Simulación con Métodos Numéricos: Nuevas Tendencias y Aplicaciones
O. Prado, M. Rao y M. Cerrolaza (Editores) © 1998 SVMNI

ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF STABILITY PROBLEMS IN GEO-ENGINEERING

D. V. Griffiths
Division of Engineering, Colorado School of Mines, Golden CO 80401-1887 U.S.A.

ABSTRACT: Slope stability analysis is a classical problem in geotechnical engineering which is usually performed using limit equilibrium methods that have remained essentially unchanged in decades. The elasto-plastic finite element method is ideally suited for tackling slope stability problems since it requires no a priori assumptions regarding the shape or location of the potential failure surface. The paper presents some examples of slope stability analysis in which the finite element method offers real advantages over traditional approaches.

INTRODUCTION

The great majority of slope stability analyses performed in geotechnical practice use limit equilibrium methods that have remained essentially unchanged for decades. Some of the best known methods include Taylor's methods (Taylor 1937), Bishop's Method (Bishop 1955, Bishop and Morgenstern 1960) and Janbu's Method (Janbu 1968) . In all these methods an initial assumption has to made regarding the shape and range of locations of the potential failure surface followed by an automated computerized search for the failure surface leading to the lowest "factor of safety". Essentially the methods return the lowest upper bound within the set of failure surfaces included in the search. The "factor of safety" (FOS) is defined (e.g. Duncan 1996) :

$$FOS = \frac{\text{Shear strength of soil}}{\text{Shear stress required for equilibrium}}$$

Many slope problems, especially those with reasonable homogeneous soils conditions, lead to rather simple circular failure surfaces which are conveniently found using the limit equilibrium methods described above. Even non-circular failure surfaces can be found using the limit equilibrium approaches although the user must decide in advance whether a search for potential failure surfaces of this type is appropriate.

The finite element method for assessing slope stability becomes a superior alternative to the traditional approaches when awkward geometries and/or non-homogeneous soil properties are encountered. In these cases, the user of limit equilibrium methods might initiate a search for an incorrect set of failure surface that led to an unconservative local minimum that was greater then the "correct" global minimum.

The finite element approach on the other hand will indicate a failure mechanism "naturally" in that the mesh will fail wherever the shear stresses due to the gravity loading exceed the shear strength of the soil. The FE method places no restriction on the shape or location of the critical failure surface.

BRIEF DESCRIPTION OF FE METHOD USED

The program is based closely on Program 6.1 in the text by Smith and Griffiths (1988)–the main difference being the ability to model more realistic geometries and better graphical output facilities. The programs are for 2-d (plane strain) analysis of elastic-perfectly plastic soils with a Mohr-Coulomb failure criterion. The programs use 8-node quadrilateral elements with reduced integration (4 Gauss-points per element) in both the stiffness and stress redistribution phases of the algorithm. A gravity 'turn-on' procedure generates nodal forces which act in the vertical direction at all nodes. These loads are applied in a single increment and generate normal and shear stresses at all the Gauss-points within the mesh. These stresses are then compared with the Mohr-Coulomb failure criterion. If the stresses at a particular Gauss-point lie within the Mohr-Coulomb failure envelope then that location is assumed to remain elastic. If the stresses lie on or outside the failure envelope, then that location is assumed to be yielding. Global shear failure occurs when a sufficient number of Gauss-points have yielded to allow a mechanism to develop.

The analysis is based on an iterative Modified Newton-Raphson method called the Viscoplastic algorithm (Zienkiewicz *et al* 1975). The algorithm forms the global stiffness matrix once only with all nonlinearity being transferred to the right hand side. If a particular zone within the soil mass is yielding, the algorithm attempts to redistribute those excess stresses by sharing them with neighboring regions that still have reserves of strength. The redistribution process is achieved by the algorithm generating self-equilibrating nodal forces which act on each element that contains stresses that are violating the failure criterion. These forces, being self-equilibrating, do not alter the overall gravity loading on the finite element mesh, but do influence the stresses in the regions where they are applied. In reducing excess stresses in one part of the mesh however, other parts of the mesh that were initially 'safe' may now start to violate the failure criterion themselves necessitating another iteration of the redistribution process. The algorithm will continue to iterate until both equilibrium and the failure criterion at all points within the soil mass are satisfied within quite strict tolerances.

If the algorithm is unable to satisfy these criteria at all yielding points within the soil mass, 'failure' is said to have occurred. Failure of the slope and numerical non-convergence occur together, and are usually accompanied by a dramatic increase in the nodal displacements. Within the data, the user is asked to provide an iteration ceiling beyond which the algorithm will stop trying to redistribute the stresses. Failure to converge implies that a mechanism has developed and the algorithm is unable to simultaneously satisfy both the failure criterion (Mohr-Coulomb) and global equilibrium.

