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NOTE

Integrating Eurocode 7 (load and resistance factor design) using nonconventional factoring strategies in slope stability analysis

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Abstract: In traditional allowable stress design, as known, the safety factor is calculated with respect to shear strength of soil(s) by dividing the available shear strength by the mobilized stresses. The limit-state method, on the other hand, compares — in the form of the inequality $E_d \leq R_d$ — the effects of all the actions, E_d, with the corresponding resistance of the ground, R_d. Although this method considers different loading conditions by using suitable combinations of design values, it is still based on direct comparison of the available shear strength with the mobilized stresses. In the present paper, various factoring strategies (in addition to the traditional one with respect to shear strength of soils) are integrated into a limit-state method framework. Eurocode 7 has been chosen for this purpose. The whole procedure aims at giving a more comprehensive insight into the design of slopes and the sensitivity of safety level of slopes to the various parameters. In addition, the proposed methodology, as shown, may result in a safety level of slopes significantly lower than the respective one obtained using the limit-state method in its traditional form. As man-made slopes that conform to design standards often fail in practice, even though conservative input values are used, these failures must be treated with more skepticism by practitioners adopting supplementary design practices such the one presented herein.

Key words: slope stability, load and resistance factor design (LRFD), factoring strategy, ultimate limit state, limit state method, Eurocode 7.

Résumé : Dans la méthode traditionnelle de calcul aux contraintes admissibles, le facteur de sécurité est calculé selon la résistance au cisaillement du (des) sol(s) en divisant la résistance au cisaillement disponible par les contraintes mobilisées. Cependant, la méthode de calcul à l'état limite compare — sous forme de l'inégalité $E_d \leq R_d$ — les effets de toutes les actions, E_d, à la résistance du sol correspondante, R_d. Malgré que cette méthode considère des conditions de chargement différentes par l'utilisation de combinaisons appropriées de valeurs de conception, elle est tout de même basée sur la comparaison directe entre la résistance au cisaillement disponible et les contraintes mobilisées. Dans cet article, plusieurs stratégies de mise en facteur (en plus de la méthode traditionnelle selon la résistance au cisaillement des sols) sont intégrées dans un protocole de calcul à l'état limite. Pour ce faire, Eurocode 7 a été choisi. La procédure entière vise à offrir des informations compréhensives sur la conception de pentes et la sensibilité du niveau de sécurité des pentes à différents paramètres. De plus, la méthodologie telle que proposée peut permettre d'atteindre un niveau de sécurité des pentes significativement inférieur à celui obtenu par la méthode de calcul de l'état limite dans sa forme traditionnelle. Puisque les pentes construites par l'homme qui sont conformes aux standards de conception en pratique cèdent souvent, même si les valeurs d'entrée utilisées sont conservatrices, ces ruptures doivent être traitées avec plus de scepticisme par les géotechniciens qui adoptent des pratiques de conception supplémentaires, telle que celle présentée dans cet article. [Traduit par la Rédaction]

Mots-clés : stabilité de pente, facteur de conception en charge et résistance (FCCR), stratégie de mise en facteur, état limite ultime, calcul de l'état limite, Eurocode 7.

Introduction

Uncertainties in the loads and resistances for a particular application are generally accounted for through the use of a single safety factor (F) in traditional allowable stress design (ASD). ASD methods suffer from the fact that uncertainty attributed to a number of different input parameters must be "lumped" together into a single factor, although such methods have been used for many years with a great deal of success (Loehr et al. 2005). This single factor, the safety factor of slopes (F), is commonly defined as the ratio of the available shear strength divided by the shear stress required to maintain a slope at the point of incipient failure; thus

$$F = \frac{\tau_f}{\tau_m} \tag{1}$$

where τ_f is the available shear strength, typically based on Coulomb's strength equation, and τ_m is the mobilized shear stress on the failure surface required to maintain equilibrium. The above expression is essentially used by all slope stability analysis methods, such as limit equilibrium methods (Fellenius 1936; Bishop 1955; Morgenstern and Price 1965; Spencer 1967), finite element methods (Griffiths and Lane 1999; Griffiths and Marquez 2007), and limit analysis approaches (Chen et al. 2005).

The manner in which the loads are combined has sometimes been unclear in the traditional use of ASD because there may be many different load types. The load and resistance factor design (LRFD), also called limit-state method (depending on the country), provides a response to this problem by specifying several load combinations with load factors selected on a probabilistic basis

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(Goble 1999). The implementation of the limit-state method uses any of the above slope stability analysis methods and it is based on the application of partial factors to the actions (or their effects) and soil properties or resistances (or to both soil properties and resistances). Design codes based on this method (European Committee for Standardization 2004; Driscoll et al. 2008; German Institute for Standardization 2009) are already in use by many countries, where the following inequality (or a similar one) must be satisfied when examining the ultimate limit state (ULS):

$$E_d \leq R_d \tag{2a}$$

where E_d is the design value of the effects of all the actions and R_d is the design value of the corresponding resistance of the ground.

Equation (2a) can also be expressed as a single factor, similar to the safety factor expression of eq. (1), as follows:

$$F = \frac{R_d}{E_d} \tag{2b}$$

where a safety factor value greater than or equal to unity is required.

Pantelidis and Griffiths (2012) have shown that the safety factor with respect to shear strength (eqs. (1) and (2b)) should not be considered as an absolute procedure in slope stability analysis, proposing alternative factoring strategies. Based on this idea, various factoring strategies are integrated in the present paper into a limit-state method framework. Eurocode 7 (European Committee for Standardization 2004) has been chosen for this purpose as being one of the most widely used applications of the concept of limit state. The whole procedure aims at giving a more comprehensive insight into the design of slopes and the sensitivity of safety level of slopes as a function of the various parameters. It should also be mentioned that the proposed methodology may result in a safety level of slopes significantly lower than the respective one obtained by the use of the limit-state method in its traditional form (that is, based on the safety factor with respect to shear strength), especially in the case of extreme events (e.g., seismic loading, excessive pore water pressures). As man-made slopes that conform to design standards often fail in practice, even though conservative input values are used, these failures must be treated with more skepticism by practitioners adopting supplementary design practices.

