in Fig. 1 was meshed by a string of 100 elements attached end to end. The total depth is 1.0. The system is subjected to an instantaneous compressive load of 100.0 at the drained end. Deformations are free to occur at the drained end and are fixed to zero at the undrained end.

The 2D profile shown in Fig. 1 with W=1.0 and D=1.0 was meshed with 400 elements (20×20) and subjected to an instantaneous compressive load of 100.0 at the drained top boundary. As in the 1D case, deformations are free to occur at the drained top boundary and are fixed at the bottom with rollers on both undrained side boundaries.

Time Step for Heterogeneous Soil

When performing Monte Carlo simulations of heterogeneous soil consolidation, the soil properties vary from simulation to simulation so a suitable time step must be reevaluated in each simulation. In the current work, a simple extrapolation/interpolation approach has been used as follows, which is performed at the beginning of each simulation.

1. i=0, initial guess value of time step Δt_0 (0.002);
2. Perform a single calculation step with θ=1.0 [fully implicit, see Smith and Griffiths (2004)] and Δt_0 to estimate the average degree of consolidation $U_{avs,0}$ [Eq. (6) or Eq. (8)] and $U_{avp,0}$ [Eq. (7)]. $U_{av,0}=\max\{U_{avp,0}, U_{avs,0}\}$;
3. $i=i+1$, estimate the time Δt_i to achieve $U_{av,i}$=0.5 using $\Delta t_i = \Delta t_{(i-1)}(0.5/U_{av,i-1})^2$;
4. Perform a single calculation step with θ=1.0 (fully implicit) and Δt_i to estimate the average degree of consolidation $U_{av,i}$;
5. If $0.45 \le U_{av,i} \le 0.55$, exit; otherwise repeat Steps 3 and 4; and
6. Set $\Delta t = \Delta t_i/100$ and use θ=0.5 [Crank-Nicolson, see Smith and Griffiths (2004)] to perform the simulation.

This algorithm ensures that a reasonable number of calculation time steps (approximately 100) will be needed for each simulation to reach an average degree of consolidation of 50%.

"Equivalent" Coefficient of Consolidation

Every simulation of Monte Carlo simulations has different distributions of m_v and k. Treating every simulation as a test sample, the equivalent coefficient of consolidation may be determined by the log-time method (Casagrande 1936) or by the root-time method (Taylor 1948).

In the log-time method (Casagrande 1936) the coefficient of consolidation is determined by

$$c_{vt50} = \frac{0.197D^2}{t_{50}} \tag{14}$$

where t_{50}=time corresponding to U_{avs}=50% or U_{avp}=50%, and in the root-time method (Taylor 1948) the coefficient of consolidation is determined by

$$c_{vt90} = \frac{0.848D^2}{t_{90}} \tag{15}$$

where t_{90}=time corresponding to U_{avs}=90% or U_{avp}=90%. Following a suite of Monte Carlo simulations, the mean value of the equivalent coefficients of consolidation is easily quantified. For example, the mean value of the equivalent coefficient of consolidation defined by excess pore pressure from the log-time method is

$$\mu_{t50p} = \frac{1}{n_{\text{sim}}}\sum_{i=1}^{n_{\text{sim}}} c_{vt50,i} \tag{16}$$

where n_{sim}=number of simulations and $c_{vt50,i}$=equivalent coefficient of consolidation obtained by Eq. (14) from the ith simulation. In the 1D model, draining pore water must flow in series through many different layers, in which case the equivalent coefficient of consolidation can be estimated by taking the harmonic average of all the coefficients of consolidation, namely

$$c_{vhm} = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{c_{vi}}} \tag{17}$$

where n=number of layers and

$$c_{vi} = \frac{k_i}{m_{vi}\gamma_w} \tag{18}$$

is the coefficient of consolidation of the ith element (layer). The mean value of the harmonic average of coefficients of consolidation following n_{sim} simulations can be obtained as

$$\mu_{hm} = \frac{1}{n_{\text{sim}}}\sum_{i=1}^{n_{\text{sim}}} c_{vhm,i} \tag{19}$$

where $c_{vhm,i}$=harmonic average of coefficients of consolidation obtained by Eq. (17) for the ith simulation. In a 2D system, water flow has more options for escaping from the system hence the geometric average may be a better estimator (e.g., Dagan 1989) of the equivalent coefficient of consolidation where

$$c_{vgm} = \left(\prod_{i=1}^{n} c_{vi}\right)^{1/n} \tag{20}$$

It should be mentioned that both the harmonic and geometric averaging methods given above ignore coupling effects by dealing only with c_{vi} from Eq. (18).

Results of Parametric Studies

Parametric studies were performed to investigate the sensitivity of the equivalent coefficients of consolidation to the statistically defined input data relating to both m_v and k. The means and coefficients of variation of the equivalent coefficients of consolidation will be quantified, with appropriate subscripts "*hm*," "*gm*," "*t50p*," "*t90p*," "*t50s*," and "*t90s*," where *hm*, and *gm* refer to the harmonic average and geometric average, *t50* and *t90s* refer to the log-time and root-time methods, and "*p*" and "*s*" refer to the average degree of consolidation as defined by excess pore pressure and settlement, respectively.

The following parameters have been used for the parametric studies in this paper, and for the sake of simplicity, the unit weight of water has been set to 1.0 in all cases.

- v=0.125, 0.25, 0.5, 1.0, 2.0, and 4.0 (both m_v and k);
- Θ=0.125, 0.25, 0.5, 1.0, and 2.0;
- μ=1.0 (both m_v and k); and
- Cross correlation coefficient between m_v and k, ρ=−1.0, 0.0, and 1.0.

In reality, permeability is one of the most variable soil properties and its coefficient of variation is generally much higher than that of volume compressibility. The reason for using the same value of v for both m_v and k in this paper is for comparison purposes only. From Eq. (18) and the table of parametric variations shown above, it can be seen that the coefficient of consolidation of a typical element will always equals 1.0 if m_v and k have the same coefficient of variation and are perfectly positively correlated with ρ=1.0. In an uncoupled (Terzaghi) approach, both 1D and 2D analyses with these parameters will give same equivalent coefficient of consolidation of 1.0 at each simulation; hence the mean equivalent coefficient of consolidation will also be exactly equal to 1.0 (with a variance of zero), whether it is based on the log-time, root-time, harmonic, or geometric average. This example can offer a particularly clear demonstration of the difference between coupled and uncoupled (Terzaghi) analyses.

