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An analytical solution in probabilistic rock slope stability assessment based on random fields

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ABSTRACT

An analytical solution for calculating the probability of failure of rock slopes against planar sliding is proposed. The method in based on the theory of random fields accounting for the influence of spatial variability on slope reliability. In this framework, both the cohesion and friction coefficient along a discontinuity are treated as Gaussian random fields which are fully described by their mean values ($\mu_c, \mu_{\tan\varphi}$), standard deviations ($\sigma_c, \sigma_{\tan\varphi}$), spatial correlation lengths ($\theta_c, \theta_{\tan\varphi}$), and the parameters ($\rho_{c-\tan\varphi}, \theta_{c-\tan\varphi}$) which account for the cross-correlation between cohesion and coefficient of friction. As shown by the examples presented herein, the spatial correlation of shear strength can have an important influence on slope performance expressed by the probability of failure. This is a significant observation, since ignoring the influence of spatial correlation in design may lead to unconservative estimations of slope reliability.

1. Introduction

Geotechnical engineering is the branch in civil engineering most dominated by uncertainties, as it typically deals with highly variable natural materials. The uncertainties in rock properties arise from three main sources, namely, inherent variability, statistical uncertainty and systematic uncertainties. The inherent variability results from the fact that, even in a homogeneous rock medium the rock properties exhibit variability by nature. Due to limited field sampling and laboratory testing, the statistics (such as, mean and standard deviation) of a rock property will be subject to (statistical) uncertainty. This type of uncertainty decreases with increasing number of samples. Discrepancies between the laboratory and in situ conditions, due to factors such as scale, anisotropy and water saturation are related to systematic uncertainties [1]. The present paper focuses on the inherent variability (which can be relatively large even within so-called homogeneous materials) and its influence on the failure probability. The inherent variability is treated as an aggregate property. The various components of variability, such as material inhomogeneity and discontinuity roughness, and the associated different scales are represented by a single length scale for each random field in the problem.

In common practice, deterministic design methods, as required by design codes, attempt to account for uncertainties related to rock mass by adopting conservative values for the various parameters and relatively large safety factors. Traditional approaches to the longstanding geotechnical problem of rock slope stability generally involve assuming that the rock properties are spatially constant. Common denominator of all these approaches is that the outputs (safety factor values) are based on representative discontinuity property values. However, in highly variable materials, the deviation in property values is not just a statistical number that can simply be ignored, but a parameter that gives important information regarding the performance of structures. The last can easily be confirmed by calculating the failure probability of a given slope (geometry, loading conditions and mean shear strength values) for different standard deviation values of material properties [2,3].

Slope stability analysis has probably received more attention from a probabilistic viewpoint than any other branch of geotechnical engineering. The earliest papers regarding rock slopes appeared in the late 1970s and 1980s [4–9] and have continued steadily [10–21]. Although the geotechnical profession has been quite slow to adopt probabilistic approaches to geotechnical design, especially in traditional areas such as slopes and foundations [22], an increase in their use has been visible in recent years. Evidence of this is the fact

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that several widely used proprietary slope stability codes (such as SWEDGE, ROCKPLANE, SLIDE, SLOPE/W) now include a probabilistic option, and there has been a growth in the availability of short courses on the subject for practitioners and specialty conferences. These software packages employ Monte Carlo simulations to repeatedly calculate the factor of safety with input parameters that are randomly generated according to user-defined probability distributions. The probability of failure is, then, defined as the number of Monte Carlo trials producing a factor of safety less than one divided by the total number of trials [17].

In this paper, an analytical solution for the stability assessment of rock slopes from the probabilistic point of view is proposed. The planar mode of failure is considered. Key element of the solution in question is that it is based on the theory of Random Fields taking into account the influence of spatial variability on slope reliability. Carefully planned and executed core drilling followed by detailed core analysis tests can improve the quantitative description of discontinuities (including its spatial variability) carried out using a rock exposure survey [23]. The concept of Random Fields has already been applied to various geotechnical engineering problems (e.g. including stability of soil slopes, spread foundations, pile foundation, retaining walls) as part of a finite element approach, best known as the Random Finite Element Method (RFEM) [3]. However, to the best knowledge of the authors, the present work constitutes the first *analytical approach* of random fields in geotechnical engineering. The numerical counterpart of this work, based on the Local Average Subdivision method [3,24] for the simulation of the random fields, can be found in [25].

