Desarrollos Recientes en Métodos Numéricos
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A Probabilistic Investigation of c', ϕ' Slope Stability

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Abstract

This paper investigates the stability of a c', ϕ' slope within both deterministic and probabilistic contexts. The initial portion of the study examines the individual effect of the cohesion (c'), and friction angle (ϕ') on the traditional deterministic factor of safety calculation. These results then give insight into how the spatial variability of the respective properties will affect the probability of failure in a probabilistic slope reliability analysis. The results of the study highlight the effect of cohesion and friction angle on overall slope stability, as well as lead to a direct comparison between the traditional factor of safety and the probability of failure. Effects of the statistical range of variability of both c' and ϕ' on the probability of failure are also presented and give an important insight into their effect on reliability analyses.

Introduction

The probabilistic results herein were obtained using a program which merges nonlinear elasto-plastic finite element analysis (e.g. Smith and Griffiths 1998) with random field theory (e.g. Fenton 1990). Previous work on this subject for an undrained clay slope ($\phi_u = 0$) was presented by Paice and Griffiths (1997) and more recently Griffiths and Fenton (2000), this study however, considers a slope with both cohesion and friction.

The slope, together with the mesh refinement is presented in Figure 1 and has a height H, a gradient of 1:1, and rests on a firm foundation layer at a depth of 1.5H.

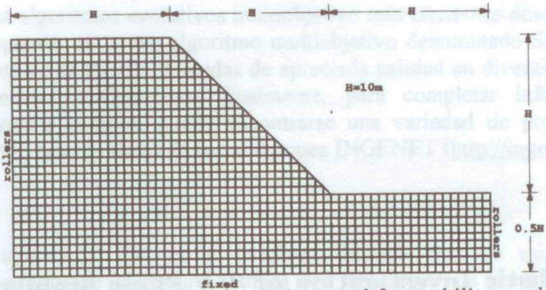

Figure 1: Mesh and slope configuration used for stability analyses.

The initial(base) property values are summarized in Table 1, and, in a deterministic analysis, lead to a Factor of Safety (FS) of 1.37, as given by Bishop and Morgenstern (1960), as well as by independent calculation.

	Parameter	Value
Cohesion	c'	10 kN/m^2
Angle of Friction	ϕ'	35°
Unit Weight	γ	20 kN/m^3
Slope Height	H	10 m

Table 1: Initial Property Values Leading to a FS = 1.37.

The code used for the present analysis generates a random field of the variable property or properties and maps it onto the finite element mesh configuration. Thus, each square of the mesh shown in Figure 1 is in itself a random variable. The properties assigned to each element are controlled by the distributions specified by the user (i.e. mean, standard deviation, and spatial correlation length), and can include normal, bounded, and lognormal distributions. In the present study, lognormal distributions are used for the purpose of avoiding negative values. Taking the logarithm of the random field yields an underlying Normal or Gaussian field. The use of lognormal distributions as a means of quantifying soil variability has been advocated by Cherubini(2000), and more recently by Schweiger *et al* (2001). The mean (μ) and standard deviations (σ) used, are expressed in terms of the coefficient of variation defined as:

$$COV = \frac{\sigma}{\mu} \tag{1}$$

The spatial variability from element to element is controlled by the spatial correlation length(θ), and can be specified separately in both the horizontal and vertical directions. A large spatial correlation length results in a smoothly varying, highly correlated property field, while a value approaching zero results in a "ragged" field with little property correlation between elements. A special case of the spatial correlation length occurs when $\theta = \infty$, where the entire slope is described by a uniform property or properties chosen at random from the given distributions. These analyses, where $\theta = \infty$, are termed the Single Random Variable approach and comprise a majority of routine probabilistic analyses in geotechnical engineering(e.g. Harr 1987, Duncan 2000). The Single Random Variable Method has obvious limitations in that it fails to account for the spatial correlation of the soil as well as gives rise to unrealistically large failure probabilities for geotechnical structures (see Mostyn and Li 1993). The present analysis proposes to address these issues by considering spatial correlation, and modeling the slope as a random field of strength properties.

Brief description of the finite element method used

The slope stability analyses use an elastic-perfectly plastic stress-strain law with a Mohr-Coulomb failure criterion. Plastic stress redistribution is accomplished using a viscoplastic algorithm which uses 8-node quadrilateral elements and reduced integration in both the stiffness and stress redistribution parts of the algorithm. The theoretical basis of the method is described more fully in Chapter 6 of the text by Smith and Griffiths (1998), and for a discussion of the method applied to slope stability analysis, the reader is referred to Griffiths and Lane (1999).

In brief, the analyses involve the application of gravity loading, and the monitoring of stresses at all the Gauss points. If the Mohr-Coulomb criterion is violated, the program attempts to redistribute those stresses to neighboring elements that still have reserves of strength. This is an iterative process which continues until the Mohr-Coulomb criterion and global equilibrium are satisfied at all points within the mesh under strict tolerances.

