Hydro-Mechanical Coupled Model of Hydraulic Fractures using the eXtended Finite Element Method

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ABSTRACT: This paper describes a hydro-mechanical coupled model for hydraulic fractures using the eXtended Finite Element Method (XFEM). To analyze the non-linear coupling behavior of hydraulic fractures, the XFEM uses local enrichment functions within a regular finite element framework. Thus, the mechanical behavior of a domain with fractures can be efficiently estimated by the method without explicitly meshing the discontinuities as would be necessary in other numerical techniques. The governing equations and coupling algorithms are presented as well as the required assumptions for hydraulic fractures. The 2D coupled XFEM program is validated by a numerical example study and shows coupled flows and their influence on stresses and fracture deformations.

INTRODUCTION

The role of hydraulic fracturing for enhanced recovery of unconventional oil and gas has grown dramatically in recent years. Realistic modeling of fracture and fluid flow in these systems is sought in order to improve physical understanding and efficiency. Such modeling remains highly challenging however, due to a number of factors including material heterogeneity, complex fracture propagation mechanisms, coupled behavior of rock stresses and fluid pressure, and interactions between pre-existing natural and artificially initiated hydraulic fractures.

For the numerical analysis of the hydraulic fractures, the discrete fracture network model (DFN) has been widely used (e.g. Xu and Dowd, 2010; Azadi and Khoei, 2011). However the numerical technique is computationally expensive to consider the moving boundary of the fracture propagation, since the fractures must be explicitly modeled as additional finite meshes. Therefore when the fractures and its geometry evolve, the entire model must be re-meshed and the global matrices reassembled and factorized in standard FE schemes.

A relatively new technique for finite element modeling of fractures without re-meshing developed by Belytschko and Black (1999) and Moës *et al*. (1999) has great promise and is explored in this paper. This technique named the eXtended Finite

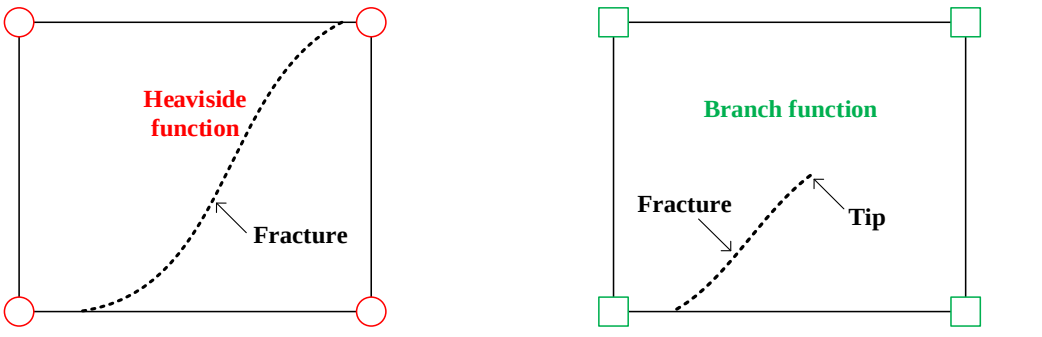
Element Method (XFEM) involves a standard displacement-based approximation, enriched near a fracture and a tip. Discontinuous basis functions are added to standard shape functions for nodes that belong to the enriched elements. A major advantage of the XFEM is that the finite element mesh does not need to be updated to track the discontinuous path. The addition of a discontinuous field allows for the entire fracture geometry to be modeled independently of the mesh, and completely avoids the need to re-mesh as the fracture propagates.

XFEM has been successfully applied in various branches of engineering regarding fractures. The method was further developed by incorporating extensions known as weak and strong discontinuities which represent the continuous and discontinuous strain distribution along fracture boundaries (e.g. Sukumar and Prevost 2003) and by considering cohesive fracture propagation, where the non-linear behavior of fracture is caused by increased strength of material and dissipation of energy at the fracture tip (e.g. Bellec and Dolbow 2003). XFEM has also been adopted for modeling plasticity and large deformation (e.g. Fagerström and Larsson 2006), composite materials (e.g. Ashari and Mohammadi 2011), and fatigue fracture elongation and failure (e.g. Pais *et al*. 2012).

