Use of Computer Algebra Systems to generate element stiffness matrices

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Abstract

Finite element matrices such those for stiffness and mass are usually generated using Gaussian quadrature because it leads to convenient formulations in terms of local coordinates. This paper describes how element matrices can be generated in *closed form* with the help of Computer Algebra Sytems (CAS) and how this can lead to improvements in run-times. The CAS approach is also shown to be able to generate matrices which would normally be obtained using reduced or selective reduced integration (SRI).

Consider a square 4-node elastic element in plane strain, fixed on two adjacent sides, with a horizontal force P applied to the free corner. Computer Algebra can be used to compute the horizontal deflection of the corner for various element stiffness integration schemes. The expressions show why locking occurs as the material approaches incompressibility with full integration.

Integration	δ_H	$\delta_H(\nu \approx 0.5)$
FULL	$\dfrac{96(3-4\nu)(1+\nu)(1-2\nu)P}{(9-16\nu)(15-16\nu)E}$	0
SRI(λ,μ)	$\dfrac{16(2-3\nu)(1+\nu)P}{3(5-6\nu)E}$	$\dfrac{2P}{E}$
SRI(G,K)	$\dfrac{144(17-25\nu)(1+\nu)P}{25(43-50\nu)E}$	$\dfrac{2.16P}{E}$
REDUCED	$\dfrac{(3-4\nu)(1+\nu)P}{(1-\nu)E}$	$\dfrac{3P}{E}$

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