Integrating LRFD approach with different factoring strategies

Depending on the case, the proposed safety factor formulas will have one of the following two general forms:

$$(3)\quad \begin{aligned} F_x &= \frac{\text{available value of } x}{\text{mobilized value of } x} && (\text{e.g., } x = c_u,\ c,\ \tan\varphi \text{ or } \tau_f) \\ F_x &= \frac{\text{maximum value of } x \text{ that the slope can stand}}{\text{available value of } x} && (\text{e.g., } x = r_u,\ k \text{ or } \gamma) \end{aligned}$$

All expressions are given in a simple form that rather reminds us of the Fellenius' method of slices with which most readers are familiar (see Fig. 1). The authors, however, would like to mention that the proposed approach can be used along with any of the existing slope stability analysis methods, such as limit equilibrium methods (including the closed-form solutions proposed by Pantelidis and Griffiths 2013a, 2013b), finite element method, and limit analysis approaches. Besides, all examples presented herein have been produced with a suitable finite element program. Three loading cases are considered. In the first loading case (LC1; see Fig. 2) the slope is considered to be at a state of an unfavorable groundwater condition, but with a quite short return period (e.g., yearly maximum), whilst seismic forces are ignored. In this loading case the interest is concentrated on the soil property values, ensuring that any common possible error in the laboratory (Lee and Singh 1968; Johnston 1969; Singh 1970) or any stochastic variation across the site will not affect the stability of a slope in practice. The influence of extreme pore-water pressures on stability is quantified in the second loading case (LC2; see Fig. 2) by factoring the pore pressure term (u or r_u), whilst seismic forces are ignored. The extreme pore-water pressures refer to a groundwater condition with large return period (e.g., 50 year return period). In the third loading case (LC3; see Fig. 2), the stability condition of the slope is assessed under the influence of seismic forces while pore water pressures do not exceed a value corresponding to a rainfall with a quite short return period (e.g., yearly maximum). Any possible error in the measurement of soil property values or any stochastic variation of soil material across the site is taken into account in LC2 and LC3 by using relevant partial factors. It is noted that loading cases similar to the ones used in this paper, but without using partial factors, are commonly met in current and past design practices (e.g., Geotechnical Engineering Office 2000; Egnatia Motorway SA 2001; WDOT 2011).

Apart from the above-mentioned loading cases (LC1, LC2, and LC3), additional loading cases could be considered, especially for external forces that are expected to be highly variable during the design life of the slope; the procedure would be analogous to the one presented below for LC1, LC2, and LC3. Favoring simplicity in the presentation of the safety factor expressions, any possible external loading (e.g., footing) has been ignored.

Loading case LC1: Slope subjected to unfavorable groundwater condition, but with short return period

This loading case aims at ensuring that any common possible error in measuring c', φ', and γ in the laboratory or any stochastic variation of soil material across the site will not be the cause for unsatisfactory design and thus, slope failure. The slope is considered to be subjected to static loads whilst the groundwater level corresponds to an unfavorable condition with a short return period. Four subcases are distinguished; namely, the factored shear strength ($\tau_f/F_{c,\tan\varphi}$), the factored unit weight of soil (γF_γ), the factored cohesion (c'/F_c), and the factored friction coefficient ($\tan\varphi'/F_{\tan\varphi}$).

Factoring the shear strength of soils ($\tau_f/F_{c,\tan\varphi}$)

This case refers to the well-known slope stability practice, where the safety factor of slopes is defined by the ratio of the resisting forces (or moments) to the driving forces (or moments). Factoring the shear strength of soil (τ_f) means that the same safety factor value, $F_{c,\tan\varphi}$, is applied to both c' and $\tan\varphi'$, whilst the design value applies to the weight of soil. Thus, the force equilibrium is as follows:

$$(4)\quad \frac{\sum \tau_f}{F_{c,\tan\varphi}} = \sum (W_{i,d} \sin a_i) \quad \text{or} \quad \sum \left[\frac{c'}{F_{c,\tan\varphi}} l_i + (W_{i,d} \cos a_i - u_i l_i) \frac{\tan\varphi'}{F_{c,\tan\varphi}} \right] = \sum (W_{i,d} \sin a_i)$$

Fig. 1. Geometry of slope and elements of the problem. The soil mass over the slip surface has been divided into slices similar to Fellenius' method of slices for the easy presentation of the equations.

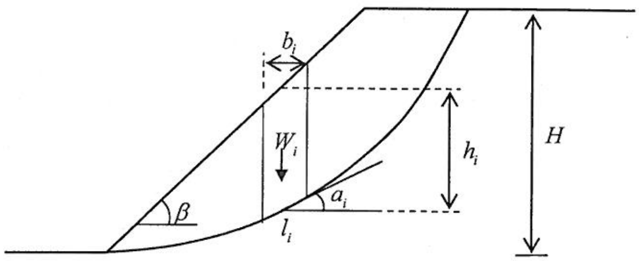
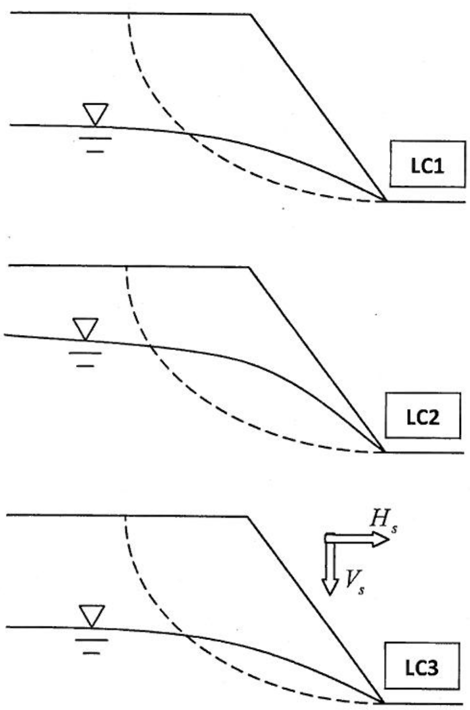