The main objective of this paper is to present the advanced coupled finite element program for hydraulic fractures. An XFEM scheme is developed based on the public domain software of Smith and Griffiths (2004) allowing for representation of fractures without any need for re-meshing. The 2D XFEM studies examine the fractured system can be deformed and coupled flows and their influence on stresses and fracture deformations. Since the surrounding rock is assumed to be very tight, fluid flow may only get through the fracture, and fluid leak off to the surrounding rock is neglected.

MECHANICAL BEHAVIOR OF FRACTURE USING THE XFEM

The XFEM uses local enrichment functions to enhance the traditional finite element shape functions, making the approximated displacement field and its derivatives discontinuous within an element. Once the fracture geometry and location are defined, different enrichment terms are selectively applied to the different element groups, namely elements divided by a fracture, or elements containing a fracture tip (See Fig. 1).



(a) Element divided by a fracture **(b) Element containing a fracture tip**

FIG. 1. Example of enriched elements

The enrichment functions include modified Heaviside and Branch functions that represent normal displacement of the fracture surfaces and the fracture tip asymptotic fields based on the Westergaard solution (Moës *et al*., 1999), respectively. The Heaviside function is first applied to elements entirely cut by a fracture and is given as below in Eq. (1).

$$H(\mathbf{x}) = \begin{cases} +1 & \varphi(\mathbf{x}) \geq 0 \\ -1 & \varphi(\mathbf{x}) < 0 \end{cases} \tag{1}$$

where $\varphi(x)$ is the signed distance function normal to the fracture. Branch functions are used to enrich elements that contain fracture tips and are given as in below.

$$\left[\Phi_\gamma(\mathbf{x})\right]_{\gamma=1}^{4} = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2}\right] \tag{2}$$

where r and θ are the polar coordinates of the point x in the coordinate system centered at the tip of the fracture with the *x*-axis aligned to the fracture direction. The sum of the Heaviside and Branch enrichment terms results in the following displacement field within a discontinuous domain:

$$\begin{aligned} \mathrm{u}(\mathbf{x}) = \sum_{I\in\mathcal{N}} N(\mathbf{x})\mathrm{u}_I + \sum_{J\in\mathcal{N}_{fr}} N(\mathbf{x})\left[H\big(\varphi(\mathbf{x})\big) - H\left(\varphi\big(\mathbf{x}_J\big)\right)\right]\mathrm{a}_J \\ + \sum_{K\in\mathcal{N}_{tip}} N(\mathbf{x}) \sum_{\gamma=1}^{4}\left[\Phi_\gamma(\mathbf{x}) - \Phi_\gamma(\mathbf{x}_K)\right]\mathrm{b}_{\gamma k} \end{aligned} \tag{3}$$

COUPLING ALGOLITM FOR A FRACTURE

For coupling purposes, a 1D fluid flow model is added to the XFEM program, and a sequential coupling scheme as suggested by Yew (1997) and Dahi (2009) using Picard iteration is applied. For the simplification of the model, an incompressible Newtonian fluid is assumed, and fluid leak off to the surround rock mass and fracture propagation are not considered. Those assumptions are reasonable when the surrounding rock has very small pore size and thus has almost zero permeability.

The weak form of the fluid mass balance equation (*e.g*. Rungamornrat *et al*. 2005; Segura and Carol 2008; Chen 2013) is applied to calculate the hydraulic pressure to the fracture and is presented as Eq. (4).

$$\int_L \frac{\Delta w}{\Delta t} N_i(x)dx + \int_L q_L(t)N_i(x)dx = \frac{1}{12\mu}\int_0^L \frac{\partial}{\partial x}\left(w^3\frac{\partial P}{\partial x}\right)N_i(x)dx \tag{4}$$

Although q_L is the fluid leak-off rate along the fracture surface, it is neglected due to the assumption that the surrounding rock is nearly impermeable. Boundary conditions are applied at each end of the fracture, for fluid injection at the wellbore

and zero fracture opening at the fracture tip. The boundary conditions are summarized in Eqs. (5) and (6).