The input parameters relating to the mean, standard deviation, and spatial correlation length are assumed to be defined at the "point" level. While statistics at this resolution are obviously impossible to measure in practice, they represent a fundamental baseline of the inherent soil variability which can be corrected through local averaging to take account of the sample size. At the point scale, one could just as easily be inside a void (very high permeability and compressibility) or inside a chunk of granite (very low permeability and compressibility). It is only when you start averaging that the range begins to reduce. In other words, a point scale v of 1.0 will be smaller after averaging (and even smaller for small correlation lengths). For the purposes of parametric studies, v was pushed up to quite high levels in some cases.

There are little data available to indicate the level of correlation between m_v and k. Strong positive correlations between the compressibility and permeability for fine-grained dredged materials were shown by Morris (2003); however, Freeze (1977) used

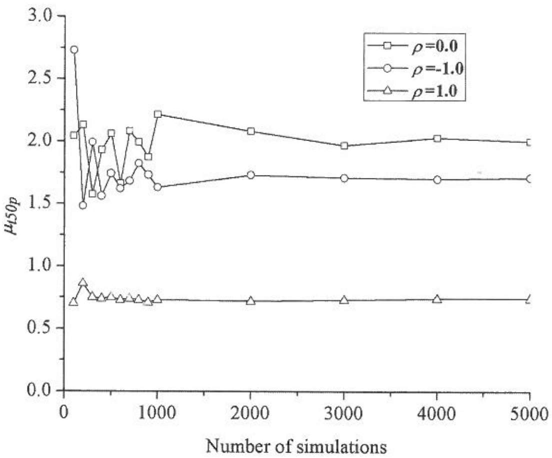

Fig. 5. Influence of the number of Monte Carlo simulations on μ_{t50p} (1D analyses, $\mu=1.0$, $v=4.0$, and $\Theta=1.0$)

$\rho=-0.9$. For the purpose of parametric study in this paper, the cross correlation coefficient between m_v and k was set to -1.0, 0.0, and 1.0.

To maintain accuracy and run-time efficiency, the sensitivity of results to the number of Monte Carlo simulations was examined. The "worst" cases with the highest input $v=4.0$ and $\Theta=1.0$ were chosen to investigate the effect of number of simulations on the output quantities. 1D analyses were conducted for this purpose. Fig. 5 shows the convergence of the mean equivalent coefficient of consolidation μ_{t50p} as the number of simulations increases. It can be seen from Fig. 5 that 2,000 simulations were enough to give reliable and reproducible estimates. For each parametric combination in the following parametric studies, 2,000 Monte Carlo simulations were performed.

Equivalent Coefficient of Consolidation Defined by Excess Pore Pressure and Settlement, $\rho=1.0$

Looking at the 1D results, it can be seen from Figs. 6 and 7 that all the mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t50p} and μ_{t90p}) are lower than 1.0. However, the mean equivalent coefficients of consolidation defined by settlement (μ_{t50s} and μ_{t90s}) tend to be greater than 1.0 for all but

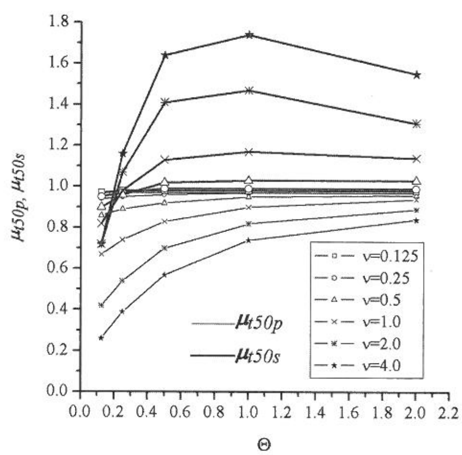

Fig. 6. 1D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t50p}) and settlement (μ_{t50s}), log-time method, $\rho=1.0$

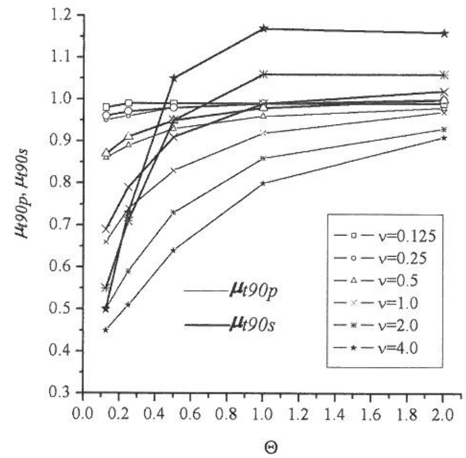

Fig. 7. 1D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t90p}) and settlement (μ_{t90s}), root-time method, $\rho=1.0$

the smallest spatial correlation lengths. For consolidation of a 1D system in which water flow is occurring in series through many different layers, these results are expected. For every simulation, one element with a low k could cause a "blockage" to the flow and a correspondingly low equivalent coefficient of consolidation based on pressure. On the other hand, the settlement of a 1D system is dominated by the high compressibility zones. One element with high volume compressibility value can contribute a large amount to the total settlement leading to a correspondingly high equivalent coefficient of consolidation based on settlement.