2. The proposed analytical solution

Assuming that the rock block may slide along a planar discontinuity (plane AB in Fig. 1), the following two cases are described. In the first and simpler case, only friction is treated as random field. In the second case, as discontinuity may have cohesion, both cohesion and friction are treated as random fields. For the sake of simplicity, any possible external loading (water pressures, seismic forces, footing etc.) has been ignored. However, all equations given below may easily be transformed according to specific loading situations.

2.1. Treating friction as a random field

Following Coulombs failure criterion, the safety factor of a rock slope against planar sliding is given by the formula

$$F=\frac{cL+\int_0^L t(x)\tan\varphi(x)dx}{W\sin\beta_d} \tag{1}$$

where $t(x)$ is the normal reaction at the base of rock block (per unit length of slope), $\tan\varphi(x)$ is the friction coefficient along the

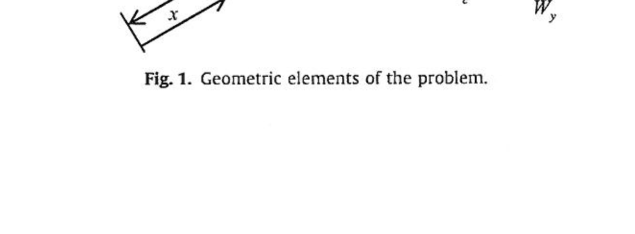

Fig. 1. Geometric elements of the problem.

discontinuity which is assumed to be a function of x (x is a distance along discontinuity on the cross-section plane measured from the lower end of discontinuity), c is the cohesion along the discontinuity which is assumed constant, L is the total length of discontinuity on the cross-section plane, β_d is the inclination angle of discontinuity considering the planar type of failure and W is the total weight of rock block. Inspired from the LAS method proposed by Fenton and Vanmarcke [24], in this respect LAS in one dimension, $\tan\varphi(x)$ is treated as a random field with specified stochastic properties.

Since friction along the discontinuity is variable, it is apparent that the normal force (or reaction) at every point along the discontinuity must be known. The rational assumption that the normal reaction varies linearly along the contact area can be made, especially in the present case where a rigid body lays on a rigid body. The stress distribution under the rock block will have a trapezoidal pattern with maximum and minimum stress values (σ_M and σ_m, respectively) as given by the following equation:

$$\sigma_{M,m}=\frac{W\cos\beta_d}{L}\left(1\pm\frac{e_c}{L/6}\right) \tag{2}$$

where, e_c is the eccentricity of the resultant force acting on the base, in this respect the eccentricity due to the self-weight of rock block (Fig. 1). It is reminded that Eq. (2) is commonly used in retaining wall and spread footing stability problems and it stands for $e_c < L/6$. If the eccentricity e_c is equal to or greater than $L/6$, the rock block is not bearing on its whole base but only on the front edge [26].

Based on the trapezoidal distribution of normal reaction below the rock block of Fig. 1, the normal stress at a given distance x on the discontinuity is

$$t(x)=\frac{x}{L}\sigma_M+\frac{L-x}{L}\sigma_m=\frac{W\cos\beta_d}{L}\hat{t}(x) \tag{3}$$

with

$$\hat{t}(x)\equiv 1+\left(\frac{x}{L}-\frac{1}{2}\right)\frac{2e_c}{L/6} \tag{4}$$

The safety factor expression of Eq. (1) can therefore be rewritten as

$$F=\frac{cL}{W\sin\beta_d}+\frac{Z}{\tan\beta_d} \tag{5}$$

where

$$Z\equiv\frac{1}{L}\int_0^L \hat{t}(x)\tan\varphi(x)dx \tag{6}$$

The friction coefficient $\tan\varphi(x)$ is assumed to be a Gaussian random field with given mean $\mu_{\tan\varphi}$, variance $\sigma^2_{\tan\varphi}$ and covariance function $C(x)$. It should be noted that, although the Gaussian nature of the field allows $\tan\varphi(x)$ to take any real value, the probability density is essentially zero outside of a (physical) range of values. Also, the Gaussian nature of the field is a structure rich enough to be non-trivial but simultaneously allowing the statistical properties of Z and F to be determined in a closed form; that would not be the case if $\tan\varphi(x)$ were log-normal, for example. Thus, the quantity Z is a Gaussian random variable with mean

$$\begin{aligned}E[Z]&=\frac{1}{L}\int_0^L \hat{t}(x)E[\tan\varphi(x)]dx\\&=\frac{1}{L}\int_0^L \hat{t}(x)\mu_{\tan\varphi}dx\\&=\mu_{\tan\varphi}\end{aligned} \tag{7}$$