SOIL MODEL

An elastic-perfectly plastic (Mohr-Coulomb) model has been used in this work consisting of a linear (elastic) section followed by a horizontal (plastic) failure section.

The soil model used in this study consists of six parameters as shown in Table 1.

Table 1: **Six–parameter model**

ϕ'	Friction angle
c'	Cohesion
ψ	Dilation angle
E	Young's modulus
ν	Poisson's ratio
γ	Unit weight

The dilation angle ψ affects the volume change of the soil during yielding. In this simple model ψ is assumed to be constant which is unrealistic. It has been shown however, that the value of ψ has little influence on the failure loads in collapse problems, especially when they are relatively unconfined such as in this case. For this reason, it is recommended that ψ is set equal to zero for slope stability analysis which corresponds to a no-volume-change condition during yield.

The parameters c' and ϕ' refer to the cohesion and friction angle of the soil. Although a number of failure criteria have been suggested for use in representing the strength of soil as an engineering material, the one most widely used in geotechnical practice is due to Mohr-Coulomb. In terms of principal stresses and assuming a compression-negative sign convention, the criterion can be written as follows:

$$F = \frac{\sigma_1' + \sigma_3'}{2} \sin\phi' - \frac{\sigma_1' - \sigma_3'}{2} - c' \cos\phi' \qquad (1)$$

where c' and ϕ' represent the shear strength parameters of the soil and σ_1' and σ_3' the major and minor principal effective stresses at the point under consideration. The failure function F can be interpreted as follows:

$F < 0$ stresses lie inside the failure envelope (elastic)
$F = 0$ stresses lie on the failure envelope (yielding)
$F > 0$ stresses lie outside the failure envelope (yielding) and must be redistributed

The unit weight γ assigned to the soil is important because it is proportional to the nodal loads generated by the gravity turn-on procedure.

In summary, the most important parameters in a finite element slope stability analysis are the unit weight γ which is directly related to the nodal forces trying to cause failure of the slope, and the shear strength parameters c' and ϕ' which measure the ability of the soil to resist failure.

DETERMINATION OF THE FACTOR OF SAFETY

The Factor of Safety (FOS) of a soil slope is defined here as the factor by which the original shear strength parameters must be reduced to bring the slope to the point of failure. The factored shear strength parameters c'_f and ϕ'_f, are therefore given by:

$$c'_f = c'/FOS \tag{2}$$

$$\phi'_f = \arctan\left(\frac{\tan\phi'}{FOS}\right) \tag{3}$$

This method has been referred to as the 'shear strength reduction technique' (e.g. Matsui and San 1992) and allows for the interesting option of applying different factors of safety to the c' and $\tan\phi'$ terms. In this paper however, the same factor is always applied to both terms. To find the 'true' factor of safety, it is necessary to initiate a systematic search for the value of FOS that will just cause the slope to fail. This is achieved by the program solving the problem repeatedly using a sequence of user-specified FOS values.

EXAMPLES OF FINITE ELEMENT SLOPES STABLITY ANALYSIS

The following three examples have been chosen to demonstrate the accuracy and versatility of the elasto-plastic finite element method in relation to both the determination of the factor of safety and the indication of the failure mechanism. The first two examples describe total stress analyses of slopes made of "frictionless" material such as undrained clay in which $\phi_u = 0$. The third example describes a slope made of an essentially cohesionless material such as sand in which $c' \approx 0$.

Example 1: Undrained clay slope with a foundation layer

This example describes a stability analysis of an undrained clay 2:1 slope (26.6°) resting on a foundation layer as shown in Figure 1a. The shear strength of the slope material (C_{u1}) has been maintained at a constant value of 5 kPa while the shear strength of the foundation layer has been varied. The relative size of the two shear strengths has been expressed as the ratio C_{u2}/C_{u1}. The mesh used for the analysis is given in Figure 1b.

Figure 2 shows the computed factor of safety for a range of C_{u2}/C_{u1} values together with classical solutions of Taylor (e.g. Craig 1997) for the two cases when $C_{u2}/C_{u1} = 1$ and $C_{u2}/C_{u1} \rightarrow \infty$. There is clearly a change of behavior occurring at $C_{u2}/C_{u1} \approx 1.5$ as indicated by the flattening out of the curve. The reason for the transition becomes clear when the mechanisms are observed at failure for different values of C_{u2}/C_{u1}. Figure 3 shows three such cases indicating that there is a transition from the deep-seated circular mechanism when $C_{u2} \ll 1.5C_{u1}$ (Figure 3a) to a shallow "toe" mechanism when $C_{u2} \gg 1.5C_{u1}$ (Figure 3c). The result corresponding to the approximate transition point when $C_{u2} = 1.5C_{u1}$ (Figure 3b) shows an ambiguous situation in which both mechanisms are trying to form at the same time.