Looking at the 2D results, it can be seen from Figs. 8 and 9

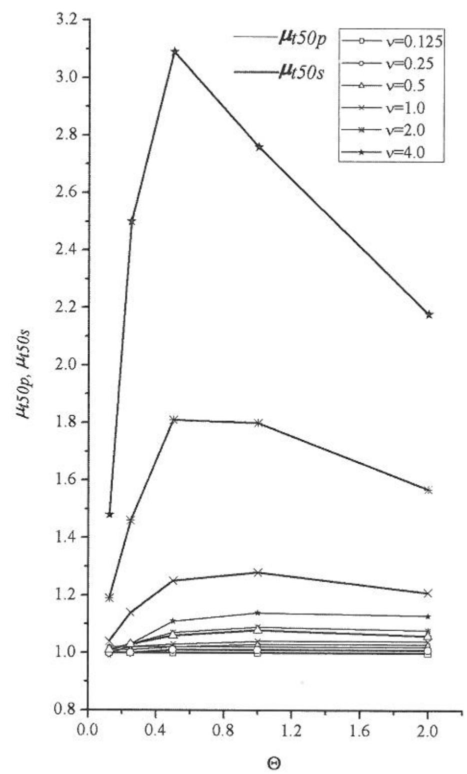

Fig. 8. 2D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t50p}) and settlement (μ_{t50s}), log-time method, $\rho=1.0$

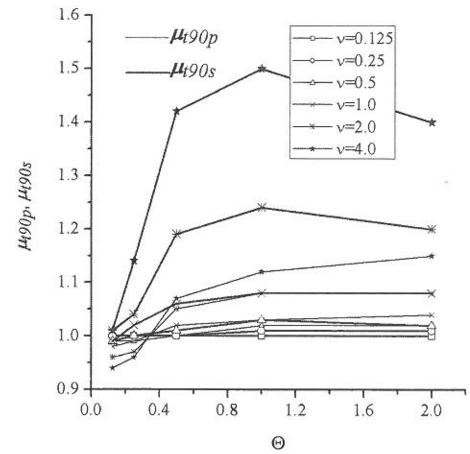

Fig. 9. 2D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t90p}) and settlement (μ_{t90s}), root-time method, $\rho = 1.0$

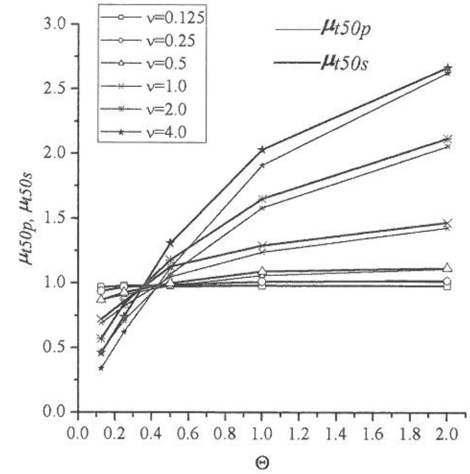

Fig. 10. 1D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t50p}) and settlement (μ_{t50s}), log-time method, $\rho = 0.0$

that the equivalent mean values, μ_{t50p} and μ_{t90p}, are lower than 1.0 only when $\Theta < 0.2$. This differs from the corresponding 1D results in that the blockages in 2D only occur when the spatial correlation length is small enough to result in low k values distributed quite uniformly in the 2D heterogeneous soil. However, all μ_{t50s} and μ_{t90s} are greater than 1.0.

Figs. 8 and 9 from 2D analyses also reveal that the mean equivalent coefficients of consolidation defined by excess pore pressure are lower than the mean equivalent coefficients of consolidation defined by settlement and the difference is greater than in 1D. It may also be noted that μ_{t50p} is much smaller than μ_{t50s} in 2D analyses as shown in Fig. 8.

Uncoupled (Terzaghi) Approach versus Coupled (Biot) Approach, $\rho = 1.0$

As mentioned before, all mean equivalent coefficients of consolidation evaluated by the uncoupled (Terzaghi) approach equal 1.0. Comparing the probabilistic coupled (Biot) results to those from the uncoupled (Terzaghi) approach, it can be seen from Figs. 6 and 7 that Terzaghi's theory will always overestimate the mean equivalent coefficient of consolidation defined by excess pore pressure for 1D systems. However, Terzaghi's theory will only overestimate μ_{t50p} and μ_{t90p} when $\Theta < 0.2$ in 2D as shown in Figs. 8 and 9.

It can also be seen from Fig. 7 in 1D that Terzaghi's theory will overestimate the mean equivalent coefficients of consolidation defined by settlement for lower spatial correlation lengths (say $\Theta < 0.2$ for μ_{t50s} and $\Theta < 0.5$ for μ_{t90s}), but underestimate it for higher values (say $\Theta > 0.2$ for μ_{t50s} and $\Theta > 0.5$ for μ_{t90s}). However, Terzaghi's theory always underestimates μ_{t50s} and μ_{t90s} in 2D as shown in Figs. 8 and 9.

As the input coefficient of variation is increased, divergence is observed between the coupled and uncoupled results. The uncoupled approach could either over- or underestimate the mean equivalent coefficients of consolidation.

The spatial correlation length leads to quite different conclusions in 1D depending on whether the equivalent coefficient of consolidation is defined by settlement or excess pore pressure. With respect to excess pore pressure, the difference between the coupled and uncoupled theories decreases as the spatial correlation length increases. As for the settlement, increasing the spatial correlation length decreases the differences between the coupled and uncoupled results for lower Θ (say $\Theta < 0.2$ for μ_{t50s} and $\Theta < 0.5$ for μ_{t90s}), but has little influence for higher values (say $\Theta > 0.2$ for μ_{t50s} and $\Theta > 0.5$ for μ_{t90s}). Unlike the 1D cases, spatial correlation length changes do not lead to any clear trends in the 2D cases.

Effect of Uncertainty in m_v and k on Mean Equivalent Coefficients of Consolidation, $\rho = 1.0$

In 1D analyses, as shown in Figs. 6 and 7, increasing the uncertainty in m_v and k will always decrease μ_{t50p} and μ_{t90p}. The reason lies in the fact that the larger the value of v, the more chance there is for a low permeability zone to exists in any given simulation. In 2D however, uncertainty in m_v and k has opposite effects on μ_{t50p} and μ_{t90p} for different Θ. For $\Theta < 0.2$, larger v leads to lower μ_{t50p} and μ_{t90p} while for $\Theta > 0.2$, larger v leads to higher μ_{t50p} and μ_{t90p}. The reason is that for higher Θ and v, there are more chances of high permeable zones interconnecting to facilitate drainage. For low Θ and high v however, low k values may be distributed quite uniformly throughout the domain and lead to regular blockages.

