Characterizing Natural-Fracture Permeability From Mud-Loss Data

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Summary

Liétard et al. (1999, 2002) have provided important insight into the mechanism and prediction of transient-state radial mud invasion in the near-wellbore region. They provided type curves describing mud-loss volume vs. time that allow the hydraulic width of natural fractures to be estimated through a curve-matching technique. This paper describes a simpler and more direct method for estimating the hydraulic width by the solution of a cubic equation, with input parameters given by the well radius r_w, the overpressure ratio $\Delta p/\tau_y$, and the maximum mud loss volume $(V_m)_{\max}$.

Introduction

Knowledge of locations and apertures of fractures crossing an oil or gas well has a strong technical and economical effect on drilling, production, and reservoir-management strategies. Most of the techniques currently used for natural-fracture detection and localization do not differentiate clearly between fractures that allow fluid flow and those that do not because they do not measure fluid-flow properties directly (Dyke 1995). Massive or small losses of drilling mud flowing from the wellbore into surrounding formations can be a good indicator of the natural-fracture permeability and can be used successfully to support and integrate other detection methodologies.

Sanfillippo et al. (1997) developed a model for Newtonian-mud propagation in a nondeformable fracture of constant aperture width with impermeable walls. The model is based on the diffusivity equation applied to mud flow radially propagating into a fracture that is perpendicularly intersecting the wellbore. The model then was used to obtain the fracture-aperture width using mud-loss measurements in wells drilled by the Agip Oil Company. However, this model is limited to Newtonian fluids and is not applicable to common drilling fluids that are non-Newtonian. The rheological behavior of the drilling fluid considerably influences the rate and volume of losses to the fracture system. Moreover, the assumption of a Newtonian mud leads to an invasion radius (r_s) of infinity, which is clearly unrealistic.

Liétard et al. (1999, 2002) developed a model based on the radial flow of a non-Newtonian Bingham-plastic fluid into an unlimited-extension fracture. The mud flow through the fracture is described by the local pressure drop from laminar flow in a slot of width w. Assuming that a constant overpressure is applied at the wellbore, a relationship for mud-invasion velocity vs. time was developed. The mud losses eventually stop when the overpressure is unable to overcome the yield stress of the drilling fluid. The ultimate volume of losses, therefore, depends on the yield value of the drilling fluid and the magnitude of the overpressure. The investigators provided type curves describing mud-loss volume vs. time, which allow the hydraulic width of natural fractures to be estimated through a curve-matching technique with field data.

Further work on this topic has been reported by Lavrov and Tronvoll (2003), who modeled the flow of Newtonian fluids into a deformable fracture. Their work was subsequently refined (Lavrov and Tronvoll 2004) to include non-Newtonian-fluid flow by incorporating a power-law model. The work was developed further to account for the influence of formation fluid by Lavrov (2006).

On the basis of the work of Liétard et al. (1999, 2002), Majidi et al. (2008a, 2008b) used the yield-power law (in contast to a Bingham fluid) to account for the formation fluid. All these methodologies, however, involved numerical solution of the governing equations, which is generally not favored by field engineers.

In this paper, the work of Liétard et al. (1999, 2002) is reviewed. It is shown that the maximum mud-loss volume $(V_M)_{\max}$ is related only to the well radius r_w, the overpressure ratio $\Delta p/\tau_y$, and the fracture width w through a cubic equation. This observation leads to a simple and direct method for determination of the fracture width from mud-loss data. Real fractures have rough walls and variable apertures. In the case of a Newtonian fluid, it is known [see, for example, Zimmerman and Bodvarsson (1996)] that the effective hydraulic aperture that governs the flow rate is roughly equal to the geometric-mean aperture, which is less than the arithmetic mean. The method presented in this paper gives an estimate of the effective hydraulic aperture on the basis of an analytical solution for a smooth-walled fracture. Rough walls and variable apertures will be investigated in future research.

Drilling-Mud Invasion Into Fractures

Liétard et al. (1999, 2002) considered mud-flow invasion into a fracture defined as a hollow cylindrical aperture of height w and internal and external radii given by r_w and $(r_s)_{\max}$, respectively, as shown in **Fig. 1**, where r_w is the wellbore radius and $(r_s)_{\max}$ is the maximum invasion radius.

