Technical Note

Stability Assessment of Slopes Using Different Factoring Strategies

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Abstract: Traditional slope stability analysis delivers the factor of safety with respect to shear strength, which is the factor by which $\tan\varphi'$ and c' must be reduced to bring a slope to the point of failure. In the present paper, alternative strategies are considered to include the factors by which destabilizing parameters, such as pore pressures and gravitational loads, must be increased to bring a slope to the point of failure. The approach described gives a more comprehensive insight into the stability of slopes and the sensitivity of failure to different input parameters. Finally, the practical use of the alternative factoring strategies is illustrated through two application examples referring to a slope under different loading combinations (pore-pressure ratio and horizontal seismic coefficient). **DOI: 10.1061/(ASCE)GT.1943-5606.0000678.**

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Introduction

The factor of safety (FS) in slope stability analysis is typically defined as the ratio of the available shear strength divided by the shear stress required to maintain a just-stable slope, thus

$$\mathrm{FS} = \frac{\tau_f}{\tau_m} \tag{1}$$

where τ_f = available shear strength, typically based on Coulomb's strength equation, and τ_m = mobilized shear stress on the failure surface needed to maintain equilibrium. The above expression is essentially used by all slope stability analysis methods.

Among these methods, limit equilibrium methods involving methods of slices are most commonly used, and these have been described in detail in other publications (Duncan and Wright 2005; Cheng and Lau 2008). A characteristic of these classical methods is that the safety factor value is assumed constant along the slip surface and is assumed to act equally on the cohesive and frictional component of strength (c' and $\tan\varphi'$, respectively), thus

$$\tau_m = \frac{\tau_f}{\mathrm{FS}} = \frac{c'}{\mathrm{FS}} + \sigma'_n \frac{\tan\phi'}{\mathrm{FS}} \tag{2}$$

where σ'_n = effective normal stress on the failure surface and ϕ' = angle of internal friction.

Finite-element methods of slope stability analysis (Zienkiewicz et al. 1975; Griffiths and Lane 1999) are rapidly growing in popularity and can offer great benefits over the traditional approaches for heterogeneous soil profiles, atypical slope geometries, and three dimensions. For conventional two-dimensional slope geometries with relatively homogeneous soil properties, however, the finite-element approach gives essentially the same results as limit equilibrium methods. The most commonly used finite-element approach involves the application of soil self-weight to the slope followed by strength reduction in which factored shear strength parameters c'_f and ϕ'_f are used in the analysis given by

$$c'_f = \frac{c'}{\mathrm{SRF}} \quad \text{and} \quad \phi'_f = \arctan\left(\frac{\tan\phi'}{\mathrm{SRF}}\right) \tag{3}$$

where SRF is the strength reduction factor. The required FS of a slope is found by gradually increasing the SRF, in ever decreasing increments, until the slope just fails. The safety factor of the slope is then equal to the SRF (FS = SRF).

Alternatively, Chen and Mizuno (1990) implemented Coulomb and Drucker-Prager models by applying gravity loading in increments until critical failure mechanisms developed. An application of the gravity induced failure approach was also presented by Swan and Seo (1999) in the context of finite elements, where it was shown that a conventional strength-reduction method [Eq. (2) or (3)] generally give significantly lower factors of safety than gravity increase methods.

In the present paper a number of different factoring strategies on various parameters are investigated for reaching slope failure. These are subsequently presented along with two application examples.

New Factoring Strategies

Consider the parameters shown in Table 1 as being some of the most important in determining the stability of slopes. These parameters are connected either to a soil property or to an action (e.g., shear strength, self-weight, pore-pressure ratio, horizontal pseudostatic acceleration coefficient).