In 1D analyses, increased uncertainty in m_v and k has the opposite effects on μ_{t50s} and μ_{t90s}, as shown, in Figs. 6 and 7. For lower Θ (say $\Theta < 0.2$ for μ_{t50s} and $\Theta < 0.5$ for μ_{t90s}), increasing v will decrease μ_{t50s} and μ_{t90s}; for higher Θ (say $\Theta > 0.2$ for μ_{t50s} and $\Theta > 0.5$ for μ_{t90s}), increasing v will increase μ_{t50s} and μ_{t90s}. Those results mean that for ragged random fields with lower spatial correlation lengths, greater uncertainty in m_v and k will lead to slower average settlement in 1D systems. For smooth random fields with higher correlation lengths, greater uncertainty in m_v and k will lead to faster average settlement in 1D systems. In 2D systems however, increasing the uncertainty in m_v and k will always lead to faster average settlement.

Equivalent Coefficient of Consolidation Defined by Excess Pore Pressure versus Settlement, $\rho = 0.0$

It can be seen from Figs. 10–13 that all the mean equivalent coefficients of consolidation defined by excess pore pressure

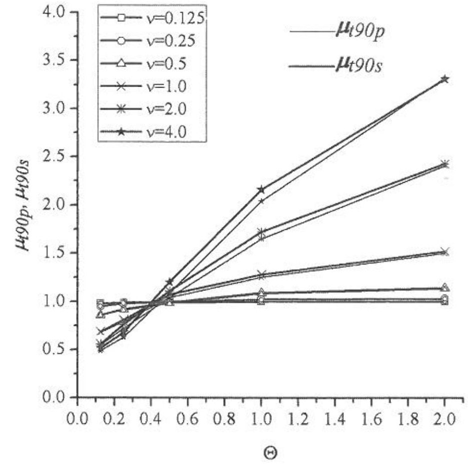

Fig. 11. 1D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t90p}) and settlement (μ_{t90s}), root-time method, $\rho=0.0$

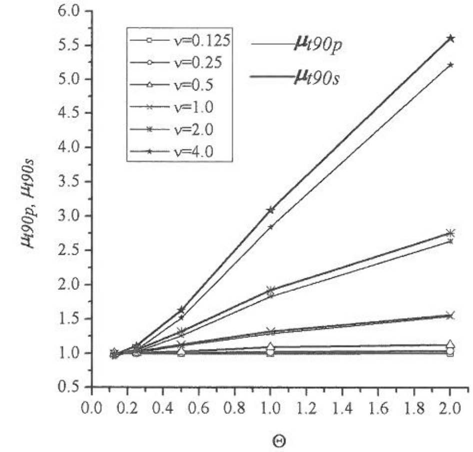

Fig. 13. 2D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t90p}) and settlement (μ_{t90s}), root-time method, $\rho=0.0$

(μ_{t50p} and μ_{t90p}) and by settlement (μ_{t50s} and μ_{t90s}) have similar trends when $\rho=0.0$. However, μ_{t50p} are always lower than μ_{t50s} and μ_{t90p} are always lower than μ_{t90p}.

Estimated Uncoupled Approach versus Coupled (Biot) Approach,$\rho=0.0$

If m_v and k are not correlated ($\rho=0.0$), every element has a different coefficient of consolidation. Although it would be possible to do an uncoupled analysis based on Terzaghi's theory with different coefficients of consolidation values assigned to every element, the analysis would strictly speaking be wrong. In this case therefore, the harmonic and geometric averages are used to represent the uncoupled results. Since the primary purpose of consolidation simulations is to calculate the rate of settlement of a system, μ_{t50s} is compared to μ_{hm} and μ_{gm} in Figs. 14 and 15. It can be seen that embodying m_v and k into a single coefficient of consolidation will nearly always give lower mean equivalent coefficients of consolidation than μ_{t50s}. The only exceptions occur when very high values of $v=4.0$ and $\Theta \geq 1.0$ are considered in 1D. Although μ_{hm} and μ_{gm} underestimate μ_{t50s}, they give similar trends of μ_{t50s} to those obtained by the coupled approach.

Effect of Uncertainty in m_v and k on Mean Equivalent Coefficients of Consolidation, $\rho=0.0$

In 1D analyses, as shown in Figs. 10 and 11, μ_{t50p}, μ_{t90p}, μ_{t50s}, and μ_{t90s} have same trends that decrease as the uncertainty in m_v and k increase for lower Θ (say $\Theta<0.4$) and increase as the uncertainty in m_v and k increase for higher Θ (say $\Theta>0.4$). Unlike the 1D cases, increasing the uncertainty in m_v and k will always increase all the mean equivalent coefficient of consolidation (μ_{t50p}, μ_{t90p}, μ_{t50s}, and μ_{t90s}), as shown in Figs. 12 and 13.

Effect of Spatial Correlation Length on Mean Equivalent Coefficients of Consolidation, $\rho=-1.0$, 0.0, and 1.0

In both 1D and 2D analyses, increasing Θ generally also increases μ_{t50p}, μ_{t90p}, μ_{t50s}, and μ_{t90s}. It can be seen from Figs. 3 and 4 that higher Θ makes the layered system more uniform, so

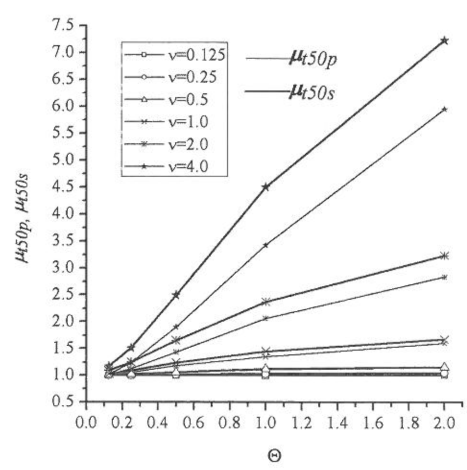

Fig. 12. 2D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by excess pore pressure (μ_{t50p}) and settlement (μ_{t50s}), log-time method, $\rho=0.0$

