Stochastic Analysis of Hydraulic Fracture Propagation using the eXtended Finite Element Method and Random Field Theory

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Abstract. Hydraulic fracturing is an important technology to increase the amount of production extracted from unconventional hydro-carbon reservoirs. In spite of the recent proliferation of the stimulation technique, the technical understanding of how fractures initiate, propagate, and interact with material heterogeneity is not well established. The need for improved scientific understanding of this methodology has motivated this research, which seeks to develop probabilistic finite element software for modeling fracture propagations in random formations. The finite element development proposed herein will combine the eXtended Finite Element Method (XFEM) with random field theory to characterize fracture propagation within heterogeneous tight hydro-carbon reservoirs without any need for re-meshing. The XFEM not only has a potential for improved modeling accuracy, but also reduces computational costs that might be needed when using a standard finite element method. Stochastic modeling of hydraulic fracturing will be used to further account for randomly distributed formation properties of the tight formation and the resulting various hydraulic fracturing patterns. The new methodology from this research will be called eXtended Random Finite Element Method (XRFEM). When combined with a Monte-Carlo simulation approach, this methodology will lead to probabilistic information on the response of various formations and enable better technical and financial risk management of unconventional reservoir stimulation. Parameters used in the XRFEM modeling of well stimulation are subject to different types and levels of uncertainties caused by inherent spatial variability in geological formations.

Keywords. hydraulic fracture propagation, eXtended Finite Element Method (XFEM), random field, stochastic analysis, eXtended Random Finite Element Method (XRFEM)

1. Introduction

Hydraulic fracturing is the most widely used stimulation technology enhancing the amount of hydro-carbon production from unconventional formations. Although the technology can significantly increase hydro-carbon production from low permeability reservoirs, the interactions and complex nature is still not fully understood.

Development of realistic simulation tools for the hydraulic fracturing process is therefore an important step towards understanding the complex, multiscale and multiphysics phenomena and developing efficient and environmentally safe hydraulic fracturing technologies during the production. However, the numerical simulation of hydraulic fracture growth remains a significant challenge due to a number of factors including material heterogeneity, complexity of fracture propagation mechanisms, and interactions between multiple hydraulic fractures.

To deal with the complex behavior of hydraulic fracturing, the main objective of this research is set to develop an advanced finite element program to provide a better analysis tool for hydraulic fractures in various formations. In this paper, an eXtended Finite Element Method (XFEM) scheme is developed allowing mechanical representation of fractures and its propagation within heterogeneous formations without any need for re-meshing. The XFEM not only has a potential for improved modeling accuracy, but also reduces computational costs that might be required when using a standard finite element method.

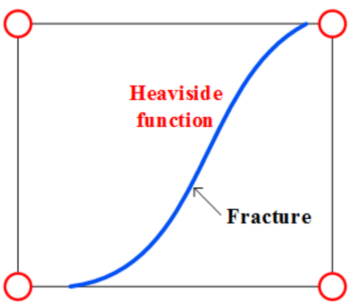
Material heterogeneity also plays a key role in determining fracture initiation and propagation. Thus a novel probabilistic approach combining the XFEM and random field theory is proposed, namely eXtended Random Finite Element Method (XRFEM). This new method will be able to model fractures and account for randomly distributed properties (e.g. stiffness and strength) within unconventional formations. This analysis will allow users to customize their analysis for different site-specific conditions and to evaluate the effectiveness of hydraulic fracturing as a form of stochastic analysis. To improve the applicability and accessibility of the XRFEM program, the codes are developed based on the "in-house" FE program written in Fortran 2003.

2. Fracture modeling within random heterogeneous formation

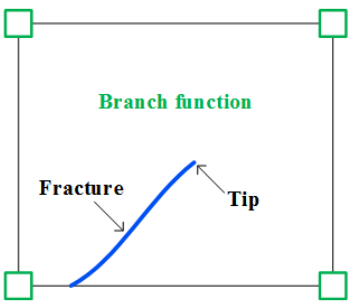
2.1. eXtended Finite Element Method (XFEM)

XFEM was first introduced by Belytschko and Black (1999) and Moës *et al.* (1999). They presented enrichment functions that could be added to the traditional finite element approximation. Therefore the numerical scheme allows discontinuous displacement along a discontinuous fracture, and therefore the displacement is entirely independent to the mesh.

To represent the discontinuous deformation along a fracture, the XFEM utilizes two different enrichment functions, namely Heaviside and Branch. The Heaviside function is applied to the elements entirely cut by a fracture as shown in Figure 1(a) and given by Eq. (1).