In this study, failure is said to occur if, for any given realization, the algorithm is unable to converge within 1000 iterations. Following a set of 4000 realizations of the Monte-Carlo process the probability of failure is defined as the proportion of these realizations that required 1000 or more iterations to converge.

While the choice of 1000 as the iteration ceiling is arbitrary, Figure 2 confirms for the case of deterministic cohesion and variable friction angle leading to various factors of safety (based on the means) that the probability of failure computed using this criterion is stable after about 1000 iterations. The convergence at median-values of the probability of failure is somewhat less certain in this case, but will not be investigated here. However it is of interest to note that at low or high values of the probability of failure the convergence is much more definite. At low probabilities of failure, most realizations rarely require more than several hundred iterations, thus drastically increasing the iteration ceiling will yield little change in the probability of failure count. At high probabilities of failure in turn, most realizations have "no hope" for convergence, thus again, increasing the iteration ceiling will yield little change in the failure count. Alternately at mid values of the probability of failure many realizations are close to either failure or non-failure, therefore, increasing the iteration ceiling will cause the most significant change in the failure count.

Figure 3 displays, for a number of the same cases analyzed in Figure 2, that the probability of failure is well stable after 4000 realizations of the Monte-Carlo process, thus no additional benefit is derived from increasing the number of realizations further.

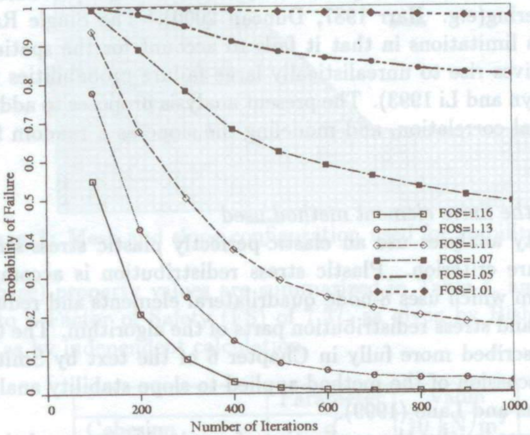

Figure 2: Probability of Failure vs. Number of Iterations for variable ϕ' ($COV_{\phi'} = 0.25$), deterministic c'.

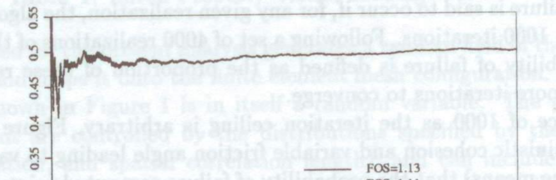

Figure 3: Probability of Failure vs. Number of Realizations for variable ϕ' ($COV_{\phi'} = 0.25$), deterministic c'.

Initial Deterministic c'–ϕ' Investigation

An initial, deterministic investigation, based on the method of slices, was performed into how c', and $\tan\phi'$ affect the factor of safety. In common geotechnical practice, both c' and $\tan\phi'$ are factored to find the overall FS, for example, in the ordinary method of slices:

$$FS = \frac{\sum c'l + \sum W \tan\phi' \cos\alpha}{\sum W \sin\alpha} \quad (2)$$

Where W is the weight of each individual slice, α is the inclination of the base of the slice to the horizontal, and l is the length of the failure surface of each slice.

However it is not apparent how the reduction of each strength component separately affects this calculation. The given slope with a factor of safety of 1.37 was analyzed using a deterministic finite element program developed by the authors. The program analyzes a Mohr-Coulomb material slope subjected to gravity and reduces the strengths of $\tan\phi'$ and c' until a user specified convergence limit is violated (usually 500 iterations) signifying slope failure. The program was modified to reduce each of the factors separately and provide a FS for each case. Equation (3) demonstrates the reduction of cohesion only.

$$FS = \frac{\sum c'l}{\sum W \sin\alpha - \sum W \tan\phi' \cos\alpha} \quad (3)$$

Conversely, Equation (4) shows the reduction of the frictional strength ($\tan\phi'/FS$) only.

$$FS = \frac{\sum W \tan\phi \ \cos\alpha}{\sum W \sin\alpha - \sum c'l} \quad (4)$$

The results are summarized in Figure 4 and show the large overestimation of the FS when each property is factored individually. More importantly, the results indicate the respective contributions of the properties c' and $\tan\phi'$ to slope stability analysis. Reduction of $\tan\phi'$ only, results in a FS of approximately 1.56, causing an overestimation of 0.19, while reduction of c' only, results in a FS of 2.44, leading to an overestimation of 1.07. This indicates that the traditional FS is much more sensitive to the value of the friction angle ($\tan\phi'$) than the cohesion (c'). Thus the reduction of the tangent of the friction angle for the given slope configuration is the overwhelming contributor to slope stability and tends to dictate the failure mechanism of a slope.