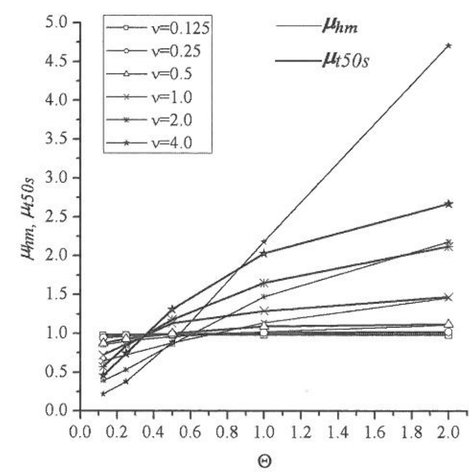

Fig. 14. 1D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by settlement and the log-time method (μ_{t50s}) and mean harmonic average (μ_{hm}), $\rho=0.0$

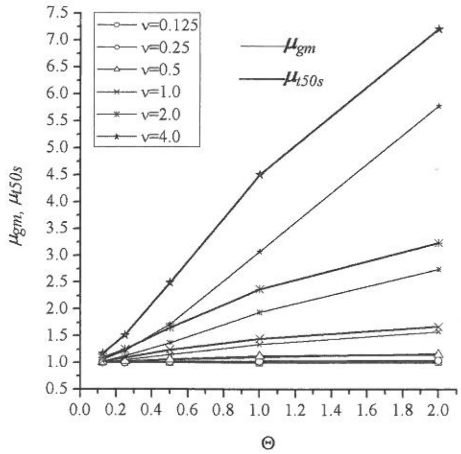

Fig. 15. 2D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by settlement and the log-time method (μ_{t50s}) and mean geometric average (μ_{gm}), ρ=0.0

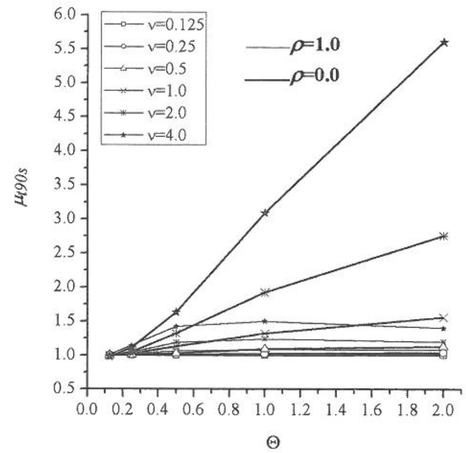

Fig. 17. 2D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by settlement and the root-time method (μ_{t90s}) obtained by ρ=1.0 and ρ=0.0

the blockage-causing low k values and low compressible zones are bunched together in only some of the simulations and therefore increase the mean equivalent coefficients of consolidation. It is interesting to note in Figs. 8 and 9 that in 2D analyses there are critical spatial correlation lengths that lead to maximum values of μ_{t50s} and μ_{t90s}. In all cases, for low variability soil ($\upsilon \leq 0.25$), the value of Θ has little effect.

Effects of Cross Correlation on Mean Equivalent Coefficients of Consolidation, ρ=−1.0, 0.0, and 1.0

The computed values of μ_{t90s} obtained with ρ=1.0 and ρ=0.0 are compared in Figs. 16 and 17. It shows that uncorrelated m_υ and k will lead to higher μ_{t90s} than when perfectly correlated. Uncorrelated m_υ and k also lead to higher μ_{t50s}, μ_{t50p}, and μ_{t90p} than when correlated, although these plots have not been included in the paper. Results with ρ=−1.0 gave a similar trend to those with ρ=0.0 so these figures have also been omitted.

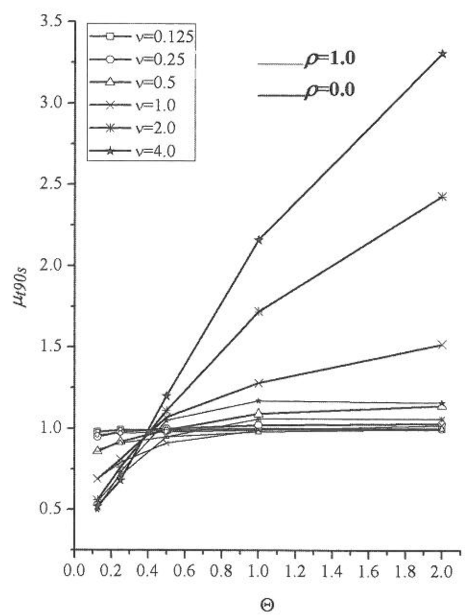

Fig. 16. 1D probabilistic results, comparison of mean equivalent coefficients of consolidation defined by settlement and the root-time method (μ_{t90s}) obtained by ρ=1.0 and ρ=0.0

Effects of Dimensionality of the Analysis on Mean Equivalent Coefficients of Consolidation, ρ=−1.0, 0.0, and 1.0

The 2D mean equivalent coefficients of consolidation are always higher than those obtained in 1D results as predicted by Rowe (1972). One can anticipate that the three-dimensional (3D) values would be higher still. As an example, the mean equivalent coefficients of consolidation defined by settlement and the root-time method (μ_{t90s}) as obtained by 1D and 2D analyses when ρ=1.0 are compared in Fig. 18.