Liétard et al. (1999, 2002) assumed the rheological behavior of the non-Newtonian drilling mud to be modeled as a Bingham fluid (Bird et al. 1960), leading to

$$\tau = -\mu_p \frac{dv}{dy} \pm \tau_y,\ |\tau| > \tau_y \qquad (1)$$

and

$$\frac{dv}{dy} = 0,\ |\tau| < \tau_y, \qquad (2)$$

in which y is the distance along the direction of width w (vertical), v is the velocity at location y, μ_p is the plastic viscosity, and τ_y is the drilling-mud yield value.

As indicated in Eq. 1, when the overpressure exceeds the drilling-mud yield value close to the wellbore, mud propagation begins to occur. The propagation slows down and eventually stops at a certain radial distance from the borehole when the overpressure falls below the drilling-mud yield value, as indicated in Eq. 2.

The local pressure drop is (Liétard et al. 1999, 2002)

$$\frac{dp}{dr} = \frac{12\mu_p v_m}{w^2} + \frac{3\tau_y}{w}, \qquad (3)$$

where v_m denotes the local velocity of the mud in the fractures under the radial-flow conditions around the well, given by

$$v_m(t) = \frac{q_m(t)}{2\pi r w}. \qquad (4)$$

If V_m represents the cumulative volume of mud loss at a given time, then the volumetric rate of mud invasion q_m is given by

$$q_m(t) = \frac{dV_m(t)}{dt}. \qquad (5)$$

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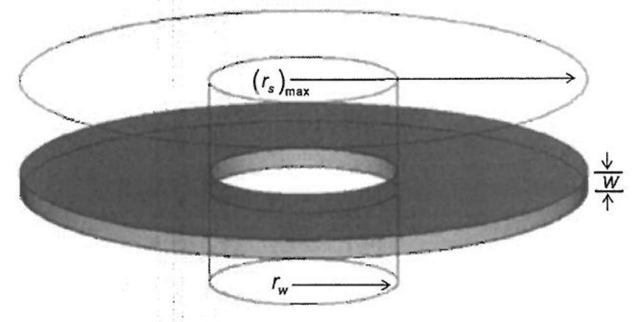

Fig. 1—Mud invasion into a slot of width *w*.

The cumulative volume of mud loss V_m is given by

$$V_m(t) = \pi w\left\{\left[r_s(t)\right]^2 - r_w^2\right\}, \quad \text{(6)}$$

where $r_s(t)$ is the invasion radius r_s at time t.

Thus, substituting Eqs. 4 and 5 into Eq. 3 results in

$$\frac{dp(t)}{dr} = \frac{6\mu_p}{\pi r w^3}\frac{dV_m(t)}{dt} + \frac{3\tau_y}{w}. \quad \text{(7)}$$

Integration of Eq. 7 over the mud-invasion region extending from the wellbore at r_w to the depth of mud-invasion radius $r_s(t)$ yields

$$\Delta p = \frac{6\mu_p}{\pi w^3}\frac{dV_m(t)}{dt}\ln\left[\frac{r_s(t)}{r_w}\right] + \frac{3\tau_y}{w}\left[r_s(t) - r_w\right], \quad \text{(8)}$$

where Δp is the difference between circulating pressure and the static reservoir pressure.

Substituting Eq. 6 into Eq. 8 yields an expression for the drilling overpressure Δp (assumed constant) as

$$\Delta p = \frac{12\mu_p}{w^2} r_s(t)\ln\left[\frac{r_s(t)}{r_w}\right]\frac{dr_s(t)}{dt} + \frac{3\tau_y}{w}\left[r_s(t) - r_w\right]. \quad \text{(9)}$$