Fig. 2. The three loading cases (LC1, LC2, and LC3) considered in the present paper.



where

$$u_i l_i = (r_u \gamma_d h_i)\left(\frac{b_i}{\cos a_i}\right) = r_u(\gamma_d h_i b_i)\frac{1}{\cos a_i} = r_u \frac{W_{i,d}}{\cos a_i} \quad (5)$$

r_u is the pore pressure ratio (Bishop and Morgenstern 1960) and $W_{i,d} = \gamma_d h_i b_i$. An assumed average r_u value for the whole failure mass is usually specified. Also, b_i and h_i are the width and height, respectively, of a soil part bounded by the failure and the free ground surface (e.g., it may correspond to the width and height of a slice, if a method of slices is used; see Fig. 1).

Substituting eq. (5) into eq. (4) and solving as for $F_{c,\tan\varphi}$

$$F_{c,\tan\varphi} = \frac{\sum\left[(c'l_i) + W_{i,d}\left(\cos a_i - \frac{r_u}{\cos a_i}\right)\tan\varphi'\right]}{\sum(W_{i,d}\sin a)} \quad (6)$$

According to Lee and Singh (1968), Johnston (1969), and Singh (1970), the human error in laboratory measurement of the shear strength values of a soil may be as high as ±50% for cohesion and ±25% for internal friction angle; these studies refer to testing of the same soil by different persons. Such errors in c' and φ' may correspond to a deviation for the mean shear strength of soil of up to ±30% (Singh 1970). Based on this, a minimum safety factor value equal to 1.30 is suggested by the authors to be used for $F_{c,\tan\varphi}$ (see Table 1). As mentioned previously, a design value is also suggested for the weight of soil(s), however, according to current design standards and the literature, a partial factor equal to unity is appropriate for the unit weight of soil (Craig 2004; European Committee for Standardization 2004; Bond and Harris 2008; Driscoll et al. 2008; German Institute for Standardization 2009).

Considering undrained conditions, where $\tau = c_u$ and $\varphi = 0°$, the equilibrium of forces is as follows:

$$\sum\left(\frac{c_u}{F_{c_u}} l_i\right) = \sum(W_{i,d}\sin a_i) \quad (7)$$

and rearranging, the safety factor is

$$F_{c_u} = \frac{\sum(c_u l_i)}{\sum(W_{i,d}\sin a_i)} = \frac{c_u}{\sum(W_{i,d}\sin a_i)/\sum(l_i)} \quad (8)$$

Craig (2004), Driscoll et al. (2008), European Committee for Standardization (2004), and German Institute for Standardization (2009) suggest that the characteristic value of c_u be divided by a partial factor equal to 1.40 (for recommended design values of soil properties see Table 2). As known, the partial factors are used to absorb the various uncertainties related to the numerical values of parameters by increasing or decreasing the characteristic values to conservative design values. On the other hand, a safety factor F_x as for a specific unfactored parameter x indicates how close to a given allowable threshold value the available value is. An F_x value equal to or greater than the respective partial factor f_x aims at ensuring that any extreme but common error in measurement–estimation of parameter x will not adversely affect the slope stability. Based on the above, the authors suggest that $F_{c_u} \geq 1.40$ (see Table 1).

Factoring the unit weight of soils (γF_γ)

Here, the calculation of the safety factor of slope has as the reference point the unit weight of soil γ (the parameter which is factored). An application of this gravity-induced method of slope stability analysis has been presented by Swan and Seo (1999), where in the framework of finite element analysis they applied gravity in increments in the slope model until critical failure mechanisms developed. The intention of this work was to compare the relative performance characteristics of the gravity and strength-reduction methods of slope stability analysis through example problems. Swan and Seo (1999) concluded that the strength-reduction method (see eq. (1)), generally gives more conservative results compared to the gravity-induced method, although there are cases where the gravity-induced method gives smaller stability factor values.

In the present paper the unit weight (or weight) of soils is factored, whilst design values are used for cohesion and the friction coefficient. It is noted that, in case of heterogeneous slopes, the same safety factor value (F_γ) applies to all soil materials. The force equilibrium, therefore, is as follows:

$$\sum[c_d l_i + (F_\gamma W_i \cos a_i - u_i l_i)\tan\varphi_d] = \sum(F_\gamma W_i \sin a_i) \quad (9)$$

Table 1. Minimum recommended safety factor values.

Loading case	Min $F_{c,\tan\varphi}$	Min F_γ	Min F_c	Min F_{c_u}	Min $F_{\tan\varphi}$	Min F_u or min F_{r_u}	Min F_k[a]
LC1							
c', φ'	1.30	1.00	1.50	—	1.25	—	—
c_u, $\varphi = 0°$	—	1.00	—	1.40	—	—	—
LC2	—	—	—	—	—	1.30[b]	—
LC3	—	—	—	—	—	1.00	1.00

[a]Refers to the case where both the vertical and horizontal components of the seismic force are taken into account. If only the horizontal component is considered, then min F_{k_h} = 1.00.
[b]A more conservative minimum recommended safety factor value could be adopted for such a variable and site-specific parameter.