The variance

$$s^2_{\tan\varphi}=\mathrm{Var}[Z]=E[Z^2]-\mu^2_{\tan\varphi} \tag{8}$$

can be calculated once a specific covariance function of the random field $\tan\varphi(x)$ has been specified. The Markovian

$$C(x)=\sigma^2_{\tan\varphi}\exp\left[-\frac{2|x|}{\theta_{\tan\varphi}}\right] \tag{9}$$

is a convenient choice. $\theta_{\tan\varphi}$ is the correlation length for $\tan\varphi(x)$ (also known as *scale of fluctuation*) of the random field [3]. This explicitly means that

$$E[\tan\varphi(x)\tan\varphi(x')]=\mu^2_{\tan\varphi}+C(x-x') \tag{10}$$

Then, as

$$\begin{aligned}E[Z^2]&=\frac{1}{L^2}\int_0^L dx\int_0^L dx'\ \hat{t}(x)\hat{t}(x')E[\tan\varphi(x)\tan\varphi(x')]\\&=\mu^2_{\tan\varphi}+\frac{1}{L^2}\int_0^L dx\int_0^L dx'\ \hat{t}(x)\hat{t}(x')C(x-x')\end{aligned} \tag{11}$$

the variance of Z is given by the integral

$$s^2_{\tan\varphi}=\frac{\sigma^2_{\tan\varphi}}{L^2}\int_0^L dx\int_0^L dx'\ \hat{t}(x)\hat{t}(x')\exp\left[-\frac{2|x-x'|}{\theta_{\tan\varphi}}\right] \tag{12}$$

Explicitly

$$s^2_{\tan\varphi}=\sigma^2_{\tan\varphi}\left\{\gamma_0(L/\theta_{\tan\phi})+\left(\frac{2e_c}{L/6}\right)^2\gamma_1(L/\theta_{\tan\phi})\right\} \tag{13}$$

where

$$\gamma_0(L/\theta)=\frac{1}{2(L/\theta)^2}(2(L/\theta)-1+e^{-2(L/\theta)}) \tag{14}$$

$$\gamma_1(L/\theta)=\frac{1}{24(L/\theta)}\left(2-\frac{3}{L/\theta}+\frac{3}{(L/\theta)^3}-\frac{3}{L/\theta}\left(1+\frac{1}{L/\theta}\right)^2 e^{-2(L/\theta)}\right) \tag{15}$$

The function $\gamma_0(L/\theta)$ is well known in Markov processes [3], where, $\sigma^2\gamma_0(L/\theta)$ is the variance of the Markov process i.e. the variance of the mean value of the process with point-variance σ^2 and correlation length θ in an interval of length L. The function $\gamma_1(L/\theta)$ is associated with the (linear) distribution of the normal stress along the discontinuity. Thus, the probability distribution of Z is now completely specified:

$$Z\sim N(\mu_{\tan\varphi}, s^2_{\tan\varphi}) \tag{16}$$

One may note that when the correlation length of the random field is large i.e. when the correlation length $\theta_{\tan\varphi}$ is adequately larger than the length of the discontinuity L, the variance $s^2_{\tan\varphi}$ becomes essentially equal to point variance σ^2, as it should. Indeed, Eq. (13) implies that

$$s^2_{\tan\varphi}=\sigma^2_{\tan\varphi}+O(L/\theta_{\tan\varphi}) \tag{17}$$

The symbol $O(x)$ is used in the usual sense to denote a quantity in the order of magnitude x. In the inverse limit, when $\theta_{\tan\varphi}$ is small the variance $s^2_{\tan\varphi}$ vanishes linearly with $\theta_{\tan\varphi}$.