This study has important implications for the probabilistic approach to this problem as well. The variability of the angle of friction is predicted to have a more important role in stability analyses within the probabilistic context than the variability of cohesion, which has been shown in a deterministic context to have less of an effect on overall slope stability.

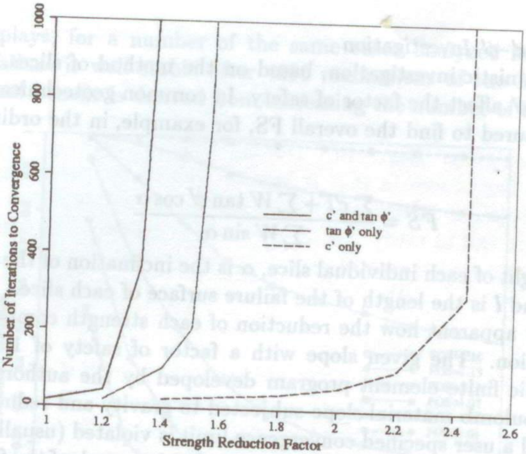

Figure 4: Number of Iterations vs Strength Reduction Factor for factorization of c' and $\tan\phi'$ together and individually.

Probabilistic c'–ϕ' Investigation

In the probabilistic studies in this section, the variability of the cohesion and friction angle was investigated seperately. Thus, one of the properties was assigned a distribution, while the other was held constant and deterministic. Deterministic factors of safety for the slope were calculated using the mean value from the varying property, and the deterministic value of the other. The coefficient of variation was held constant while the mean and standard deviation were adjusted to yield various values of the deterministic factor of safety. These factors of safety are then compared with the probability of failure calculated from the Monte-Carlo simulations. Realistic values of the coefficient of variation were used in the analysis after an extensive literature review. Typical values have been suggested (e.g. Duncan 2000, Phoon and Kulhawy 1999, Phoon and Kulhawy 1996) for the COV of both the cohesion and friction angle, and lie in the range of 0.05 – 0.5.

Spatial correlation lengths (θ) in both the horizontal and vertical directions were held constant for this portion of the study at a non-dimensionalized ($\Theta = \theta/H$) value of 0.05. For the given slope configuration and element size this spatial correlation value gives no property correlation between elements and results in a ragged property field, providing a model where soil spatial variability is high.

Variability of ϕ'

In this portion of the study, the friction angle was assigned statistical properties based on a lognormal distribution defined by the mean and standard deviation, while the cohesion remained deterministic at a value of 10 kN/m^2. The results are summarized in Figure 5, and show that a broad range of factors of safety greater than unity result in probabilities of failure greater than zero. It is evident from the figure that as the $COV_{\phi'}$

increases, failures occur at an increasingly broader range of factors of safety. This is further evidenced by the flattening out of the curves as they go from a low to a high coefficient of variation. For example, with a $COV_{\phi'}$ of 0.50, a factor of safety as high as 1.3 yielded a probabilistic failure rate of 1.28%.

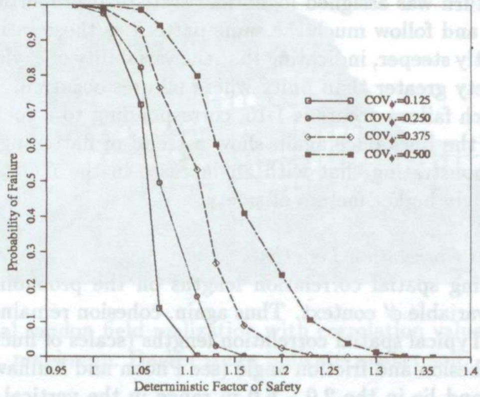

Figure 5: Factor of Safety vs Probability of Failure for variability of ϕ'.

Figure 5 also confirms previous results reported by Griffiths and Fenton (2000) where an increase in the coefficient of variation increased the probability of failure. This is the result of the weaker elements in the mesh dominating the failure mechanism of the slope. Thus, as the variability of a soil mass increases, the probability that a failure will occur increases.

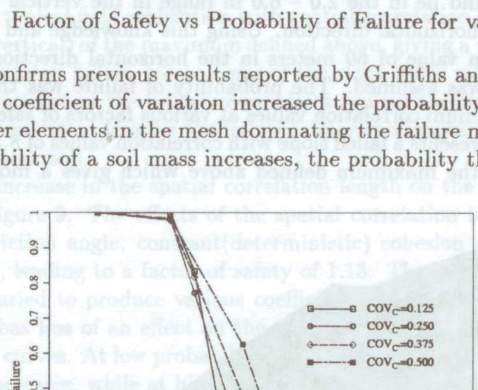

Figure 6: Factor of Safety vs Probability of Failure for variability of c'.