Table 2. Recommended design values of soil properties.

c_d	c_u	$\tan\varphi_d$	γ_d[a]
$c'/1.50$	$c_u/1.40$	$\tan\varphi'/1.25$	$\gamma/1.00$

[a]The design value for the weight of soil $W_{i,d}$ is used instead of γ_d in all formulations.

Substituting eq. (5) into eq. (9) and solving for F_γ

$$(10)\quad F_\gamma = \frac{\dfrac{\sum(c_d l_i)}{\sum\left\{A_i\left[\sin a_i - \left(\cos a_i - \dfrac{r_u}{\cos a_i}\right)\tan\varphi_d\right]\right\}}}{\gamma}$$

where

$$(11)\quad c_d = \frac{c'}{f_c} \quad \text{and} \quad \tan\varphi_d = \frac{\tan\varphi'}{f_{\tan\varphi}}$$

Suggested values for the partial factors applied to the shear strength parameters are:

- Following the design standards CIRIA C641 (Driscoll et al. 2008), European Committee for Standardization (2004), and German Institute for Standardization (2009): $f_c = 1.25$ and $f_{\tan\varphi} = 1.25$. The fact that these two partial factors have the same value ($f_c = f_{\tan\varphi}$ in all design approaches considered in the above standards) indicates the use of the common practice of factoring shear strength τ_f (that is, $f_\tau = f_{c,\tan\varphi} = 1.25$), where the same factor is simultaneously applied to cohesion ($f_c = f_{c,\tan\varphi}$) and friction coefficient ($f_{\tan\varphi} = f_{c,\tan\varphi}$). Besides, as shown immediately below, independent treatment of cohesion requires an f_c value much greater than 1.25.
- Following Craig (2004): $f_c = 1.60$ and $f_{\tan\varphi} = 1.25$ (see eq. (11)).
- Following Hansen (1970): $f_c = 1.75$ and $f_{\tan\varphi} = 1.25$ (see eq. (11)).
- Based on Singh (1970): $f_c = 1.50$ and $f_\varphi = 1.25$ (here, the partial factor f_φ refers to the internal friction angle φ'; see eq. (12)).

$$(12)\quad \varphi_d = \frac{\varphi'}{f_\varphi}$$

A value greater than or equal to 1.00 is, generally, suggested for F_γ (see Table 1). With regard to the partial factors for cohesion and friction coefficient, the authors suggest $f_c = 1.50$ and $f_{\tan\varphi} = 1.25$, respectively (see Table 2).

Factoring the cohesion of soils (c'/F_c)

This case focuses on the cohesion of soils, which is factored, while design values are used for their unit weight and friction coefficient (γ_d and $\tan\varphi_d$ respectively; see Table 2 for suggested design values).

Therefore, the force equilibrium is

$$(13)\quad \sum\left[\frac{c'}{F_c}l + (W_{i,d}\cos a_i - u_i l_i)\tan\varphi_d\right] = \sum(W_{i,d}\sin a_i)$$

Substituting eq. (5) into eq. (13) and solving for F_c

$$(14)\quad F_c = \frac{c'}{\dfrac{\sum\left\{W_{i,d}\left[\sin a_i - \left(\cos a_i - \dfrac{r_u}{\cos a_i}\right)\tan\varphi_d\right]\right\}}{\sum(l_i)}}$$

An interesting observation is that the right-hand side of eq. (14) is similar to the one in eq. (10), differing only on where design values apply.

Apparently, it can be said that in case of homogenous slopes, where c' is constant along the failure surface, eq. (14) follows the definition of safety factor given by Taylor (1948)

$$(15)\quad F_c = \frac{c'}{c_{dev}}$$

where c_{dev} is the average value of the developed cohesion which, according to eq. (14), is given as

$$(16)\quad c_{dev} = \frac{\sum\left\{W_{i,d}\left[\sin a_i - \left(\cos a_i - \dfrac{r_u}{\cos a_i}\right)\tan\varphi_d\right]\right\}}{\sum l_i}$$

Factoring the friction coefficient ($\tan\varphi'/F_{\tan\varphi}$)

This case focuses on the friction coefficient of soils, which is factored, while design values are used for their unit weight and cohesion (γ_d and c_d, respectively; see Table 2 for suggested design values).

Therefore, the force equilibrium is

$$(17)\quad \sum\left\{c_d l_i + (W_{i,d}\cos a_i - u_i l_i)\frac{\tan\varphi'}{F_{\tan\varphi}}\right\} = \sum(W_{i,d}\sin a_i)$$

Substituting eq. (5) into eq. (17) and solving for $F_{\tan\varphi}$

$$(18)\quad F_{\tan\varphi} = \frac{\tan\varphi'}{\dfrac{\sum(W_{i,d}\sin a_i - c_d l_i)}{\sum\left[W_{i,d}\left(\cos a_i - \dfrac{r_u}{\cos a_i}\right)\right]}}$$

Based on the recommendations by Craig (2004), Driscoll et al. (2008), European Committee for Standardization (2004), and German Institute for Standardization (2009), it is suggested that min $F_{\tan\varphi} = 1.25$.

Apparently, it can be said that in the case of homogenous slopes, where φ' is constant along the failure surface, eq. (18) follows the definition of safety factor given by Taylor (1948)

$$(19)\quad F_{\tan\varphi} = \frac{\tan\varphi'}{\tan\varphi_{dev}}$$

where φ_{dev} is the average value of the developed friction angle, which, according to eq. (18), is given as

$$\varphi_{dev} = \arctan\left\{\frac{\sum(W_{i,d}\sin a_i - c_d l_i)}{\sum[W_{i,d}(\cos a_i - r_u/\cos a_i)]}\right\} \tag{20}$$

Loading case LC2: Slope subjected to extreme pore-water pressures

This loading case covers the condition of maximum pore-water pressures expected to occur in the design life of a slope or within a given return period; e.g., according to Geotechnical Engineering Office (2000) slopes in Hong Kong are designed for the groundwater conditions corresponding to a 10 year return period rainfall, whilst according to Egnatia Motorway S.A. (2001) a return period of 50 years is considered. On the other hand, EN 1997-1 (European Committee for Standardization 2004) suggests that design values for groundwater pressures shall represent the most unfavorable values that could occur during the design life of the structure. Seismic forces are ignored as two extreme unfavorable conditions are rather improbable to take place at the same time.