(a) Element divided by a fracture



(b) Element containing a fracture tip

Figure 1. Example of enriched elements (Youn and Griffiths 2014)

$$H(\mathbf{x}) = \begin{cases} +1 & \psi(\mathbf{x}) \geq 0 \\ -1 & \psi(\mathbf{x}) < 0 \end{cases} \tag{1}$$

where $\psi(\mathbf{x})$ is the signed distance function normal to the fracture. The Heaviside enrichment function represents fracture aperture changes between the fracture surfaces. Branch functions are used to enrich elements that contain fracture tips as shown in Figure 1(b) and given by Eq.(2).

$$\left[\Phi_\gamma(\mathbf{x})\right]_{\gamma=1}^{4} = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2}\right] \tag{2}$$

where r and θ are the polar coordinates of the point $\mathbf{x}$ in the coordinate system centered on the tip of the fracture with the *x*-axis aligned in the fracture direction (See Figure 2). The Branch enrichment function is used to represent fracture asymptotic fields suggested by Westergaard fracture tip stress and displacement analysis (Moës *et al.*, 1999).

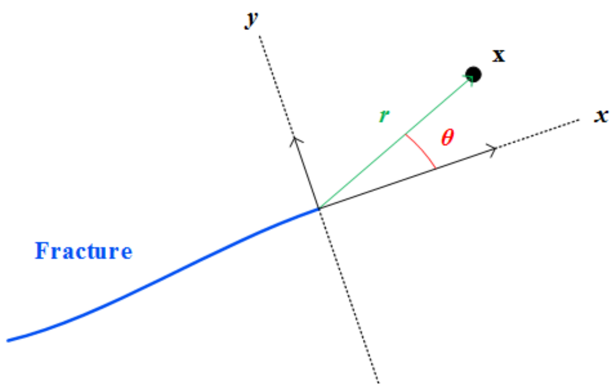


Figure 2. Fracture tip polar coordinates, r and θ (Youn and Griffiths 2014)

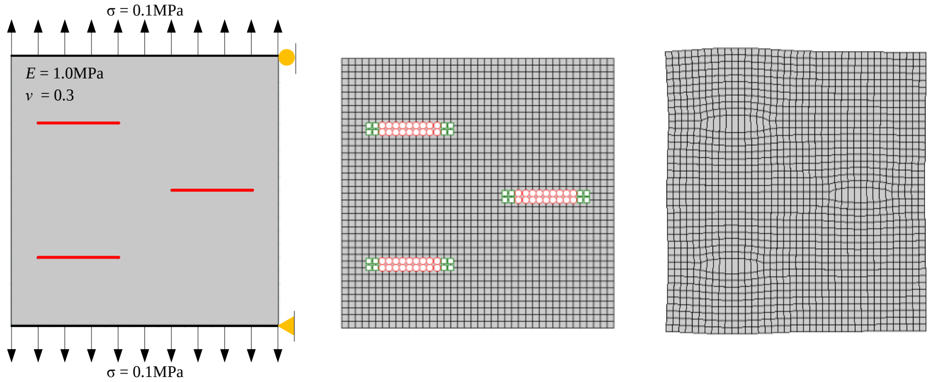
Inclusion of the Heaviside and Branch enrichment terms results in the following displacement field within a continuous domain:

$$\begin{aligned} \mathbf{u}^{XFEM}(\mathbf{x}) = & \sum_{I\in\boldsymbol{\mathcal{N}}} N^I(\mathbf{x})\mathbf{u}_I + \sum_{J\in\boldsymbol{\mathcal{N}}_{cr}} \hat{N}^J(\mathbf{x})\left[H\big(\psi(\mathbf{x})\big) - H\Big(\psi(\mathbf{x}_J)\Big)\right]\mathbf{a}_J \\ & + \sum_{K\in\boldsymbol{\mathcal{N}}_{tip}} \hat{N}^K(\mathbf{x}) \sum_{\gamma=1}^{4}\left[\Phi_\gamma(\mathbf{x}) - \Phi_\gamma(\mathbf{x}_K)\right]\mathbf{b}_{\gamma k} \end{aligned} \tag{3}$$

where $\mathbf{u}_I$ is the nodal displacement vector for the continuous part of the normal finite element solution, $N^I, \hat{N}^J$ and $\hat{N}^K$ are finite element shape functions, $\boldsymbol{\mathcal{N}}$ is the set of all nodes in the mesh, $\boldsymbol{\mathcal{N}}_{cr}$ is the set of nodes of the divided element by the fracture (red circles in Figure 1(a)), $\boldsymbol{\mathcal{N}}_{tip}$ is the set of nodes surrounding an element contains a fracture tip (green squares in Figure 1(b)) and $\mathbf{a}_J$ and $\mathbf{b}_{\gamma k}$ are the nodal enriched degrees of freedom vectors for Heaviside and Branch enrichment functions, respectively.