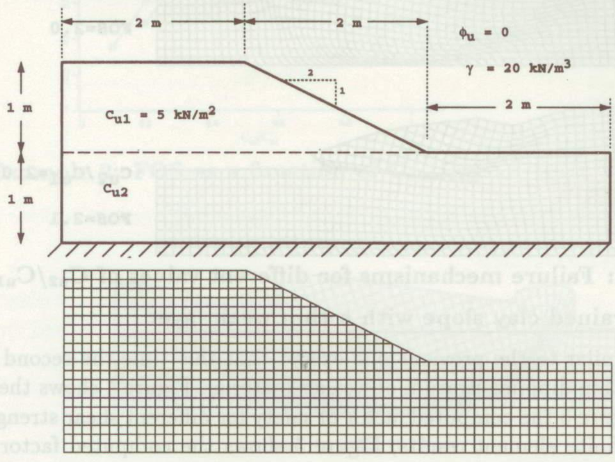

Fig 1. Example 1: a) Two-layer undrained clay slope and b) finite element mesh.

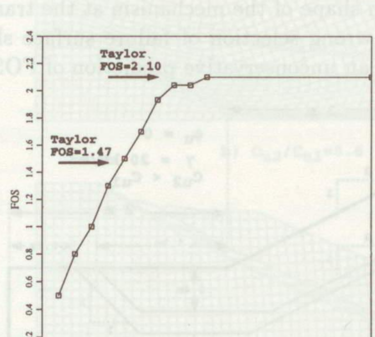

Fig 2. Example 1: FOS as a function of C_{u2}/C_{u1}.

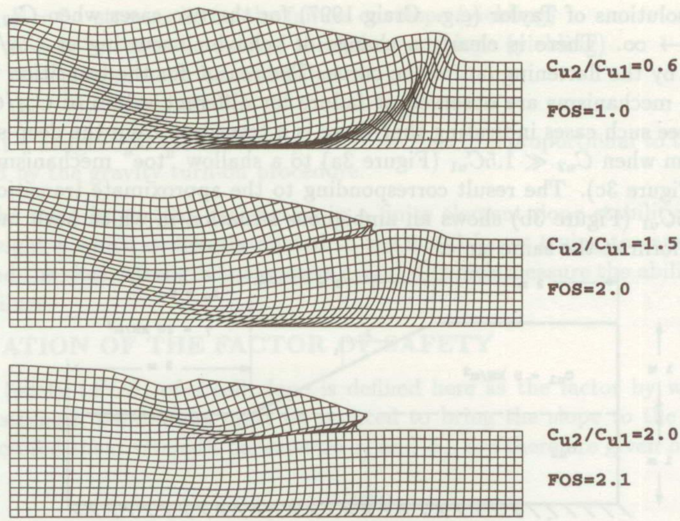

Fig 3. Example 1: Failure mechanisms for different values of C_{u2}/C_{u1}.

Example 2: Undrained clay slope with a thin weak layer

This examples is similar to the previous one except that this time the second soil layer takes the form of a thin dipping stratum of weaker material. Figure 4 shows the layout of the problem and Figure 5 the computed factor of safety for different shear strength ratios. In addition to the finite element results, Figure 5 shows the computed factor of safety given by a limit equilibrium package in which circular and then wedge-shaped mechanisms were assumed. It is clear from the failure mechanisms shown in Figure 6 that the finite element results detect the change in shape of the mechanism at the transition point that occurs around $C_{u2}/C_{u1} \approx 0.6$. A wrong selection of failure surface shape in the limit equilibrium approach could lead to an unconservative prediction of FOS.

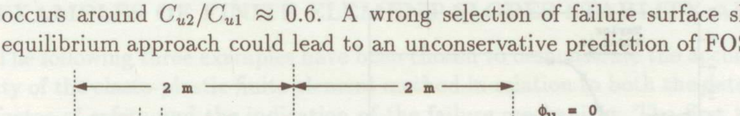

Fig. 4. Example 2: Undrained clay slope with a foundation layer including a thin weak layer.