$$\frac{1}{12\mu}\left(w^3\frac{\partial P}{\partial x}\right)N_i(x)\Big|_{well} = Q_i N_1(0) \tag{5}$$

$$P|_{tip} = 0 \tag{6}$$

Once the fluid pressure along the fracture is estimated, it is then applied to the global force vector, and the modified force vector is used by the XFEM program to calculating the trial solution of fracture aperture. The explicit fracture opening may be estimated using the additional degrees of freedom as suggested by Dahi (2009) and Chen (2013)

$$w(x) = \begin{cases} 2\sum_{J\in\mathcal{N}_{cr}} N(x)\mathrm{a}_J\,, & \text{For fracture body} \\ 2\sqrt{r}\sum_{K\in\mathcal{N}_{tip}} N(x)\mathrm{b}_{1K}\,, & \text{For fracture tip} \end{cases} \tag{7}$$

The entire process is repeated for each time step, and the resulting two trial solutions (w_k and $w_{k+1/2}$) for the fracture opening are compared with each other to enable an updating as shown in Eq. (8).

$$w_{k+1} = (1-\alpha)w_k + \alpha w_{k+1/2} \tag{8}$$

Convergence is determined by comparing the relative error between w_k and w_{k+1} for all elements along the fracture as shown in Eq. (9).

$$\varepsilon \geq \frac{\sum|w_{k+1} - w_k|}{\sum|w_k|} \tag{9}$$

Iterations stop when the relative error is less than 10^{-6}, at which time the fracture aperture and fluid pressure are taken as the converged solution.

SIMULATION RESULTS AND VALIDATION

A simple hydraulic fracturing example is presented in this section. The coupled fracture model analyzes only the half of the original fracture geometry due to the horizontally symmetric fracture shape, and the fracture tip is located at the center of the domain. Rollers are used to constrain lateral deformations along the left and right edges of the entire domain, and the right bottom corner is fully fixed. The details of the example model including the problem mesh, boundary condition, selection of the enriched nodal points are presented in Fig. 2.

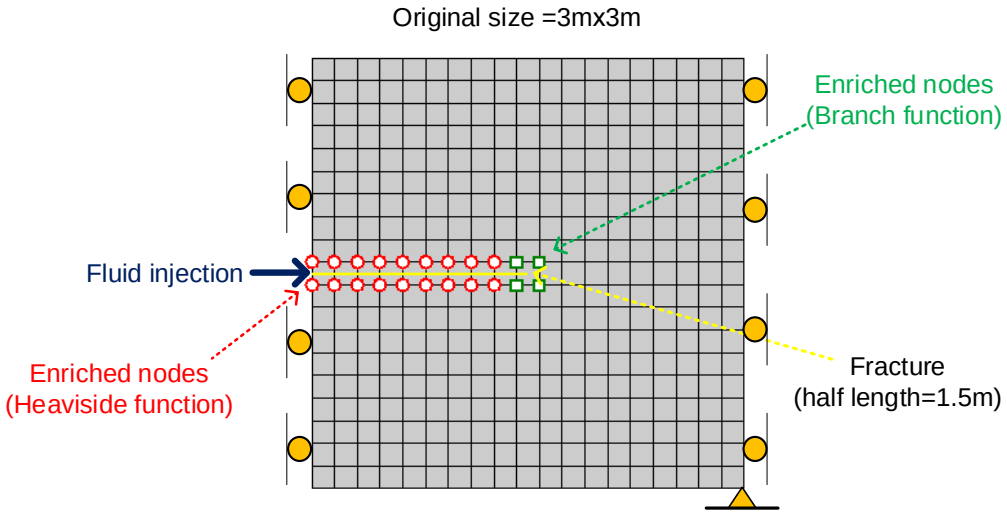


FIG. 2. Finite mesh for the example study

The hydraulic pressure is applied along the fracture and yields additional displacements along the enriched elements successfully. The recommended range of the Picard iteration coefficient ($0.1 < \alpha < 0.3$) reported by Yew (1997) and Dahi (2009) should be maintained, otherwise convergence may be slowed down or not achieved at all.