The probability of failure can now be calculated. The safety factor F is a linear function of Z, therefore F is another Gaussian variable and the probability of failure can be immediately computed:

$$\begin{aligned}P(F<1)&=P\left(Z<\tan\beta_d\left(1-\frac{cL}{W\sin\beta_d}\right)\right)\\&=\Phi\left(\frac{\tan\beta_d-\mu_{\tan\varphi}-cL/(W\cos\beta_d)}{s_{\tan\varphi}}\right)\end{aligned} \tag{18}$$

It is instructive to write the probability of failure in terms of the 'deterministic' value of the safety factor:

$$\overline{F}\equiv E[F]=\frac{cL}{W\sin\beta_d}+\frac{\mu_{\tan\varphi}}{\tan\beta_d} \tag{19}$$

In terms of this quantity the probability of failure reads

$$P(F<1)=\Phi\left(\frac{\tan\beta_d}{s_{\tan\varphi}}(1-\overline{F})\right) \tag{20}$$

For given input parameters, Eqs. (18) and (20) provide explicitly the probability of failure. Nonetheless, apart from the obvious usefulness of these equations in a risk assessment based design, the same equation may be used inversely, from the *safety factor* point of view. In this framework, the probability of failure is less than any given value $p_{f\max}$ for

$$\overline{F}\geq 1-z\frac{s_{\tan\varphi}}{\tan\beta_d} \tag{21}$$

where z is determined by the normal distribution: $\Phi(z)=p_{f\max}$. The variable z as a function of $p_{f\max}$ can be found in numerous textbooks in table form [27]; graphically is shown in Fig. 2. If, for example, it is required that the probability of failure should not exceed 10% then Eq. (21) advises that the deterministic safety factor should better satisfy the above equality for $z=-1.28$. Both directions may give important information about the input parameters (e.g. allowable slope face gradient β_s).

2.2. Treating both friction and cohesion as (correlated) random fields

In this case, cohesion varies along the discontinuity and the safety factor is given by

$$F=\frac{\int_0^L\{c(x)+t(x)\tan\varphi(x)\}dx}{W\sin\beta_d} \tag{22}$$

The Gaussian random fields $c(x)$ and $\tan\varphi(x)$ have expectation values μ_c and $\mu_{\tan\varphi}$ respectively. The expectation value $\overline{F}\equiv E[F]$ of the safety factor reads

$$\begin{aligned}\overline{F}&=\frac{1}{W\sin\beta_d}\int_0^L\{E[c(x)]+t(x)E[\tan\varphi(x)]\}dx\\&=\frac{1}{W\sin\beta_d}\int_0^L\left\{E[c(x)]+\frac{W\cos\beta_d}{L}\hat{t}(x)E[\tan\varphi(x)]\right\}dx\\&=\frac{\mu_c L}{W\sin\beta_d}+\frac{\mu_{\tan\varphi}}{\tan\beta_d}\end{aligned} \tag{23}$$

In the proposed solution the possible correlation between cohesion and friction is also taken into account. The correlation functions of the random fields are

$$E[\tan\varphi(x)\tan\varphi(x')]=\mu^2_{\tan\varphi}+C_{\tan\varphi}(x-x') \tag{24}$$

$$E[c(x)c(x')]=\mu_c^2+C_c(x-x') \tag{25}$$

$$E[\tan\varphi(x)c(x')]=\mu_{\tan\varphi}\mu_c+C_{c-\tan\varphi}(x-x') \tag{26}$$

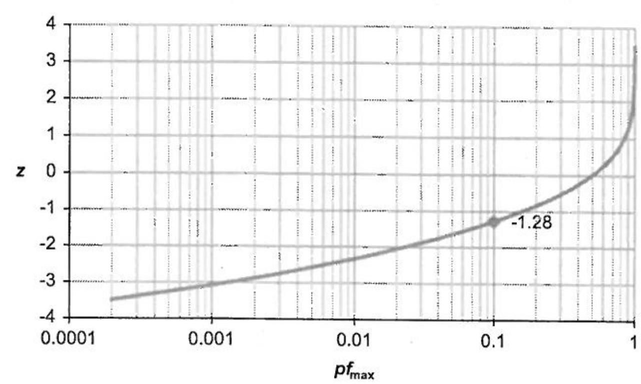

Fig. 2. The variable z of the standard normal distribution as a function of $pf_{\max}$.