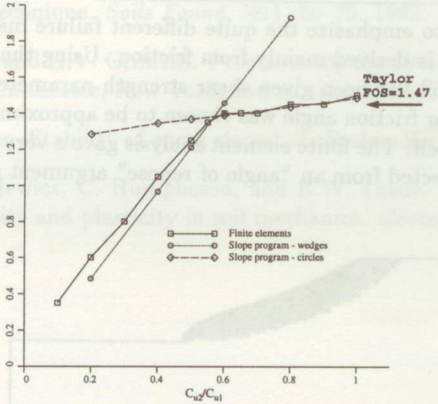

Fig 5. Example 2: FOS as a function of C_{u2}/C_{u1}.

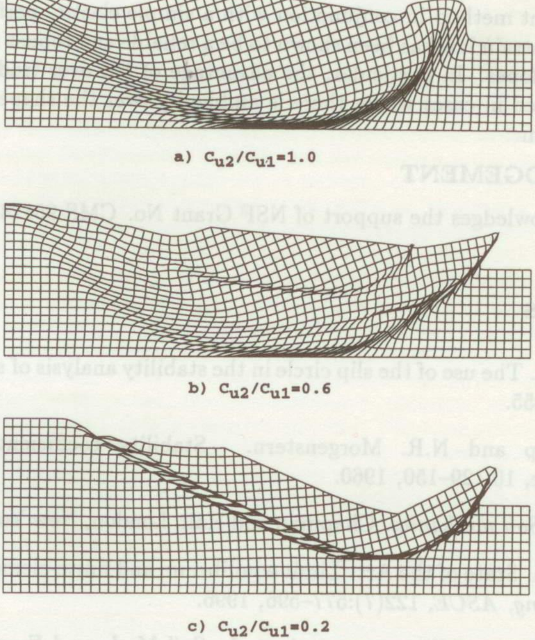

Fig 6. Example 2: Failure mechanisms for different values of C_{u2}/C_{u1}.

Example 3: Homogeneous slope with almost no cohesion

The final examples is included to emphasize the quite different failure mechanisms that are observed if the soil strength is derived mainly from friction. Using the same mesh as in the previous examples, the soil has been given shear strength parameters of $c' = 0.01$ kPa and $\phi' = 25^o$. The particular friction angle was chosen to be approximately the same as the inclination of the slope itself. The finite element analysis gave a very shallow failure mechanism which might be expected from an "angle of repose" argument and a factor of safety just greater than unity.

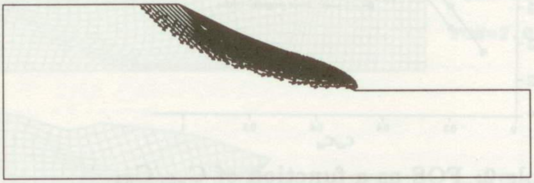

Fig 7. Example 3: Failure mechanism for an essentially cohesionless slope..

CONCLUDING REMARKS

The finite element method in conjunction with a simple elastic- perfectly plastic (Mohr-Coulomb) stress strain law has been shown to be a reliable and robust method for assessing the stability of slopes. In particular, the advantage of FE over traditional methods has been demonstrated in cases where there is ambiguity over the shape and location of the failure mechanism.

ACKNOWLEDGEMENT

The writer acknowledges the support of NSF Grant No. CMS-9512434.

References

[1] A.W. Bishop. The use of the slip circle in the stability analysis of slopes. *Géotechnique*, 5(1):7–17, 1955.

[2] A.W. Bishop and N.R. Morgenstern. Stability coefficients for earth slopes. *Géotechnique*, 10:129–150, 1960.

[3] R.F. Craig. *Soil mechanics.* Chapman and Hall, London, New York, 6th edition, 1997.

[4] J.M. Duncan. State of the Art: Limit equilibrium and finite-element analysis of slopes. *J Geotech Eng, ASCE*, 122(7):577–596, 1996.

[5] N. Janbu. Slope stability computations. In *Soil Mech. and Found. Engrg. Rep.* Technical University of Norway, Trondheim, Norway, 1968.

[6] T. Matsui and K-C. San. Finite element slope stability analysis by shear strength reduction technique. *Soils Found*, 32(1):59–70, 1992.

[7] I.M. Smith and D.V. Griffiths. *Programming the Finite Element Method*. John Wiley and Sons, Chichester, New York, 2nd edition, 1988. Third edition due in 1998.

[8] D.W. Taylor. Stability of earth slopes. *J. Boston Soc. Civ. Eng.*, 24:197–246, 1937.

[9] O.C. Zienkiewicz, C. Humpheson, and R.W. Lewis. Associated and non-associated viscoplasticity and plasticity in soil mechanics. *Géotechnique*, 25:671–689, 1975.