Alternative ways of calculating the safety margin of slopes based on these parameters are shown in Table 2. This is not a comprehensive

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list, because other parameters could affect slope stability (e.g., slope geometry, loading at the crest); however, only those parameters shown in Table 1 will be considered in this paper. Only Option 1 (Table 2) will lead to the generally accepted FS delivered by traditional slope stability analysis. The other will lead to different values that should not be used in practice, unless it is clearly understood how they were obtained. The values obtained by the nontraditional factoring strategies (Option 2–6), however, will give insight into the relative importance of the parameters listed in Table 1 and emphasize some differences between strength and load factoring applied to slopes and other geotechnical applications. This has implications for Load and Resistance Factor Design (LRFD) applications to slope stability analysis, which is complicated by the fact that gravity appears in both the denominator and numerator of the FS equation.

Application Examples

Example 1

The stability of a homogeneous 6-m-high earth slope having gradient 1V:1.5H (Fig. 1) has been examined using the different factoring strategies shown in Table 2.

Table 1. Parameters Affecting Stability of Slopes

Parameter	Explanation
$\tan\phi'$	Coefficient of friction
c'	Cohesion
c_u	Undrained shear strength ($\varphi_u = 0°$)
γ	Unit weight
r_u	Pore-pressure ratio
k_h	Horizontal pseudostatic acceleration coefficient

Table 2. Different Factoring Strategies

Option	Parameter factored	Factoring strategy
1a[a]	$\tau_f = c' + \sigma'\tan\varphi'$	Both c' and $\tan\ \phi'$ are factored down ($c'/\mathrm{FS}_{c',\tan\varphi'}$ and $\tan\varphi'/\mathrm{FS}_{c',\tan\varphi'}$)
1b[a]	$c_u(\varphi = 0°)$	Factored down (c_u/FS_{c_u})
2	c'	Factored down ($c'/\mathrm{FS}_{c'}$)
3	$\tan\phi'$	Factored down ($\tan\varphi'/\mathrm{FS}_{\tan\varphi'}$)
4	γ	Factored up ($\gamma\mathrm{FS}_{\gamma}$)
5	r_u	Factored up ($r_u\mathrm{FS}_{r_u}$)
6	k_h	Factored up ($k_h\mathrm{FS}_{k_h}$)

[a]Conventional approach.

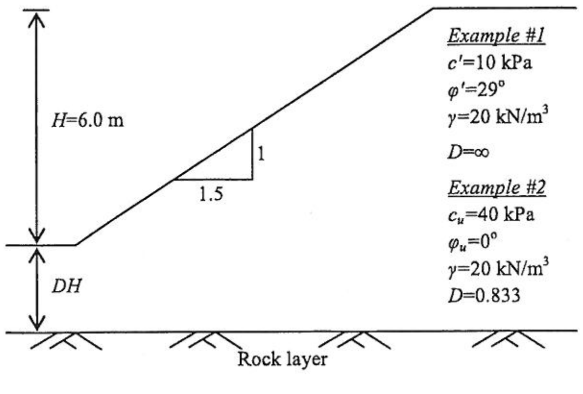

Fig. 1. Slope geometry and properties

All safety margin values have been obtained by the freely available finite-element program slope1_fs (see http://www.mines.edu/~vgriffit/4th_ed/Software), which allows the factoring strategies presented in Table 2 to be implemented. The characteristic values of soil properties are $c' = 10$ kPa, $\phi' = 29°$, and $\gamma = 20\ \mathrm{kN/m^3}$. In addition to analysis of the slope with the parameters shown in Fig. 1, the slope is also examined under two groundwater conditions expressed by the pore-pressure ratio ($r_u = 0.05$ and $r_u = 0.20$) and two horizontal seismic loading conditions ($k_h = 0.1$ and $k_h = 0.2$). The results for this example have been summarized in Table 3.

Example 2

The stability of the 6-m-high earth slope of Fig. 1 has also been examined considering undrained conditions and a horizontal rock layer at 5 m below the toe ($D = 0.833$). The characteristic values of the soil properties are now assumed to be $c_u = 40$ kPa, $\phi_u = 0°$, and $\gamma = 20\ \mathrm{kN/m^3}$. Five horizontal seismic coefficients were considered ($k_h = 0$, $k_h = 0.1$, $k_h = 0.2$, $k_h = 0.251$, and $k_h = 0.3$). The results for this example have been summarized in Table 4.