Eqs. (24) and (25) express the autocorrelation of the respective random fields, while Eq. (26) expresses the cross-correlation between the two fields. Each covariance function assumes the general Markovian form

$$C_{\tan\varphi}(x)=\sigma_{\tan\varphi}^2\exp\left[-\frac{2|x|}{\theta_{\tan\varphi}}\right] \tag{27}$$

$$C_c(x)=\sigma_c^2\exp\left[-\frac{2|x|}{\theta_c}\right] \tag{28}$$

$$C_{c-\tan\varphi}(x)=\rho_{c-\tan\varphi}\sigma_c\sigma_{\tan\varphi}\exp\left[-\frac{2|x|}{\theta_{c-\tan\varphi}}\right] \tag{29}$$

where $\sigma_{\tan\varphi}$ and σ_c are the point variances of the friction coefficient and cohesion respectively and $\theta_{\tan\varphi}$ and θ_c are the respective correlation lengths. The parameter $\rho_{c-\tan\varphi}$ is the correlation coefficient and $\theta_{c-\tan\varphi}$ is the cross-correlation length. When cross-correlation is regarded as uniform, as it is usually the case, the cross-correlation length is infinite, $\theta_{c-\tan\varphi}=\infty$, and the associated covariance function is reduced to a constant: $C_{c-\tan\varphi}(x)=\rho_{c-\tan\varphi}\sigma_c\sigma_{\tan\varphi}$. Then the variance of the safety factor F can be calculated explicitly:

$$\begin{aligned}E[F^2]=&\left\{\mu_{\tan\varphi}^2+\frac{1}{L^2}\int_0^L dx\int_0^L dx'\,\hat{t}(x)\hat{t}(x')C_{\tan\varphi}(x-x')\right\}\frac{1}{\tan^2\beta_d}\\&+\left\{\mu_c^2+\frac{1}{L^2}\int_0^L dx\int_0^L dx'\,C_c(x-x')\right\}\left(\frac{L}{W\sin\beta_d}\right)^2\\&+\left\{\mu_{\tan\varphi}\mu_c+\frac{2}{L^2}\int_0^L dx\int_0^L dx'\,\hat{t}(x)C_{c-\tan\varphi}(x-x')\right\}\frac{L\cos\beta_d}{W\sin^2\beta_d}\end{aligned} \tag{30}$$

thus,

$$s_F^2\equiv \mathrm{Var}[F]=\frac{1}{\tan^2\beta_d}s_{\tan\varphi}^2+\left(\frac{L}{W\sin\beta_d}\right)^2 s_c^2+2\frac{L\cos\beta_d}{W\sin^2\beta_d}s_{c-\tan\varphi}^2 \tag{31}$$

where,

$$s_{c-\tan\varphi}^2=\rho_{c-\tan\varphi}\sigma_{\tan\varphi}\sigma_c\,\gamma_0(L/\theta_{c-\tan\varphi}) \tag{32}$$

$$s_c^2=\sigma_c^2\,\gamma_0(L/\theta_c) \tag{33}$$

$s_{\tan\varphi}^2$ has been given in Eq. (13). The function $\gamma_0(L/\theta)$ is defined in Eq. (14). The probability of failure now reads

$$P(F<1)=\Phi\left(\frac{1-\overline{F}}{s_F}\right) \tag{34}$$

When the stochastic nature of the field $c(x)$ is irrelevant, i.e. when $s_c=0$ and $s_{c-\tan\varphi}=0$, Eq. (34) is reduced to Eq. (20).

2.3. The dimensionless form of the results

The number

$$\Lambda_{c\varphi}=\frac{(W/L)\cos\beta_d\mu_{\tan\varphi}}{\mu_c} \tag{35}$$

is defined here as a natural analogue of the dimensionless parameter $\lambda_{c\varphi}$ which appears in soil slope stability analysis [28,29]. This definition allows one to re-write Eq. (23) for the expectation value of the safety factor in the form

$$\overline{F}=\frac{\mu_{\tan\varphi}}{\tan\beta_d}\left(1+\frac{1}{\Lambda_{c\varphi}}\right) \tag{36}$$