Figure 3 shows an example of an XFEM simulation using the “in-house” FE code developed at CSM and based on the public domain software of Smith *et al.* (2014).

This is part of an initial study of XFEM to understand how enrichment functions are implemented. Figure 3(a) shows three horizontal 1D line fractures spread over a square domain which is then subjected to a uniform tensile stress applied to the upper and lower boundaries of the domain. Once the fracture geometry and location are defined, different enrichment terms are selectively applied to the different element groups, namely elements without a fracture, elements divided by a fracture, or elements containing a fracture tip.



(a) Schematic of the example study (b) Enriched node selection (c) Deformed mesh

Figure 3. An example study using XFEM and COMSOL

Due to the enrichment functions applied to the regular mesh geometry, the additional displacements by XFEM along the fractures are developed as shown in Figure 3(c).

To consider fracture propagation, a constant propagation length is added to the previous fracture tip with the orientation calculated using stress intensity factors (SIFs) based on the assumption that the energy release rate *G* is equal to the *J*-integral. The fracture direction is determined using Eq. (4) given by Moës *et al*. (1999).

$$\theta_f = 2\arctan\frac{1}{4}\left(\frac{K_I}{K_{II}} \pm \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right), \quad -\pi < \theta_f < \pi \tag{4}$$

where θ_f is relative orientation of fracture propagation comparing to the angle of the previous fracture tip and K_I and K_{II} are the SIFs for mode 1 and mode 2 fracture displacements, respectively.

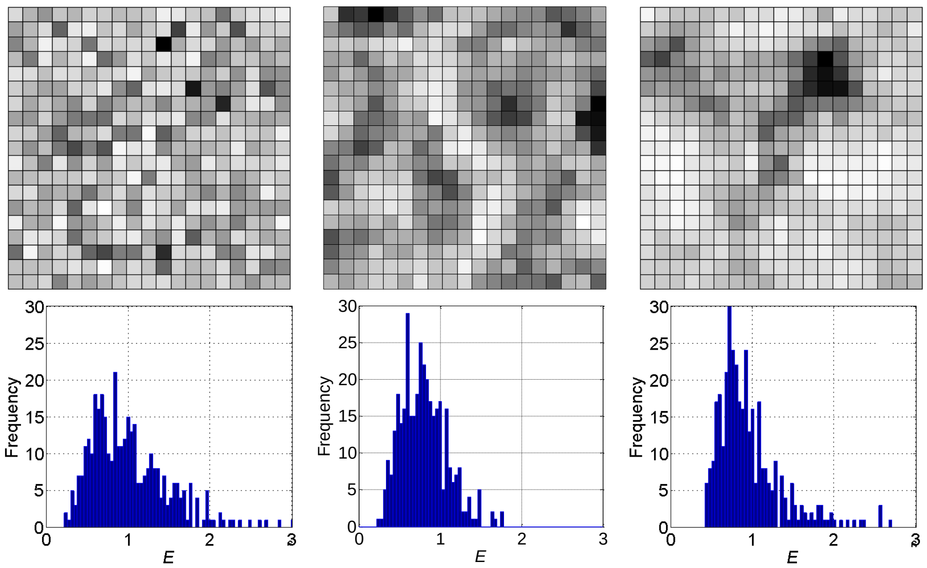
2.2. Random field and spatial correlation

The combined use of random fields and finite elements in a Monte-Carlo framework was first introduced in the early 1990s (Griffiths and Fenton 1993). The method provides a systematic way of introducing material variability with statistically defined properties given by a mean, a standard deviation and a spatial correlation length. Any appropriate formation property distribution such as elastic modulus or fracture toughness can be characterized this way.

The spatial correlation length represents the distance over which the rock property is reasonably well-correlated to its neighbors. The stochastic concept can be used to model anisotropic correlation structures by applying different correlation lengths in the horizontal and vertical directions. In this work, a Markovian correlation function is used where the spatial correlation is assumed to decay exponentially with distance (Vanmarcke 1984) as in Eq. (5).

$$\rho = e^{-2|\tau|/\theta} \tag{5}$$

where $|\tau|$ is the absolute distance between any two points in the random field. The influence of θ on a wide range of geotechnical systems has been assessed through parametric studies (e.g. Griffiths *et al*. 2009, Klammer *et al*. 2010, Huang *et al*. 2010, Kasama and Whittle 2011, Chen and Zhang 2013,) and has been shown to have a significant influence on probabilistic output quantities under considerations. Furthermore, the correlation length has been shown to affect the nature and extent of failure zones which is an important aspect of this research in relation to fracturing.