Table 1. Input Parameters

Young's modulus E	**Poisson's ratio** ν	**Injection rate** Q_i	**Fluid viscosity** μ
$4 \times 10^6 \text{Pa}$	0.25	$5\text{m}^3/\text{min}$	$2 \times 10^{-5} \text{Pa} \cdot \text{min}$

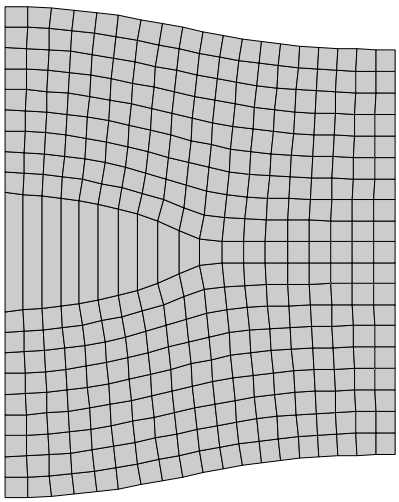


FIG. 3. Deformed mesh (deformation scale factor=200)

Figure 4 presents the fracture aperture convergence in the example study. The convergence coefficient is equal to 0.25, and it takes a total of 17 iterations, with the final relative error of about $6.30 \cdot 10^{-7}$.

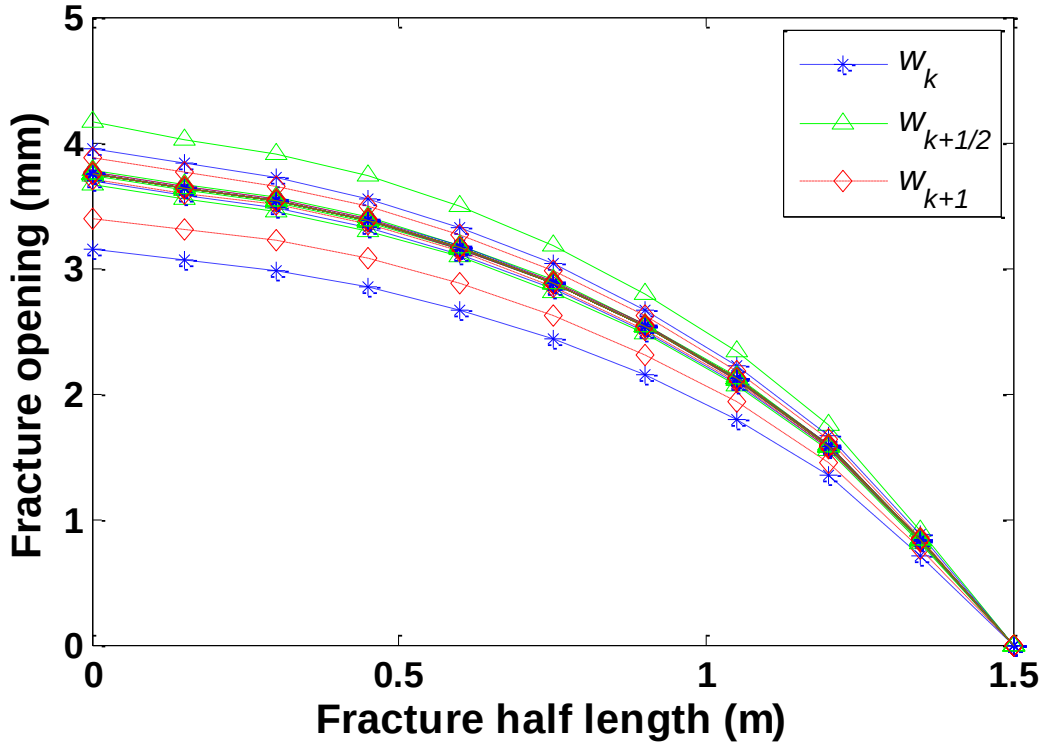


FIG. 4. Convergence of the fracture aperture

For the validation of the current numerical program , the fracture opening at the injection point from the simulation is compared to the analytical solution as in Eq. (10) given by Geertsma and de Klerk (1969). The simulation results matched quite well (the relative error is about 1.62%).