Coefficient of Variations of Equivalent Coefficient of Consolidation, ρ=−1.0, 0.0, and 1.0

The 1D and 2D coefficients of variations of the equivalent coefficient of consolidation by the log-time method are compared in Figs. 19–22. The results given by the root-time method are similar

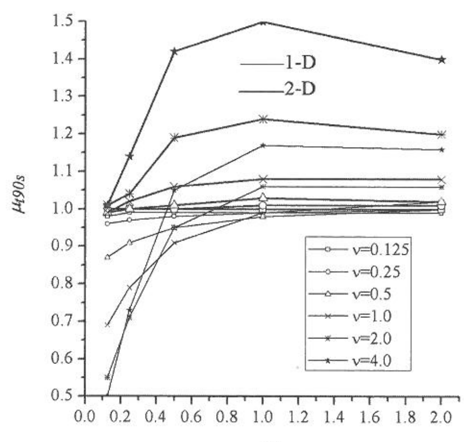

Fig. 18. Comparison of mean equivalent coefficients of consolidation defined by settlement and the root-time method (μ_{t90s}) obtained by 1D and 2D analyses, ρ=1.0

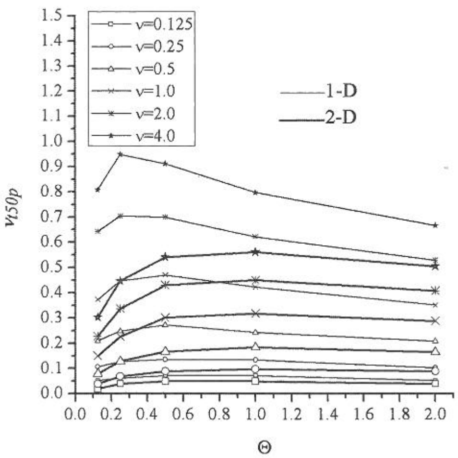

Fig. 19. Comparison of coefficient of variation of equivalent coefficients of consolidation defined by excess pore pressure (v_{t50p}) by 1D and 2D analyses, log-time method, ρ=1.0

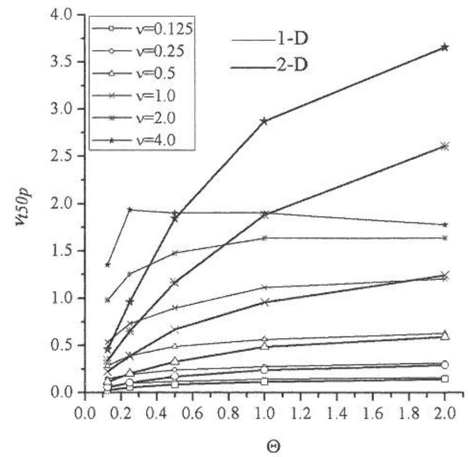

Fig. 21. Comparison of coefficient of variation of equivalent coefficients of consolidation defined by excess pore pressure (v_{t50p}) by 1D and 2D analyses, log-time method, ρ=0.0

and have been omitted. The coefficients of variations of the equivalent coefficient of consolidation from 2D analyses were not always smaller than those obtained in 1D. This contradicts the observations by Hwang and Witczak (1984), who stated that the uncertainty of the excess pore pressure decreased with increasing dimensionality. The computed coefficient of variations in 1D is greater than that in 2D only when m_v and k are perfectly positively correlated (ρ=1.0).

It should be noted that the computed coefficient of variations of the equivalent values were largest when ρ=−1.0. This might be expected on examination of Eq. (18) where the range of potential c_v values will be exaggerated if a low k tends to come with high m_v and vice versa.

Probabilistic Interpretation

Figs. 23 and 24 show the 90% confidence intervals on equivalent coefficients of consolidation defined by excess pore pressure and settlement (c_{vt50p} and c_{vt50s}) for 1D and 2D analyses when v =0.5 and ρ=1.0. In spite of the appearance of these figures and the 90% confidence intervals, it cannot be stated that c_{vt50s} is always larger than c_{vt50p}.

Figs. 25 and 26 show histograms in the 1D and 2D cases of the equivalent coefficients of consolidation following 2,000 simulations for the case where v=0.5, ρ=1.0, and Θ=1.0. Fitted lognormal distributions are also plotted and seen to match the histograms well. The parameters of the fitted distribution are estimated from the suite of simulations and given in the plots.

As mentioned before, all mean equivalent coefficients of consolidation evaluated by the uncoupled (Terzaghi) approach with ρ=1.0 equal 1.0. Referring to the particular case shown in Fig. 25 (1D analysis), the estimated probability that the uncoupled approach overestimates the rate of settlement (unconservative) is given by the following calculation, which assumes that the equivalent coefficient of consolidation is lognormally distributed:

$$P[c_{v50s} < 1.0] = 1 - \Phi\left(\frac{\mu_{\ln(c_{v50s})} - \ln(1.0)}{\sigma_{\ln(c_{v50s})}}\right) = 1 - \Phi\left(\frac{-0.077 - \ln(1.0)}{0.46}\right) = 57\% \quad (21)$$

A similar calculation in 2D analysis, referring to Fig. 26, gives the

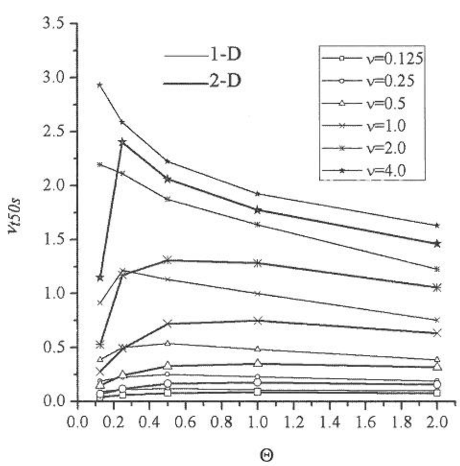

Fig. 20. Comparison of coefficient of variation of equivalent coefficients of consolidation defined by settlement (v_{t50s}) by 1D and 2D analyses, log-time method, ρ=1.0

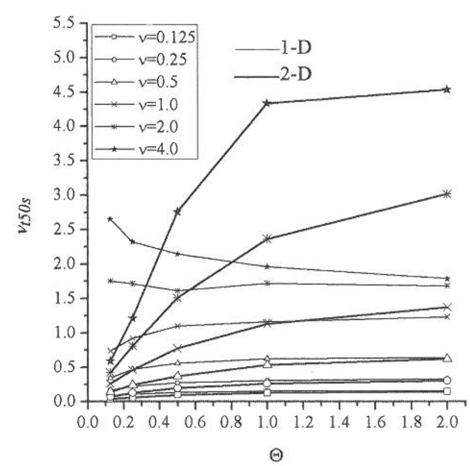

Fig. 22. Comparison of coefficient of variation of equivalent coefficients of consolidation defined by settlement (v_{t50s}) by 1D and 2D analyses, log-time method, ρ=0.0