On the basis of Eq. 9 and noting that $(r_s)_{max} >> r_w$, the maximum invasion radius $(r_s)_{max}$ is given by (Liétard 1999)

$$(r_s)_{max} = \frac{w\Delta p}{3\tau_y}. \quad \text{(10)}$$

The dimensionless mud-invasion radius and time are defined as (Liétard et al. 1999, 2002)

$$r_D = \frac{r_s}{r_w} \quad \text{(11)}$$

and

$$t_D = \frac{t}{t_c}, \quad \text{(12)}$$

respectively, in which the characteristic time scale is taken as

$$t_c = \frac{3\mu_p}{\Delta p\left(w / r_w\right)^2}. \quad \text{(13)}$$

In addition, a new parameter α_D, referred to as the dimensionless mud-invasion factor, is defined as

$$\alpha_D = \frac{3r_w\tau_y}{w\Delta p}. \quad \text{(14)}$$

Following Liétard et al. (1999, 2002), substitution of Eqs. 11 through 14 into Eq. 9 yields a dimensionless ordinary-differential equation in terms of the mud-invasion radius r_D as a function of dimensionless time t_D:

$$\frac{dr_D}{dt_D} = \frac{1 - \alpha_D\left(r_D - 1\right)}{4r_D\ln r_D}, \quad \text{(15)}$$

with initial conditions given by

$$r_D = 1 \text{ when } t_D = 0. \quad \text{(16)}$$

Solving Eqs. 15 and 16 leads to the analytical solution (Civan and Rasmussen 2002)

$$t_D = 4r_{D\max}\left(r_{D\max} - 1\right)\left\{ -\left(\ln r_D\right)\left[\frac{r_D}{r_{D\max}} + \ln\left(1 - \frac{r_D}{r_{D\max}}\right)\right] + \sum_{i=2}^{\infty}\frac{1}{i^2}\left[\left(\frac{1}{r_{D\max}}\right)^i - \left(\frac{r_D}{r_{D\max}}\right)^i\right]\right\}, \quad \text{(17)}$$

in which $r_{D\max}$ is the maximum dimensionless mud-invasion radius, given by (Liétard 1999; Sawaryn 2001)

$$r_{D\max} = 1 + \frac{1}{\alpha_D}. \quad \text{(18)}$$

Mud Loss and Type Curves

Liétard et al. (1999, 2002) first expressed Eq. 6 in dimensionless form by substituting Eqs. 11 and 12 into it, leading to the definition of a parameter X, where

$$X \equiv \frac{V_m(t_D)}{\pi r_w^2} = w\left\{\left[r_D(t_D)\right]^2 - 1\right\}, \quad \text{(19)}$$

which, after taking logs, gives

$$\log_{10} X = \log_{10} w + \log_{10}\left\{\left[r_D(t_D)\right]^2 - 1\right\}. \quad \text{(20)}$$

A second parameter Y can be derived from Eqs. 12 and 13, where

$$Y = \frac{t\Delta p}{3\mu_p r_w^2} = \frac{t_D}{w^2}, \quad \text{(21)}$$

which, again after taking logs, gives

$$\log_{10} Y = -2\log_{10} w + \log_{10} t_D. \quad \text{(22)}$$

By means of the analytical solution given by Eq. 17, and the parameters defined later in Eqs. 19 and 21, a series of type curves relating $\log_{10}\{[r_D(t_D)]^2-1\}$ to $\log_{10}t_D$ can be constructed, as shown in **Fig. 2.** These curves can be used for estimation of the fracture hydraulic width.

As explained by Liétard et al. (1999, 2002), when the field data are plotted over the type curves, the field data will shift by $\log_{10}w$ negatively along the abscissa and by $2\log_{10}w$ positively along the ordinate direction, from Eqs. 20 and 22. This allows determination of the fracture width w by means of the type curves.