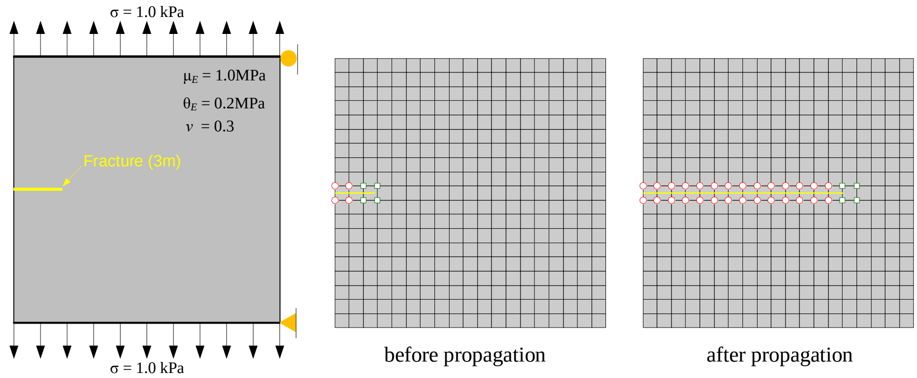
(a) correlation length=0m (b) correlation length=5m (c) correlation length=10m

Figure 4. Random field examples and histograms of Young's modulus with different correlation lengths (mean=1MPa, standard deviation=0.5MPa)

Figure 4 shows examples of random fields and histograms in terms of Young's modulus (*E*) distribution. The lighter zones indicate smaller Young's modulus, so that the finite elements are more compressible and vice versa. A lognormal distribution is applied to generate the random property distributions, which is used in the example studies given below. The random field examples (20m×20m) present the effect of different spatial correlation lengths, while the mean and standard deviation are kept to constant (mean=1MPa and standard deviation=0.5MPa). As shown in Figure 4(a), a zero correlation length generates a fully independent distribution for each neighboring element. As the correlation length increases however, elements with similar property values are grouped together, and the size of the group gets larger.

3. Simulation results

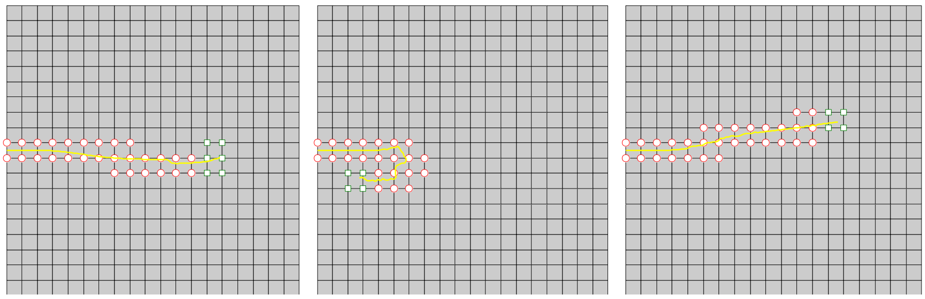
To evaluate the applicability of XRFEM and the effect of random property distributions on fracture propagation, an example study is performed with an initial horizontal fracture located along the middle of the domain (See Figure 5a). A 3m length fracture is located on the left part of the square domain (20m×20m), and the entire domain is subjected to a uniform tensile stress applied to the upper and lower boundaries of the domain.



(a) Schematic of the example study (b) Enriched node selections and fracture geometry

Figure 5. An example study using XRFEM

To run the XRFEM, randomly distributed Young's modulus are calculated by the random field program, and then those calculated random parameter is assigned into the each finite element independently. The Young's modulus distributions used in the above section 2 (See Figure 4) are applied for these case studies.