Generally, it is assumed that there is no seepage and the pore pressures are hydrostatic. Alternatively, a seepage analysis could be conducted and the pore pressure can be determined from flownet or finite element analysis. This approach is more reasonable, but is less commonly adopted in practice due to the extra effort required to perform a seepage analysis (Cheng and Lau 2008).

The parameter that is factored, here, is the pore-water pressure, u (or the pore-water pressure ratio r_u) whilst design values are used for the cohesion, friction coefficient, and weight of soil (c_d, $\tan\varphi_d$ and W_d, respectively). The force equilibrium, therefore, is as follows:

$$\sum[c_d l_i + (W_{i,d}\cos a_i - F_{r_u} u_i l_i)\tan\varphi_d] = \sum(W_{i,d}\sin a_i) \tag{21}$$

Substituting ul from eq. (5) into eq. (21) and solving for F_{r_u}

$$F_{r_u} = \frac{\dfrac{\sum\{c_d l_i + (W_{i,d}\cos a_i \tan\varphi_d - \sin a_i)\}}{\sum\left(\dfrac{W_{i,d}}{\cos a_i}\tan\varphi_d\right)}}{r_u} \tag{22}$$

From eq. (22) it is inferred that an exponential relationship between F_{r_u} and r_u exists. Indeed, F_{r_u} becomes infinite when $r_u = 0$, whilst the minimum F_{r_u} corresponds to a fully saturated slope without water table above the slope surface (Michalowski 2009). An infinite value is a totally reasonable outcome as the safety factor expression in question has the meaning of how many times the pore pressures can be increased so that the slope remains marginally stable ($F_{r_u} u$). Of course, if no pore pressures are expected to act in the slope, the examination of this loading case has no sense and should be ignored. In a fully homogenous saturated slope, where, $h_{w,i} = h_i$ and thus $r_u = \gamma_w h_{w,i}/(\gamma h_i) = \gamma_w/\gamma$, the minimum safety factor value with respect to pore pressures is

$$\min F_{r_u} = \frac{\sum[c_d l_i + W_{i,d}(\cos a_i \tan\varphi_d - \sin a_i)]}{\dfrac{\gamma_w}{\gamma}\sum\left(\dfrac{W_{i,d}}{\cos a_i}\tan\varphi_d\right)} \tag{23}$$

The effective determination of the F_{r_u} value depends on how representative the estimated pore pressure, u, or pore pressure ratio, r_u, value is. As, usually, there are no available data referring to past years (years prior to the design of the slope) or, even more, to large return periods of reoccurrence of the extreme event, a quite conservative value is suggested to be taken into account. Driscoll et al. (2008), European Committee for Standardization (2004), German Institute for Standardization (2009), and Egnatia Motorway S.A. (2001) suggest that a safety factor value greater than or equal to 1.30 be used for variable unfavorable actions (including pore-water pressures); see also Table 1 for minimum recommended safety factor values.

Loading case LC3: Slope subjected to seismic loading

Seismic slope stability is usually assessed performing "pseudo-static" analysis where the failure mass is assumed to be horizontally and vertically (often only horizontally) accelerated. The horizontal seismic coefficient, k_h, is appropriately chosen for the expected seismicity of the site (value usually taken from earthquake seismic zone maps or tables). With regard to the vertical seismic coefficient, based on common practice, k_v is expressed by the horizontal seismic coefficient multiplied by a positive, but smaller than unity, coefficient (i.e., $k_v = \lambda k_h$, where, $0 \le \lambda \le 1$).

This loading case refers to slopes that are subjected to seismic forces, whilst an unfavorable groundwater level corresponding to a quite short return period of reoccurrence (e.g., yearly maximum) is considered. Besides, two extreme unfavorable conditions (both with long return period of reoccurrence) are rather improbable to take place at the same time.

Factoring the seismic coefficients

The parameters that are factored, here, are the two seismic coefficients k_h and k_v (horizontal and vertical, respectively), whilst design values are used for cohesion, friction coefficient, and weight of soil mass (c_d, $\tan\varphi_d$, and W_d, respectively). The same safety factor, F_k, can be used for both seismic coefficients (that is, $F_k k_h$ and $F_k k_v$), although in the force equilibrium relation given below, the horizontal component of earthquake has been ignored to avoid lengthy equations:

$$\sum[c_d l_i + (W_{i,d}\cos a_i - u_i l_i)\tan\varphi_d] + F_k k_v \sum(W_{i,d}\cos a_i \tan\varphi_d) = \sum(W_{i,d}\sin a_i) + F_k k_v \sum(W_{i,d}\sin a_i) \tag{24}$$