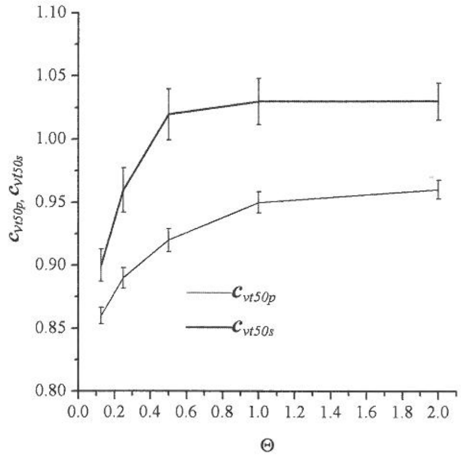

Fig. 23. 90% confidence intervals on equivalent coefficients of consolidation (c_{vt50p} and c_{vt50s}) for 1D analyses when $v=0.5$ and $\rho=1.0$

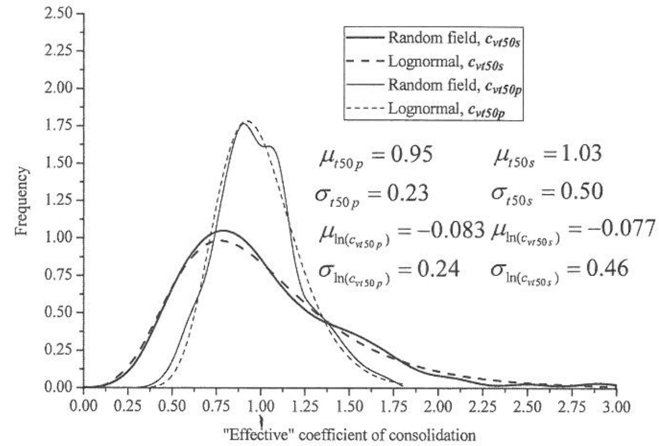

Fig. 25. Histograms of simulated c_{vt50p} and c_{vt50s} following 2,000 simulations along with fitted lognormal distribution for 1D analyses when $v=0.5$, $\rho=1.0$, and $\Theta=1.0$

estimated probability that the uncoupled approach overestimates the rate of settlement (unconservative) as 48%.

If $c_{vt50p} > c_{vt50s}$, using c_{vt50p} will overestimate the rate of settlement (unconservative). It should be noted that the probability that the equivalent coefficient of consolidation defined by excess pore pressure is larger than that by settlement cannot be obtained directly from the results shown in Figs. 25 and 26. It can only be obtained by counting the number of simulations in which $c_{vt50p} > c_{vt50s}$ and divide it by the total number of simulations. The results were found to be 48% in 1D analysis (Fig. 25) and 51% in 2D analysis (Fig. 26), which means using the average degree of consolidation defined by excess pore pressure to predict settlement will lead to misleading results which could either overestimate or underestimate the rate of settlement.

Blockages of Consolidation in Heterogeneous Soils

Increasing input variance will generally increase the mean equivalent coefficient of consolidation and its variance, but there are several exceptions which are summarized in Table 2. In cases where increasing the input variance decreases the mean equivalent coefficient of consolidation, the explanation lies in the occurrence of blockages where regions of low permeability soil are bunched together. The draining pore water cannot avoid blockages in 1D; however blockages in 2D can be avoided in some cases when flow passes through the surrounding higher permeability zones. From Table 2 it can be concluded that blockages are more likely to occur in 1D than in 2D, when $\rho=1.0$ rather than when $\rho=0.0$ and when the equivalent coefficient of consolidation is based on U_{avp} rather than U_{avs}.

Concluding Remarks

This paper has used the RFEM to investigate the influence of the standard deviation and correlation length of k and m_v and their cross correlation on various measures of the equivalent coefficients of consolidation. Both 1D and 2D analyses were performed to investigate the effect of model dimensionality. The following concluding remarks can be made.

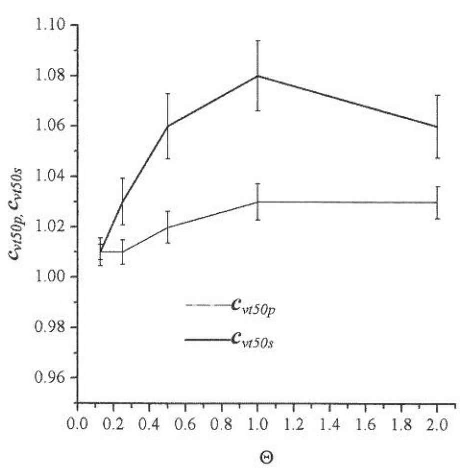

Fig. 24. 90% confidence intervals on equivalent coefficients of consolidation (c_{vt50p} and c_{vt50s}) for 2D analyses when $v=0.5$ and $\rho=1.0$

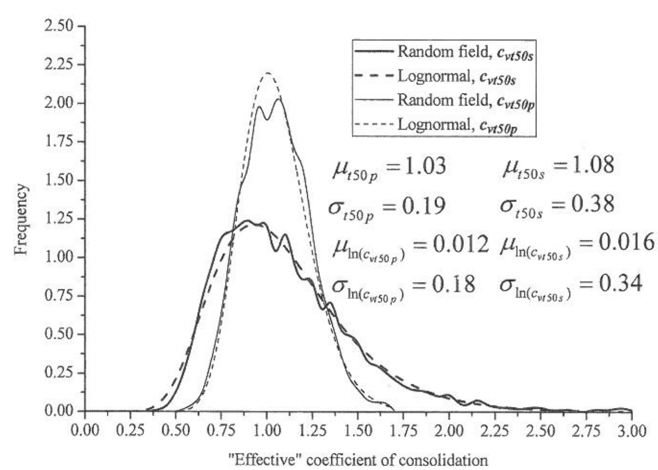

Fig. 26. Histograms of simulated c_{vt50p} and c_{vt50s} following 2,000 simulations along with fitted lognormal distribution for 2D analyses when $v=0.5$, $\rho=1.0$, and $\Theta=1.0$