Introducing also the coefficient of variance for each random field by $\sigma_{\tan\varphi}=\mathrm{COV}_{\tan\varphi}\mu_{\tan\varphi}$ and $\sigma_c=\mathrm{COV}_c\mu_c$ the complete formula for the variance of the safety factor, given by Eq. (31), reads

$$\begin{aligned}s_F^2=&\frac{\mu_{\tan\varphi}^2}{\tan^2\beta_d}\left[\mathrm{COV}_{\tan\varphi}^2\left\{\gamma_0(L/\theta_{\tan\phi})+\frac{4e_c^2}{(L/6)^2}\gamma_1(L/\theta_{\tan\phi})\right\}\right.\\&\left.+\frac{\mathrm{COV}_c^2}{\Lambda_{c\varphi}^2}\gamma_0(L/\theta_c)+2\rho_{c-\tan\varphi}\mathrm{COV}_{\tan\varphi}\frac{\mathrm{COV}_c}{\Lambda_{c\varphi}}\gamma_0(L/\theta_{c-\tan\phi})\right]\end{aligned} \tag{37}$$

By these quantities the probability of failure, given in Eq. (34), can be calculated. Inspecting Eq. (37), one observes that the cohesion field weighs in by the factor $\mathrm{COV}_c/\Lambda_{c\varphi}$ while the friction field weighs in by $\mathrm{COV}_{\tan\varphi}$. That is, when the level of uncertainty is similar between cohesion and friction, a relatively large factor $\Lambda_{c\varphi}$ renders the uncertainty of the cohesion practically unimportant, as far as the estimation of the probability of failure is concerned.

3. Application examples

3.1. Treating friction as random field

Two examples of the probability of failure as a function of the correlation length θ are given below for various μ and σ values of the field $\tan\varphi(x)$, where, the probability of failure p_f has been plotted against the normalized correlation length θ/L; see Figs. 3 and 4. Cohesion is assumed constant along the discontinuity (deterministic value). Therefore, different values of the mean μ correspond to different values of the deterministic safety factor F. The curves are labelled according to both the associated value of the deterministic safety factor F and μ. The following data stand for both examples: $\beta_d=30^\circ$, $c=40$ kPa, $L=10$ m, $e_c=0.5$ m and $W=9000$ kN /m.

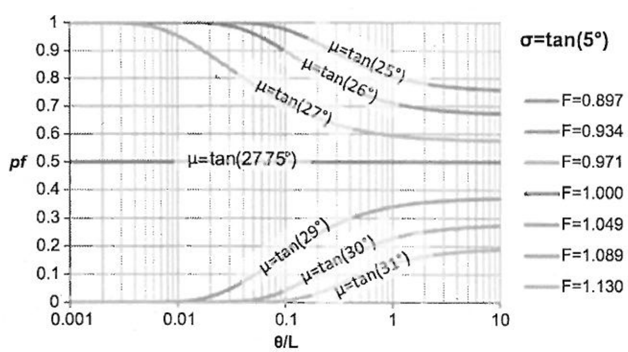

Fig. 3. Example: pf vs θ/L plot for various μ values and for $\sigma=\tan(5^\circ)$.

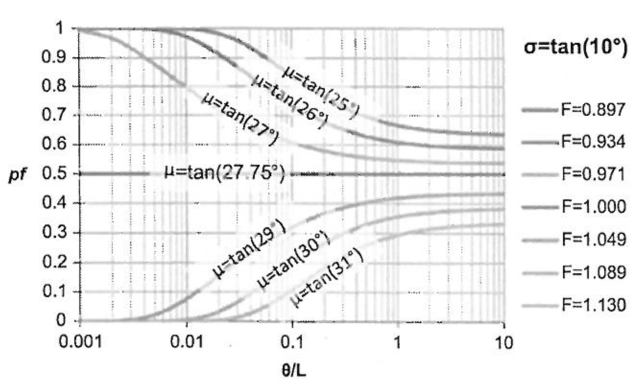

Fig. 4. Example: pf vs θ/L plot for various μ values and for $\sigma=\tan(10^\circ)$.