$$w_{well}^{anal}(t) = \left(\frac{5376}{\pi^3}\frac{\mu q_0^3}{E'}\right)^{\frac{1}{6}} t^{\frac{1}{3}} = 2.36\left(\frac{\mu q_0^3}{E'}\right)^{\frac{1}{6}} t^{\frac{1}{3}} = 3.69 \text{ mm} \quad (10)$$

$$w_{well}^{XFEM}(t) = 3.75\text{mm} \quad (11)$$

Figures 5 and 6 show the converged fluid pressure and fracture opening after the convergence, respectively.

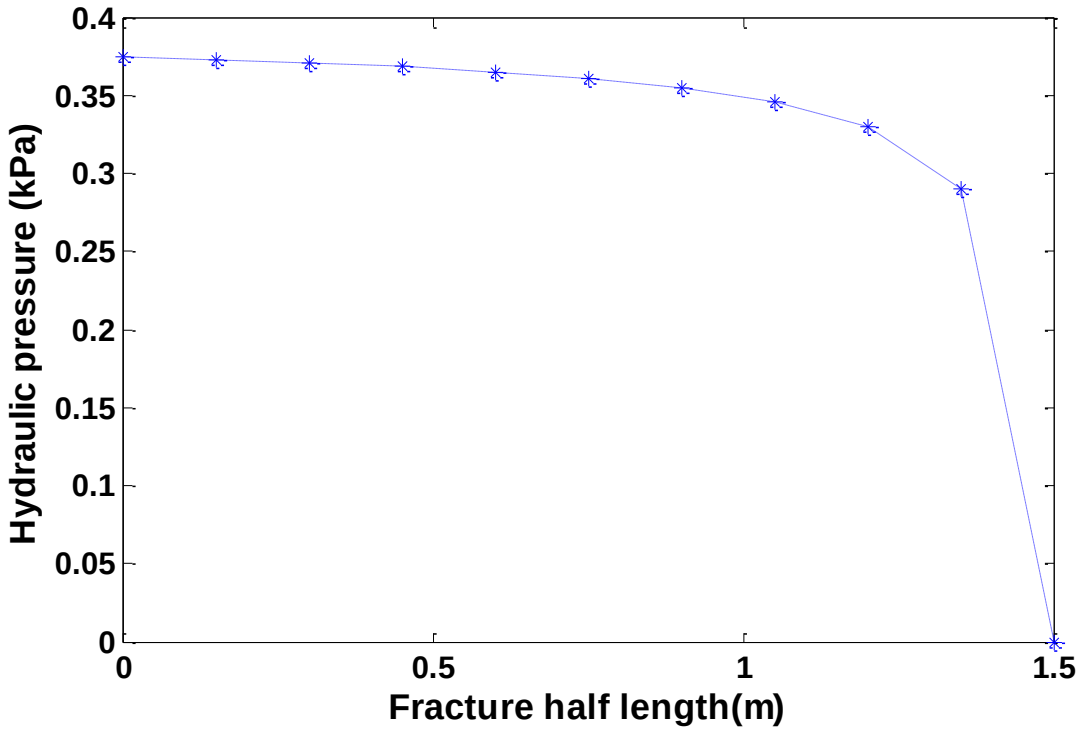


FIG. 5. Converged hydraulic pressure along the fracture

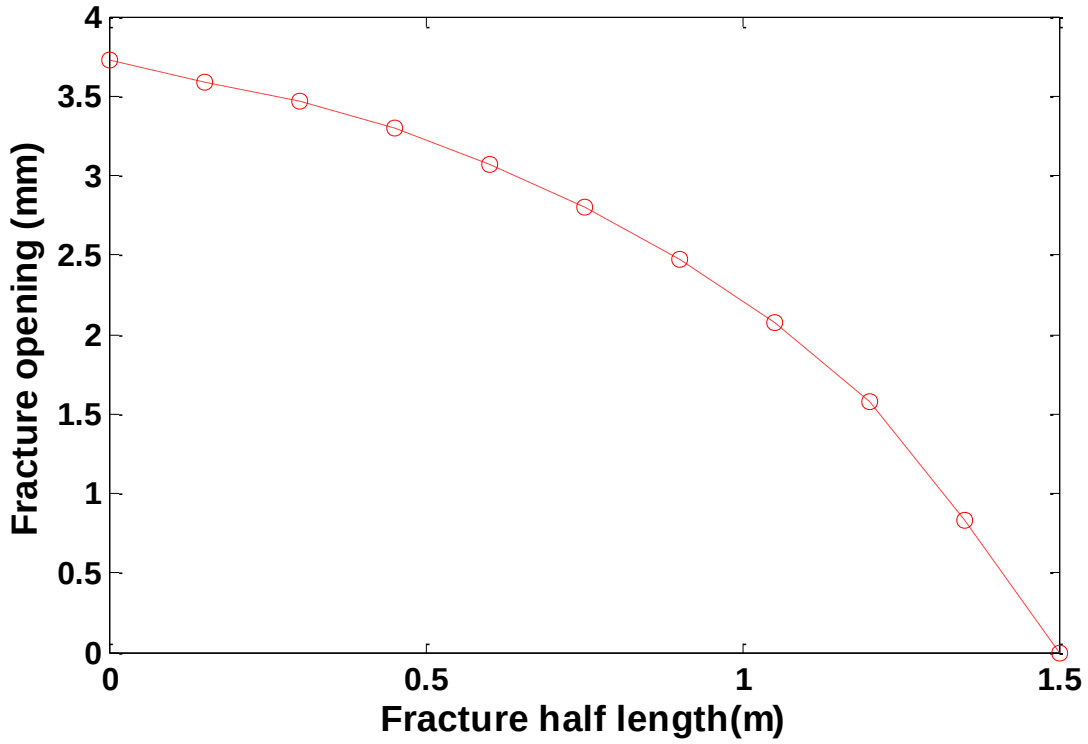


FIG. 6. Converged fracture aperture distribution

CONCLUSIONS

An algorithm for hydro-mechanical sequential coupling has been successfully developed, and the simulation results presented. This numerical model yields reasonable estimation of the key parameters, fluid pressure and fracture opening without compromising computational efficiency which can be easily caused by other numerical technique such as DFN. For the examples presented, convergence was achieved in a acceptable number of iterations by starting with a reasonable initial guess as to the fracture aperture and fluid pressure.

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NOTATION

a_J: additional degrees of freedom matched to Heaviside enrichment function
$b_{\gamma K}$: additional degrees of freedom matched to Branch enrichment function
E and v: Young's modulus and Poisson's ratio
E': effective Young's modulus
$H(x)$: Heaviside enrichment function
I, J and K: order of nodes
L: fracture domain
m: number of element in each element
N: shape function
$\mathcal{N}$, $\mathcal{N}_{fr}$ and $\mathcal{N}_{tip}$: groups of regular, Heaviside enriched and Branch enriched nodes
P: hydraulic pressure
Q_i: fluid injection rate at wellbore
q_L: fracture surface leak-off rate
q_0: fluid injection rate over fracture height
r: fracture tip distance in polar coordinate system
t: time
u: displacement (standard degrees of freedom)
u(x): displacement approximation from the XFEM program
w: fracture aperture
w_k: k^{th} trial solution of fracture aperture
w_{well}^{anal}: fracture opening at well bore measured from the analytical solution
w_{well}^{XFEM}: fracture opening at well bore measured from the XFEM program
x: 1D coordinate along fracture
x: Cartesian coordinate vector = (x, y)
α: convergence coefficient for Picard iteration
ε: convergence criterion
θ: fracture tip angle in polar coordinate system
μ: fluid dynamic viscosity

$\varphi(x)$: signed distance function normal to the fracture
$\Phi_\gamma(x)$: Branch enrichment function

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