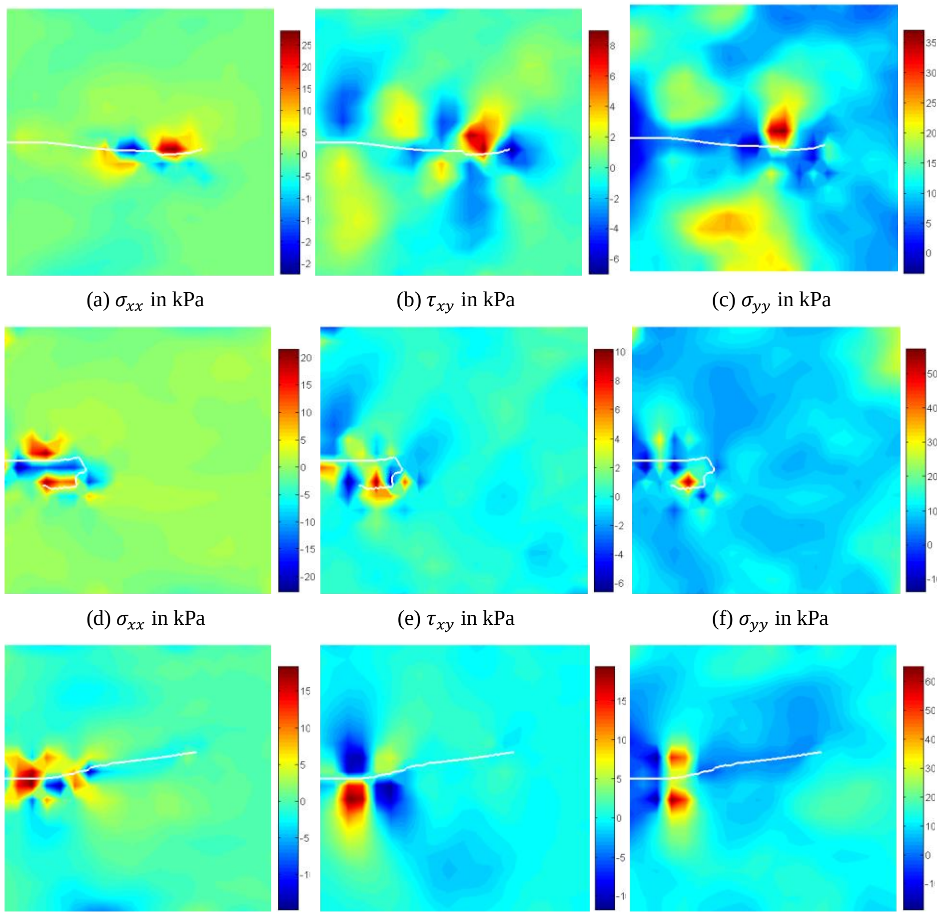


(a) correlation length=0m (b) correlation length=5m (c) correlation length=10m

Figure 6. Fracture propagation pattern within random formations

Although the regular XFEM with constant properties yield a straight propagation as in Figure 5, a randomly distributed Young's modulus creates irregular concentration of stress near the fracture and tip, so that the fracture pattern differs significantly from the constant property case (See the yellow line indicating the fracture geometry in Figures 5 and 6).

As shown in Figure 7, the fracture propagations tend to move through the brighter zone, but only the element near the vicinity of the fracture tip could directly affect the direction of the propagation. This can be mainly due to the size of the zone used to calculate the stress intensity factor. Therefore the element having the maximum quantity of Young's modulus does not significantly contribute to the direction of propagation, where the distance between the element and the fracture tip is far from each other.



(a) σ_{xx} in kPa (b) τ_{xy} in kPa (c) σ_{yy} in kPa

(d) σ_{xx} in kPa (e) τ_{xy} in kPa (f) σ_{yy} in kPa

(g) σ_{xx} in kPa (h) τ_{xy} in kPa (i) σ_{yy} in kPa

Figure 7. Stress contours (in kPa) within random fields of different correlation lengths (correlation length=0m for Figures 7(a)-7(c), correlation length=5m for Figures 7(d)-7(f), and correlation length=10m for Figures 7(g)-7(i))

By performing a series of Monte-Carlo simulation with this methodology, the final achievable data will be the average extension of the hydraulic fracture measured from the location of the wellbore or injection point. In this paper, however only several example cases are tested as an initial step in the XRFEM development. This study will be further extended by investigating different probability functions, correlations between different material parameters, and the introduction of hydro-mechanical coupling scheme.

4. Result and discussion

The XFEM program has been combined with random property generation based on random field theory and will be used to investigate the stochastic behavior of hydraulic fracturing within various geological formations. Parameters used in the XRFEM modeling of hydraulic fracture are subject to different types and levels of uncertainties caused by inherent spatial variability in geological formations. Initial parametric studies were carried out by controlling the key probabilistic parameters, spatial correlation length. It is clearly shown that the combination of those two different numerical schemes works well, and the randomness of the rock property directly affects the final pattern of fracture propagation, which may in turn significantly affect efficiency of hydraulic fractures.

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