Proposed Method

The analytical solution (Eq. 17) gives the transient radial mud-loss invasion from a borehole into a fracture plane. Recall that the solution is obtained by solving the differential equation (Eq. 15) subject to the initial condition (Eq. 16), where the maximum invasion radius is defined in Eq. 18. Eq. 18 indicates that the mud losses will eventually stop because of the overpressure eventually reaching the yield stress of the drilling fluid. The ultimate invasion radius, thus, depends on the wellbore radius, the yield value of the drilling fluid, and the amount of overpressure, as indicated in Eq. 18, which could be written as

$$(r_s)_{max} = 1 + \frac{w\Delta p}{3r_w\tau_y}. \quad \text{(23)}$$

The maximum mud-loss volume is given by

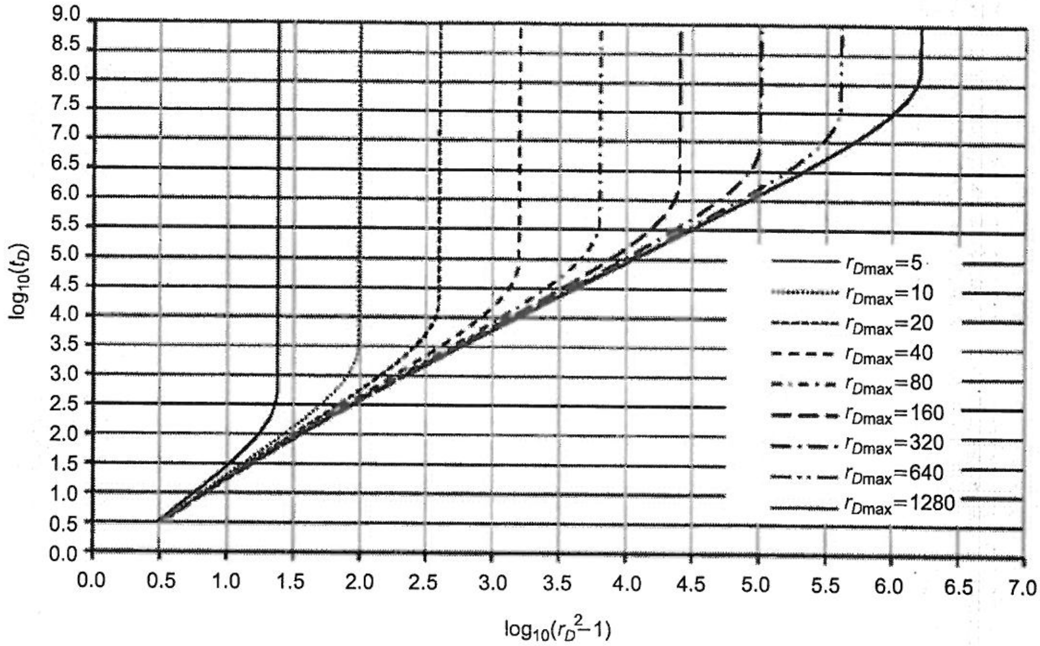

Fig. 2—Type curves.

$$\left(V_m\right)_{\max} = \pi w\left[\left(r_s\right)^2_{\max} - r_w^2\right]. \quad (24)$$

Substituting Eq. 23 into Eq. 24 gives

$$\left(\frac{\Delta p}{\tau_y}\right)^2 w^3 + 6r_w\left(\frac{\Delta p}{\tau_y}\right)w^2 - \frac{9}{\pi}\left(V_{\mathrm{m}}\right)_{\max} = 0, \quad (25)$$

which is a cubic equation in the fracture width w, with coefficients dependent on the well radius r_w, the overpressure ratio $(\Delta p/\tau_y)$, and the maximum mud-loss volume $(V_m)_{\max}$. Solution of this equation for w (discarding physically meaningless roots) is a simpler and more direct way of determining the fracture width than the curve-fitting method described previously.

Only a real, positive value of the aperture w is physically meaningful. First note that $f(w = 0) = -9(V_m)_{\max}/\pi < 0$. But, for large values of w, the cubic term dominates and $f(w) > 0$. So, there must be at least one real positive root. To check if there can be more than one positive root, we need to see if $f(w > 0)$ is monotonic. We can do this by checking the derivative:

$$f'(w) = 3\left(\frac{\Delta p}{\tau_y}\right)^2 w^2 + 12r_w\left(\frac{\Delta p}{\tau_y}\right)w, \quad (26)$$

which will always be positive for all $w > 0$. So, there will always be exactly one positive root of Eq. 25.