The general features of the probability of failure curves shown in the Figs. 3 and 4 can be described as follows. As the correlation length θ of the random field $\tan\varphi(x)$ becomes smaller the system tends to behave more in a deterministic way, that is, the probability of failure tends to 0 or 1 depending on whether the deterministic value of the safety factor is above or below the value 1. When the correlation length θ becomes comparable to length of the discontinuity, then the entire random field $\tan\varphi(x)$ tends to behave like a single Gaussian random variable. Indeed, by Eq. (9), in the limit of large θ ($\theta\to\infty$) the covariance function $C(x)$ approaches everywhere the constant value σ^2. Explicitly, as mentioned in Eq. (17) the variance s^2 that enters the probability of failure formulas (18) or (20), tends to σ^2 for large correlation lengths. Thus, in the limit of large θ the probability of failure approaches an asymptotic value that depends only on the point variance σ^2 of the random field $\tan\varphi(x)$. The effect of greater variance σ^2, observed by comparing the Figs. 3 and 4, is to introduce stronger deviations from the deterministic answers for the probability of failure, 0 or 1, on the left part of the curves, and a stronger convergence towards the indecisive 0.5 value of the probability of failure, on the right part of the curves. Finally, it is mentioned that, both plots are in agreement with those obtained by Griffiths and Fenton [2] for soil slopes.

3.2. Treating both friction and cohesion as random fields

In Fig. 5, the probability of failure p_f has been plotted against both $\theta_{\tan\varphi}/L$ and θ_c/L in a 3D graph for the same data given above but only for the case of $\mu_{\tan\varphi}=\tan(26°)$ and $\mu_{\tan\varphi}=\tan(30°)$ setting $\sigma_{\tan\varphi}=\tan(5°)$ (common in both cases) and $\mu_c=40$ kPa, $\sigma_c=20$ kPa for the cohesion field. The cross-correlation of the two fields is assumed uniform with $\rho_{c-\tan\varphi}=0.2$.

As with the case of the single random field, when the correlation lengths of the coefficient of friction and the cohesion random fields approach zero the curves tend towards the deterministic answers, probability of failure 0 or 1. When, on the other hand, the correlation lengths are comparable to or larger than the length L of the discontinuity then the probability of failure, given by Eq. (34), tends to towards asymptotic values that depends on the variances $\sigma^2_{\tan\varphi}$, σ^2_c and $\rho_{c-\tan\varphi}$. Indeed, in the limit of large correlation lengths the variance s_F^2 of the safety factor, given by Eq. (31), tends towards a fixed value given by

$$\frac{1}{\tan^2\beta_d}\sigma^2_{\tan\varphi}+\left(\frac{L}{W\sin\beta_d}\right)^2\sigma_c^2+2\frac{L\cos\beta_d}{W\sin^2\beta_d}\rho_{c-\tan\varphi}\sigma_{\tan\varphi}\sigma_c \tag{38}$$

which determines the asymptotic value of the probability of failure via Eq. (34).

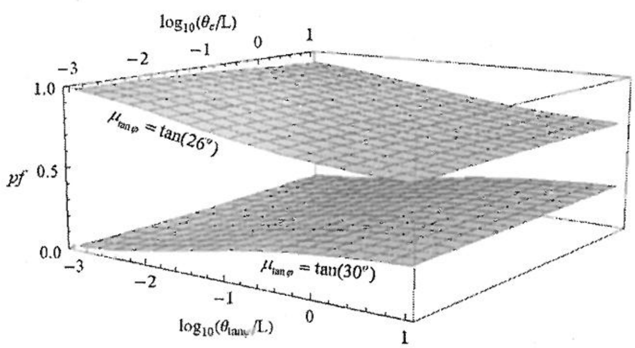

Fig. 5. pf vs $\theta_{\tan\varphi}/L$ and θ_c/L plot.

3.3. The effect of the cohesion uncertainties

As explained in Section 2.3 the significance of the cohesion uncertainties in the estimation of the probability of failure are determined by the number $\Lambda_{c\varphi}$, or more specifically by the relative size of $\mathrm{COV}_{\tan\varphi}$ and $\mathrm{COV}_c/\Lambda_{c\varphi}$. In the examples of the Sections 3.1 and 3.2 a moderate value for the cohesion was used, which corresponds to the relatively large number $\Lambda_{c\varphi}\sim 20$. The effect of cohesion becomes significant for much smaller values of this number. In the present example such cases are presented. Case 1: $\mu_{\tan\varphi}=\tan(13°)$ and $\mu_c=240$ kPa; Case 2: $\mu_{\tan\varphi}=\tan(17°)$ and $\mu_c=240$ kPa. Cases 1 and 2 correspond to $\Lambda_{c\varphi}$ equal to 0.67 and 0.50 respectively. The Cases 1 and 2 are contrasted for random friction with $\mathrm{COV}_{\tan\varphi}=0.5$ and deterministic cohesion and for random friction and random cohesion with $\mathrm{COV}_{\tan\varphi}=0.5$, $\mathrm{COV}_c=0.5$ and correlation coefficient $\rho_{c-\tan\varphi}=0.3$. The results are shown in Fig. 6.