Table 2. Influence of Input Variance

<table>
<tr><th colspan="4">1D</th><th colspan="4">2D</th></tr>
<tr><th colspan="2">ρ=1.0</th><th colspan="2">ρ=0.0</th><th colspan="2">ρ=1.0</th><th colspan="2">ρ=0.0</th></tr>
<tr><th>U_{avp}</th><th>U_{avs}</th><th>U_{avp}</th><th>U_{avs}</th><th>U_{avp}</th><th>U_{avs}</th><th>U_{avp}</th><th>U_{avs}</th></tr>
<tr><td>Increasing input variance will always decrease the mean equivalent coefficient of consolidation</td><td colspan="4">Increasing input variance will decrease mean equivalent coefficient of consolidation when Θ is small. Increasing input variance will increase the mean equivalent coefficient of consolidation when Θ is large</td><td colspan="3">Increasing input variance will always increase the mean equivalent coefficient of consolidation</td></tr>
</table>

1. Both the coefficient of volume compressibility and the soil permeability play important roles in the consolidation of heterogeneous soil and cannot be embodied into a single coefficient of consolidation. The paper has treated both m_v and k as independent random variables in conjunction with a Biot coupled consolidation analysis;
2. The average degree of consolidation defined by excess pore pressure and settlement is different in heterogeneous soil. Use of the average degree of consolidation defined by excess pore pressure to predict settlement will lead to misleading results which could either overestimate or underestimate the rate of settlement;
3. Increasing the spatial correlation length will increase the mean equivalent coefficient of consolidation and its variance when the spatial correlation length is small. Large spatial correlation length has little influence on the mean equivalent coefficient of consolidation and its variance. The spatial correlation length has little influence on the mean equivalent coefficient of consolidation and its variance when the input variance is small ($v \leq 0.25$);
4. Increasing the input variance will generally increase the mean equivalent coefficient of consolidation and its variance, with a few exceptions;
5. Positively correlated m_v and k are more likely to cause blockages leading to lower mean equivalent values of the coefficient of consolidation; and
6. The 2D mean equivalent coefficients of consolidation are always higher than the corresponding values in 1D. 3D values could be expected to be still higher.

Acknowledgments

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Notation

The following symbols are used in this paper:

c_v = coefficient of consolidation;
c_{vi} = coefficient of consolidation of the ith element (layer);
c_{vgm} = geometric average of coefficients of consolidation;
c_{vhm} = harmonic average of coefficients of consolidation;
c_{vt50} = equivalent coefficient of consolidation determined by the log-time method;
c_{vt90} = equivalent coefficient of consolidation determined by the root-time method;
c_{vt50p} = equivalent coefficient of consolidation determined by the log-time method and excess pore pressure;
c_{vt50s} = equivalent coefficient of consolidation determined by the log-time method and settlement;
D = maximum drainage path;
E' = effective elastic Young's modulus;
$g_{\ln k}$ = standard Gaussian random field of lognormal distributed permeability;
$g_{\ln m_v}$ = standard Gaussian random field of lognormal distributed compressibility;
h_i = thickness of ith layer;
i = layer number, element number;
k = permeability;
m_v = coefficient of volume compressibility;
n = number of layers;
n_{sim} = number of simulations;
s_t = settlement at time t;
s_u = long term (ultimate) settlement;
T = dimensionless "time factor";
t = time;
t_{50} = time corresponding to average degree of consolidation of 50%;
t_{90} = time corresponding to average degree of consolidation of 90%;
U = average degree of consolidation;
U_{avp} = average degree of consolidation defined by excess pore pressure;
U_{avs} = average degree of consolidation defined by settlement;
u_w = excess pore pressure;
u_x = displacements in the x-direction;
u_y = displacements in the y-direction;
u_0 = initial (uniform) excess pore pressure;
v = coefficient of variation;
v_k = coefficient of variation of permeability;
v_{t50p} = coefficient of variation of equivalent coefficient of consolidation by the log-time method and excess pore pressure;
v_{t50s} = coefficient of variation of equivalent coefficient of consolidation by the log-time method and settlement;
W = width;
γ_w = unit weight of water;
Δt = time step;
Θ = dimensionless spatial correlation length;

$\Theta_{\ln k}$ = dimensionless spatial correlation length of permeability;
θ = time interpolation parameter, $0 \leq \theta \leq 1$;
μ_{gm} = mean geometric average of coefficients of consolidation;
μ_{hm} = mean harmonic average of coefficients of consolidation;
$\mu_{\ln k}$ = mean of underlying normal distribution of permeability;
$\mu_{\ln(c_{vt50p})}$ = mean of fitted underlying normal distribution of equivalent coefficient of consolidation by the log-time method and excess pore pressure;
$\mu_{\ln(c_{vt50s})}$ = mean of fitted underlying normal distribution of equivalent coefficient of consolidation by the log-time method and settlement;
μ_{t50p} = mean equivalent coefficient of consolidation determined by the log-time method and excess pore pressure;
μ_{t90p} = mean equivalent coefficient of consolidation determined by the root-time method and excess pore pressure;
μ_{t50s} = mean equivalent coefficient of consolidation determined by the log-time method and settlement;
μ_{t90p} = mean equivalent coefficient of consolidation determined by the root-time method and settlement;
ρ = cross correlation coefficient;
$\sigma_{\ln k}$ = standard deviation of underlying normal distribution of permeability;
$\sigma_{\ln(c_{vt50p})}$ = standard deviation of fitted underlying normal distribution of equivalent coefficient of consolidation by the log-time method and excess pore pressure;
$\sigma_{\ln(c_{vt50s})}$ = standard deviation of fitted underlying normal distribution of equivalent coefficient of consolidation by the log-time method and settlement;
σ'_x = effective normal stress in the x-direction;
σ'_y = effective normal stress in the y-direction;
τ_{xy} = shear stress; and
ν' = effective Poisson's ratio.

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