Because r_w usually is approximately 0.1 m and w will usually be at least 100 μm, the quadratic term in Eq. 25 will be negligible if the overpressure ratio is at least 10^5. A useful estimation of w could be obtained:

$$w = \left[\frac{9\left(V_m\right)_{\max}/\pi}{\left(\Delta p/\tau_y\right)^2}\right]^{1/3}. \quad (27)$$

Validation of Proposed Method on the Basis of Published Data

In this section, we will test the validity of the proposed method by comparing it with the curve-fitting technique using field data from the Machar 18z and Machar 20z wells reported by Liétard et al. (1999, 2002). In both cases, the drill bit was 8.5 in. in diameter, thus

$$r_w = 0.36 \text{ ft}. \quad (28)$$

Machar 18z. The completion of Well Machar 18z is described in detail elsewhere (Liétard et al. 1999, 2002). A total of 240 bbl of mud losses was monitored during 11 events over 1,990 ft of drilling across the pay zone. The average mud loss was

$$\left(V_m\right)_{\max} = \frac{240}{11} = 21.8 \text{ bbl}, \quad (29)$$

and the overpressure ratio (Liétard et al. 1999, 2002) was

$$\frac{\Delta p}{\tau_y} = 364{,}420. \quad (30)$$

Substituting Eqs. 28, 29, and 30 into Eq. 25 and converting to SI units gives the cubic solution

$$\begin{aligned}&\left(364{,}420^2\right)w^3\\&+(6)(0.36\times0.3048)(364{,}420)w^2\\&-\frac{9}{\pi}(21.8\times0.15898) = 0.\end{aligned} \quad (31)$$

The only real root of the cubic after solution is $w = 4.21\times10^{-4}$ m (421 μm), which can be compared with the $w = 420$ μm obtained by Liétard et al. (1999, 2002). It should be mentioned that an estimate of $w = 4.21\times10^{-4}$ m (421 μm) could be obtained using Eq. 27, which agreed with the value found by Eq. 25, revealing that the quadratic term in Eq. 25 is negligible in this case.

Machar 20z. Machar 20z is the twin of Well Machar 18z. It was completed and stimulated the same way as Well Machar 18z. As reported by Liétard et al. (1999, 2002), a total of 2,844 bbl of mud loss was monitored during eight events over 1,130 ft of drilling across the pay zone. The average mud loss was

$$\left(V_m\right)_{\max} = \frac{2{,}844}{8} = 355.5 \text{ bbl}, \quad (32)$$

and the overpressure ratio was

$$\frac{\Delta p}{\tau_y} = 828{,}320. \quad (33)$$

Once more, solving Eq. 25 with these parameters gives $w = 618$ μm, which can be compared with the $w = 620$ μm obtained by

Liétard et al. (1999, 2002). An excellent estimation of w = 620 μm could also be obtained using Eq. 27.

Concluding Remarks

It has been shown in this paper that the width of a natural fracture can be determined from mud-loss data by solving a cubic equation with input parameters given by the well radius r_w, the overpressure ratio $\Delta p/\tau_y$, and the maximum mud-loss volume $(V_m)_{max}$. The method presented in this paper offers a simpler and more direct alternative to existing curve-matching methods while giving essentially the same results.

Nomenclature

p = pressure
q_m = volumetric rate of mud invasion
r = radius
r_D = dimensionless invasion radius
$r_{D\max}$ = the maximum dimensionless invasion radius
r_s = invasion radius
$(r_s)_{\max}$ = the maximum invasion radius
$r_s(t)$ = invasion radius at time t
r_w = well radius
t_D = dimensionless time
v = velocity at location y
v_m = average of v
V_m = cumulative volume of mud loss
$(V_m)_{\max}$ = the maximum mud-loss volume
w = fracture width
y = distance along the direction of width w (vertical)
Δp = overpressure, difference between circulating pressure and the static reservoir pressure
μ_p = plastic viscosity
τ = shear stress
τ_y = drilling-mud yield value

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SI Metric Conversion Factors

bbl × 1.5898	E–01 = m^3
ft × 3.048*	E–01 = m
in. × 2.54*	E–02 = m

*Conversion factor is exact.

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