Discussion of Different Factoring Strategies

Tables 3 and 4 show that quite different safety margin values, both higher and lower than the conventional value, can be obtained by the various factoring strategies for the same slope and loading conditions. Each safety margin value in the tables indicates the factor by which a parameter has to be increased or reduced relative to its characteristic value to bring the slope to the point of failure, with all other parameters being held constant at their characteristic values. For example, in the second row of Table 3, a value of $FS_{r_u} = 6.45$ indicates that the slope will be brought to the point of failure by increasing the pore-pressure ratio r_u by a factor of 6.45, i.e., $r_u = 6.45 \times 0.050 = 0.323$. Similarly, a value of $FS_{k_h} = 2.88$ (also see the second row in Table 3) indicates the slope will be

Table 3. Example 1: Stability Factor Values by Various Factoring Strategies

Loading combination		$\mathrm{FS}_{c',\tan\varphi'}$	$\mathrm{FS}_{c'}$	$\mathrm{FS}_{\tan\varphi'}$	FS_{γ}	FS_{r_u}	FS_{k_h}
		Option according to Table 2					
r_u	k_h	1a	2	3	4	5	6
0.00	0.0	1.76	7.22	3.09	7.22	—	—
0.05	0.1	1.38	2.27	1.89	5.53	6.45	2.88
0.05	0.2	1.16	1.38	1.31	3.86	3.57	1.44
0.20	0.1	1.17	1.36	1.41	4.27	1.61	1.88
0.20	0.2	0.97	0.94	0.92	2.70	0.89	0.94

Table 4. Example 2: Stability Factor Values by Various Factoring Strategies

	FS_{c_u}	FS_{γ}	FS_{k_h}
	Option according to Table 2		
Horizontal seismic coefficient, k_h	1b	4	6
0.000	1.91	1.91	—
0.100	1.42	1.58	2.52
0.200	1.13	1.20	1.27
0.251	1.00	1.00	1.00
0.300	0.92	0.80	0.84

brought to the point of failure by increasing the pseudostatic horizontal acceleration coefficient k_h by a factor of 2.88, i.e., $k_h = 2.88 \times 0.1 = 0.288$. Generally, it stands that an increase in the characteristic value of r_u or k_h by a factor of n reduces the respective safety margin (FS_{r_u} or FS_{k_h}) by the same factor and vice versa (Table 3).

In Table 3, the safety margin with respect to the horizontal pseudostatic acceleration coefficient is in close agreement with the critical acceleration factor k_c developed by Sarma and Bhave (1974), i.e., $k_c \approx F_{k_h} k_h$. When $k_h = 0$ and $r_u = 0$ in Example 1 and Example 2, $FS_{c' \text{ or } c_u} = FS_\gamma$, that is, the amount by which cohesion must be factored down to cause failure, is the same as the amount that the unit weight must be factored up. This is of course to be expected from the dimensionless groups $c'/(\gamma H \tan \phi')$ or $c_u/(\gamma H)$ (Chen 1975; Michalowski 2010) that have been shown to be fundamentally related to the safety level of slopes.

Finally, if any of the safety margin values (using the different factoring strategies) for a given slope is equal to unity, then all the other safety margin values obtained by the different factoring strategies are also equal to unity (compare FS values for $k_h = 0.251$ in Table 4).

Summary and Concluding Remarks

Practitioners have numerous methods at their disposal to perform slope stability analysis, but common to all of them is the definition of the safety factor with respect to shear strength, which has remained unchanged for decades. Each soil strength parameter (e.g., c_u, c', $\tan \varphi'$, or γ) or action (e.g., k_h or r_u) is considered as a parameter that can be factored either up or down (e.g., $c'/\mathrm{FS}_{c'}$, $k_h \mathrm{FS}_{k_h}$) until failure is achieved. A finite-element approach has been used to generate safety margins for a range of parameters applied to a couple of simple slope examples. The safety margins are seen to vary quite widely and give insight into the sensitivity of stability to different load and resistance parameters. For the frictional slope considered, increasing gravity gave significantly higher safety margins than those given in the conventional approach of factoring strength down.

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