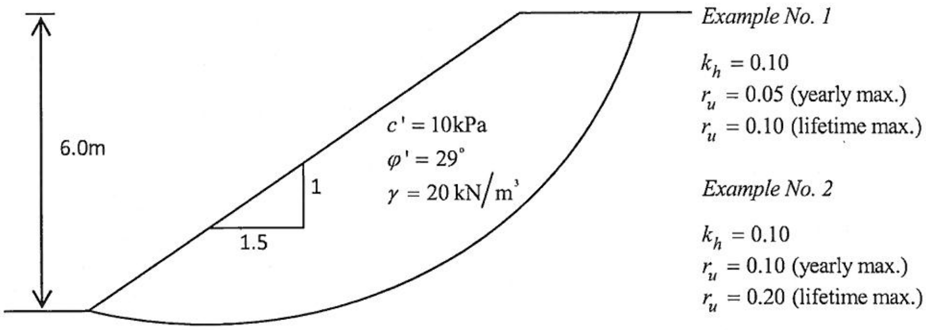
Substituting ul from eq. (5) into eq. (24) and solving for F_k

$$F_k = \frac{\dfrac{\sum\left\{c_d l_i + W_{i,d}\left[\left(\cos a_i - \dfrac{r_u}{\cos a_i}\right)\tan\varphi_d - \sin a_i\right]\right\}}{\sum[W_{i,d}(\sin a_i - \cos a_i \tan\varphi_d)]}}{k_v} \tag{25}$$

From eq. (25) it is inferred that an exponential relationship between F_k and k_v exists. Indeed, F_k becomes infinite when $k_v = 0$. An infinite value is a totally reasonable outcome as the safety factor expression in question has the meaning of how many times the vertical seismic force can be increased so that the slope remains marginally stable ($F_k k_v$). It is obvious that the examination of this loading case has no sense in seismically inactive areas and should be ignored.

A safety factor greater than or equal to unity ($F_k \ge 1.0$) can be considered adequate (Table 1) as the seismic coefficients k_h and k_v considered are already conservative. This is also in agreement with Driscoll et al. (2008), European Committee for Standardization (2004), German Institute for Standardization (2009), and other design regulations (Egnatia Motorway SA 2001) regarding the

Fig. 3. Example Nos. 1 and 2: geometry of slope, soil characteristics, and loading.



safety factor of slopes under accidental loadings (such as seismic forces).

Factoring the pore pressure term (u or r_u)

The pore pressure term can be factored following a procedure similar to the one given for the second loading case (LC2). Ignoring the horizontal component of seismic force for the sake of brevity, the force equilibrium is as follows:

$$(26)\quad \sum[c_d l_i + (W_{i,d}\cos a_i - F_{r_u} u_i l_i)\tan\varphi_d] + k_v \sum(W_{i,d}\cos a_i \tan\varphi_d) = \sum(W_{i,d}\sin a_i) + k_v \sum(W_{i,d}\sin a_i)$$

Substituting *ul* from eq. (5) into eq. (26) and solving for F_{r_u}

$$(27)\quad F_{r_u} = \frac{\dfrac{\sum\langle c_d l_i + W_{i,d}\{\cos a_i \tan\varphi_d - [\sin a_i + k_v(\sin a_i - \cos a_i \tan\varphi_d)]\}\rangle}{\sum\left(\dfrac{W_{i,d}}{\cos a_i}\tan\varphi_d\right)}}{r_u}$$

A safety factor greater than or equal to unity ($F_{r_u} \geq 1.0$) can be considered adequate (Table 1) for the reason explained in the previous paragraph.

Application examples and discussion

The stability of a 6 m high earth slope having gradient 1V:1.5H (Fig. 3), where V is vertical and H is horizontal, has been examined using three different design procedures and more specifically, (*i*) the proposed methodology based on different factoring strategies and partial factor values, (*ii*) the conventional procedure where the safety factor is calculated with respect to shear strength (eq. 1) and without using partial factor values, and (*iii*) the EN 1997-1:2003 (Eurocode 7; European Committee for Standardization 2003), which is based on the use of partial factors presenting four loading cases (mentioned as design approaches DA1, DA2, DA3, and "accidental design situation"). Table 3 shows which design approach has been adopted by each European country. The combination of partial factors used by each design approach of EN 1997-1:2003 are summarized in Table 4.

All safety factor values have been obtained by the freely available finite element program slope1_fs (see www.mines.edu/~vgriffit/) developed by D.V. Griffiths, which allows the different factoring strategies presented in the previous section to be implemented directly. Alternatively, any conventional slope stability software can be used; however, trial solutions are necessary. For example, the safety factor with respect to friction coefficient $F_{\tan\varphi}$ of eq. (19) is obtained by simply performing a number of solutions for various trial $\tan\varphi'$ values. The $\tan\varphi'$ value that gives a safety factor with respect to shear strength (conventional approach) equal to unity is the $\tan\varphi_{dev}$ value that will be used in eq. (19). It is noted that design values are used, where necessary, as described in the previous paragraphs.

The characteristic values of soil properties are: $c' = 10$ kPa, $\varphi' = 29°$, and $\gamma = 20$ kN/m^3 The stability of a slope under two groundwater conditions (thus, two different pore pressure ratios will be used) is examined. In example No. 1, a pore pressure ratio of $r_u = 0.05$ represents an unfavorable but quite common groundwater condition (e.g., yearly maximum), whilst a value of $r_u = 0.10$ represents the maximum pore-water pressure ratio expected to occur during the design lifetime of the slope. The above-mentioned pore pressure ratios have been doubled in example No. 2 ($r_u = 0.10$ and 0.20, respectively).

A horizontal seismic coefficient $k_h = 0.10$ was taken into account in both examples for the calculation of the safety factor of slopes under accidental design situation. The calculated safety factor values are presented in Tables 5 and 6. In addition, the following can be observed:

Table 3. Design approach of EN 1997-1:2003 (European Committee for Standardization 2003) for slopes adopted by each country (Bond and Harris 2008).

Design approach	Country[a]
DA1 (both combinations)	Belgium, Ireland,[b] Italy, Portugal, United Kingdom
DA2	Ireland,[b] Spain
DA3	Austria, Germany, Finland, France, Greece, Ireland,[b] Netherlands, Poland, Romania, Slovenia, Slovakia, Switzerland
Accidental design situation	All the above countries

[a]Unconfirmed for countries other than the ones listed herein (Bond and Harris 2008).
[b]Ireland has adopted all three design approaches.