3.4. Determining the minimum safety factor for given probability of failure

This example illustrates the content of Eq. (21). For any desired maximum value for the probability of failure there is an associated minimum necessary value for the safety factor, which depends on the correlation length. Considering the example of Section 3.1 for the case $\sigma_{\tan\varphi}=\tan(5°)$, the dependence of the minimum necessary safety factor has been plotted as a function of the normalized correlation length θ/L for the cases $p_{f\,\max}=10\%$, 1%, 0.1%. This is shown in Fig. 7.

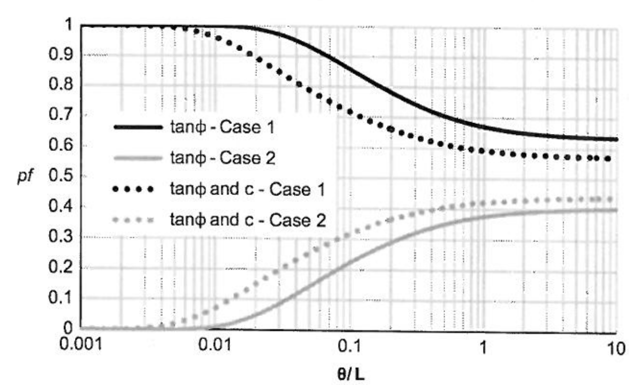

Fig. 6. Example: pf vs θ/L plot showing the effect of cohesion uncertainties on the probability of failure.

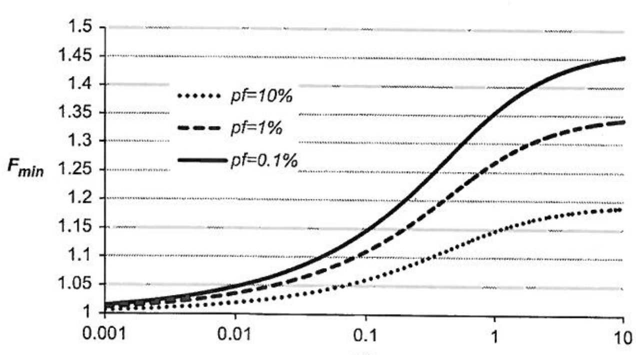

Fig. 7. Minimum necessary safety factor $F_{\min}$ vs θ/L for maximum desired probability of failure $p_{f\,\max}=10\%$, 1%, 0.1%. Clearly the value of $p_{f\,\max}$ associated with the curves decreases upwards.

4. Summary and conclusions

Soils and rocks are among the most variable of all engineering materials, and as such are highly amenable to probabilistic treatment [30]. Acknowledging the significance of spatial variability of shear strength along discontinuities, an analytical solution based on the theory of random fields for the calculation of the probability of failure of rock slopes against planar sliding is proposed. In this respect, both cohesion and friction coefficient of discontinuity are treated as Gaussian *random fields* fully described by their mean values ($\mu_c, \mu_{\tan\varphi}$), point standard deviations ($\sigma_c, \sigma_{\tan\varphi}$), correlation lengths ($\theta_c, \theta_{\tan\varphi}$) and the parameters ($\rho_{c-\tan\varphi}$, $\theta_{c-\tan\varphi}$) which account for the possible dependence between cohesion and the coefficient of friction.

The examples presented herein highlight the strong influence of scale of fluctuation of both cohesion and coefficient of friction on slope performance. Indeed, different correlation length values may correspond to totally different probability of failure values. Simplified probabilistic analyses, in which spatial variability is ignored by assuming perfect correlation, can lead to unconservative estimates of the probability of failure. This effect is most pronounced at relatively low factors of safety or when the coefficient of variation ($COV = \sigma/\mu$) of the discontinuity strength is relatively high. The above are in full agreement with the results presented by Griffiths and Fenton [2] for soil slopes using the Random Finite Element Method (RFEM).

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