Table 4. Partial factors f_x for actions, materials, and resistances according to EN 1997-1:2003 (European Committee for Standardization 2003).

Design approach	Permanent actions, f_G[a]	Variable actions, f_Q[b]	Soil parameters f_c	$f_{\tan\varphi}$	f_γ	Resistances, f_R
DA1: Combination 1	1.35	1.50	1.00	1.00	1.00	1.00
DA1: Combination 2	1.00	1.30	1.25	1.25	1.00	1.00
DA2	1.35	1.50	1.00	1.00	1.00	1.10
DA3	1.00	1.30	1.25	1.25	1.00	1.00
Accidental design situation[a]	1.00	1.00	1.00	1.00	1.00	1.00

[a]Applies to the weight of soil and the pore-water pressure.
[b]Applies to pore-water pressure in the case of water rising from its highest normal to its highest possible level.

Table 5. Examples: Safety factor values based on the proposed and conventional design methods.

Loading case	Proposed methodology[a,b] $F_{c,\tan\varphi}$	F_γ	F_c	$F_{\tan\varphi}$	F_{r_u}	F_k	$[F_{c,\tan\varphi}]$[c]
Example No. 1							
LC1	1.685✓	1.974✓	2.545✓	1.859✓	—	—	[1.685]✓
	{1.296}	{1.974}	{1.697}	{1.487}	—	—	{1.204}
LC2	—	—	—	—	2.317✓	—	[1.604]✓
	—	—	—	—	{1.782}	—	{1.234}
LC3	—	—	—	—	1.017✓	0.993✗	[1.383]✓
	—	—	—	—	{1.017}	{0.993}	{1.383}
Example No. 2							
LC1	1.604✓	1.870✓	2.228✓	1.719✓	—	—	[1.604]✓
	{1.234}	{1.870}	{1.485}	{1.375}	—	—	{1.146}
LC2	—	—	—	—	1.159✗	—	[1.441]✓
	—	—	—	—	{0.892}	—	{1.108}
LC3	—	—	—	—	0.509✗	0.723✗	[1.313]✓
	—	—	—	—	{0.509}	{0.723}	{1.313}
Minimum required *F* values (Table 1)							
LC1	1.30	1.00	1.50	1.25	—	—	1.40[d]
LC2	—	—	—	—	1.30	—	1.30[d]
LC3	—	—	—	—	1.00	1.00	1.00[d]

[a]✓ denotes safety factor value greater than or equal to the minimum required, whilst ✗ denotes safety factor value lower than the minimum required.
[b]For direct comparison with Table 6, the derived safety factor values have been divided by the minimum required *F*. These values appear inside curly brackets.
[c]Values in square brackets indicate that safety factor values have been calculated using the *conventional method* of analysis; that is, with respect to shear strength and using characteristic soil property values.
[d]Values taken from Egnatia Motorway S.A. (2001).

- Each safety factor value obtained, in essence, indicates how many times the characteristic value of the parameter of interest (factored parameter) is greater than a critical value corresponding to a just-stable slope ($F = 1.00$). For example, the value $F_c = 2.545$ in Table 5 (see loading case LC1 of example No. 1) means that the characteristic value of cohesion c' is by 2.545 times greater (favorable) than the cohesion value giving a just-stable slope (i.e., $F = 1.00$ for $c' = 10/2.545 = 3.92$ kPa). The above safety factor value with respect to cohesion has been obtained using design values for φ' and γ (i.e., φ_d and γ_d).
- Both EN 1997-1:2003 (Eurocode 7) and the conventional method of analysis consider that the slope of example No. 1 is safe under all loading situations (including the accidental one). Based on the proposed methodology, the same slope satisfies all conditions except for one of the two conditions of loading case 3 (LC3) giving $F_k = 0.993 < 1.0$. The slope of example No. 2 is considered safe according to both the conventional method (all loading cases) and EN 1997-1:2003 (all design approaches except for the accidental design situation). The slope fails only to conform with the "accidental design situation" of EN 1997-1:2003 (Eurocode 7) $F = 0.958 < 1.0$. Based on the proposed methodology the same slope appears unsafe in two out of the three loading cases (LC2 and LC3). Indeed, the calculated safety factor values are much lower than the minimum required ones ($F_{r_u} = 1.159 < 1.3$ for LC2 and $F_{r_u} = 0.509 < 1.0$ and $F_k = 0.723 < 1.0$ for LC3). A safety factor value of $F_{r_u} = 0.509$ means that for the given seismic coefficient ($k_h = 0.1$) the slope can only withstand 50.9% of the design pore pressure ratio value ($r_u = 50.9\%(0.10) = 0.051$ for a just-stable slope). Correspondingly, a safety factor value of $F_k = 0.723$ means that for the given pore pressure ratio ($r_u = 0.1$) the slope can only withstand 72.3% of the design horizontal seismic force ($k_h = 72.3\%(0.10) = 0.072$ for a just-stable slope).
- The proposed methodology may give *F* values much greater than unity, even infinity. In the second loading case (LC2), for example, where the pore pressure parameter *u* is factored, if $u = 0$ then the slope can withstand infinite times this value ($F_{r_u} = \infty$). In such a case the investigation of LC2 is superfluous and should be skipped.

Treating pore pressures as a variable action instead of permanent one with Eurocode 7 does not result in a remarkable difference in the safety factor values, at least for the two examples presented in Table 6.

Summary and concluding remarks

Practitioners have numerous methods at their disposal to perform a slope stability analysis. A common characteristic, however,

Table 6. Examples: Safety factor values using EN 1997-1:2003 (Eurocode 7; European Committee for Standardization 2003); to be compared with the values in curly brackets of Table 5.

	$F = R_d/E_d$ ($F_{min} = 1$)	
Design approach	Treating water as permanent action[a]	Treating water as variable action[a,b]
Example No. 1		
DA1: Combination 1	1.355✓	1.330✓
DA1: Combination 2	1.259✓	1.222✓
DA2	1.232✓	1.209✓
DA3	1.259✓	1.222✓
Accidental design situation[c]	1.313✓	1.313✓
Example No. 2		
DA1: Combination 1	1.138✓	1.090✓
DA1: Combination 2	1.133✓	1.058✓
DA2	1.035✓	0.991✗
DA3	1.133✓	1.058✓
Accidental design situation[c]	0.958✗	0.958✗

[a]✓ denotes safety factor value greater than or equal to the minimum required, whilst ✗ denotes safety factor value lower than the minimum required.

[b]Refers to pore-water pressure corresponding to a situation of water rising from its highest normal to its highest possible level.

[c]Accidental design situation, here, refers to a maximum seismic action expected to occur during the lifetime of the slope.

of all these methods is the expression of safety factor with respect to shear strength of soil(s), which has remained unchanged over time. In the present paper, in addition to the shear strength (τ_f), each soil property (c', $\tan\varphi'$ or γ) or action (e.g., pore pressure, seismic force) is considered as a parameter that can be factored (e.g., c'/F_c, uF_{r_u}). Based on this idea, an approach combining different rational safety factor expressions with widely acceptable loading cases in a limit-state framework has been presented allowing a more comprehensive insight into stability of slopes. The three loading cases considered are (*i*) slope subjected to unfavorable groundwater conditions corresponding to a short return period, (*ii*) slope subjected to extreme pore-water pressures, and (*iii*) slope subjected to seismic loading in combination with unfavorable groundwater condition corresponding to a short return period. Each safety factor, F_x, obtained by the proposed approach indicates how close to a given allowable threshold value is the available value of the parameter x.

Comparison examples have shown that widely used design codes and procedures based either on the limit-state method (use of design values) or the more traditional allowable stress design approach (use of characteristic values) may underestimate the failure hazard of slopes. The interest is concentrated mainly around the factors that are responsible for the vast majority of landslide events; namely, precipitation and earthquakes (Wieczorek and Jäger 1996; Koukis et al. 1997; Pantelidis 2009, 2010), expressed in the analysis by the pore pressure ratio and the seismic coefficient(s), respectively. Among the main findings is that the safety level of slopes expressed by various safety factor expressions with respect to soil properties (shear strength, cohesion, friction coefficient, and unit weight of soil) indicates a significant amount of satisfactory performance compared to the safety level of the same slope as calculated based on the safety factor expressions related to pore-water pressures and seismic loading. This is of particular importance as slopes usually fail in practice during or immediate after (extreme) unfavorable events, such as intense or prolonged rainfalls and earthquakes, and because all current design approaches are based on the unilateral traditional safety factor expression with respect to shear strength of soils. The proposed procedure treats each and every parameter involved in slope stability equally, giving a more comprehensive insight into their relative importance.

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List of symbols

A_i area of the ith slice (used to calculate the weight of slice W_i)
a_i average inclination angle of the base of the ith slice in degrees
b_i width of the ith slice in metres
c cohesion
c' cohesion with respect to effective stresses in kPa (characteristic value)
c_d cohesion with respect to effective stresses in kPa (design value)
c_{dev} developed cohesion
c_u undrained shear strength (characteristic value)
E_d design value of the effects of all the actions
F safety factor
F_c safety factor with respect to cohesion
$F_{c,\tan\varphi}$ safety factor with respect to shear strength
F_{c_u} safety factor with respect to the undrained shear strength of soil
F_k safety factor with respect to seismic coefficient
F_{r_u} and F_u safety factor with respect to the pore pressure ratio and the pore pressure term, respectively
$F_{\tan\varphi}$ safety factor with respect to friction coefficient
F_x safety factor with respect to x. Subscript x indicates the factored parameter (e.g., if the parameter that is factored is the cohesion of soil, then the safety factor with respect to cohesion is given as F_c)
F_γ safety factor with respect to the unit weight of soil
f_x partial factor of a characteristic geotechnical parameter or action X (e.g., f_c is the partial factor for cohesion)
H slope height in metres
H_s horizontal component of the seismic force in kN
h_i average height of the ith slice in metres
$h_{w,i}$ average height of water table above the failure surface at the ith slice in metres
k seismic coefficient
k_h, k_v horizontal and vertical seismic coefficient, respectively
l_i length of the base of the ith slice in metres
R_d design value of the corresponding resistance of the ground
r_u pore pressure ratio
$\tan\varphi'$ friction coefficient of soil
u pore pressures in kPa
V_s vertical component of the seismic force in kN
W_i weight of the ith slice in kN ($W_i = \gamma A_i$)
$W_{i,d}$ weight of the ith slice in kN (design value)
X characteristic geotechnical parameter or action
X_d design value of magnitude X ($X_d = X/f_x$ or $X_d = f_x X$)
x unfactored parameter
β slope angle in degrees
γ unit weight of soil in kN/m^3 (characteristic value)
γ_d unit weight of soil in kN/m^3 (design value)
γ_w unit weight of water (9.81 kN/m^3)
λ coefficient for obtaining k_v from k_h
τ shear strength
τ_f available shear strength along the failure surface in kPa
τ_m mobilized shear stress along the failure surface in kPa
φ friction angle
φ' friction angle with respect to effective stresses in degrees (characteristic value)
φ_d friction angle with respect to effective stresses in degrees (design value)
φ_{dev} developed friction angle