Variability of c′

The study was continued by keeping the friction angle deterministic at a value of 35° while the cohesion in turn was assigned lognormal statistical properties. The results are displayed in Figure 6, and follow much the same pattern as those in Figure 5. The lines however, are significantly steeper, indicating that the variability of c' yields a much smaller range of factors of safety greater than unity where failures occurred. In fact the highest factor of safety at which failure occurs is 1.16, corresponding to a coefficient of variation of 0.50. Additionally, the lines once again show a trend of flattening out as the $COV_{c'}$ increases to 0.50, demonstrating that with an increase in the $COV_{c'}$, failures can still occur even at increasingly higher factors of safety.

Investigation of Spatial Correlation Lengths

The effects of varying spatial correlation lengths on the probability of failure were investigated within a variable ϕ' context. Thus again, cohesion remained deterministic at a value of 10 kN/m^2. Typical spatial correlation lengths (scales of fluctuation) have been summarized for the cohesion and friction angle (see Phoon and Kulhawy 1999, Cherubini 2000, DeGroot 1996) and lie in the 2.0 – 6.0 m range in the vertical direction and 10.0 – 60.0 m range in the horizontal direction. Using this knowledge and previous research, a maximum correlation value of 60 meters in the horizontal direction and 6 meters in the vertical direction was assumed. The probability of failure was then compared to a percentage of the maximum correlation values at various factors of safety and coefficients of variation. Figure 7 presents a failed slope with correlation values of 8.3% (5m horizontal, and 0.5m vertical) of the maximum defined above which gives a more rapidly varying friction angle field.

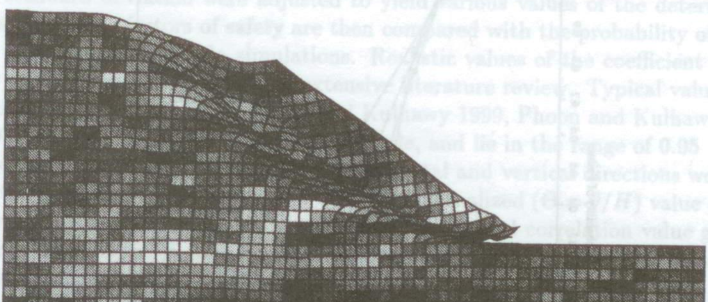

Figure 7: Typical randon field realization with correlation values at 8.3% of the maximum. Darker zones indicate weaker soil.

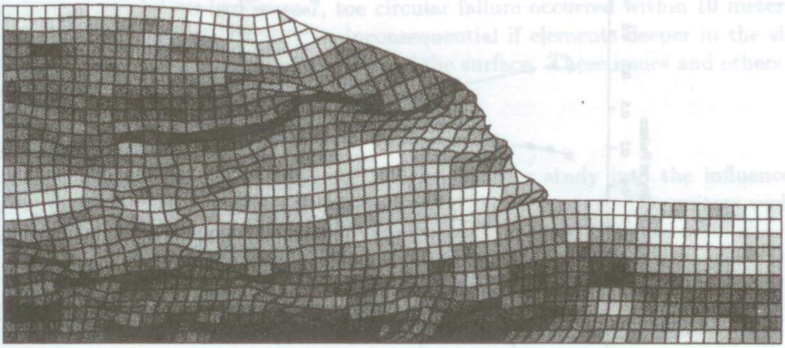

Figure 8: Typical randon field realization with correlation values at 83.3% of the maximum. Darker zones indicate weaker soil.

Figure 8 shows a typical field realization with spatial correlation values of 83.3% (50m horizontal, and 5m vertical) of the maximum defined above, giving a more gradual element variability. The associated failure mechanisms of the slope differ markedly as well. With a small amount of correlation the failure resembles a typical toe circle failure. However, with larger spatial correlation lengths the failure is more horizontal, and mainly oriented along benches of weaker soil.

The effect of an increase in the spatial correlation length on the probability of failure is summarized in Figure 9. The effects of the spatial correlation lengths were analyzed within a variable friction angle, constant(deterministic) cohesion context with a mean friction angle of 28°, leading to a factor of safety of 1.13. The standard deviation of the friction angle was varied to produce various coefficients of variation. As the correlation length increases, it has less of an effect on the probability of failure as evidenced by the flattening out of the curves. At low probabilities of failure, the probability increases as the correlation length increases, while at high probabilities of failure the probability shows a slight increase before decreasing. The initial increase is currently being investigated by the authors. Results showing the same pattern were reported by Griffiths and Fenton (2000) for an undrained clay slope ($\phi_u = 0$). The flattening out of the curve may be the result of the slope configuration which has a maximum width in the horizontal direction of 30m. Thus, consideration of correlation lengths greater than the slope configuration itself may have little additional impact on the probability of failure.

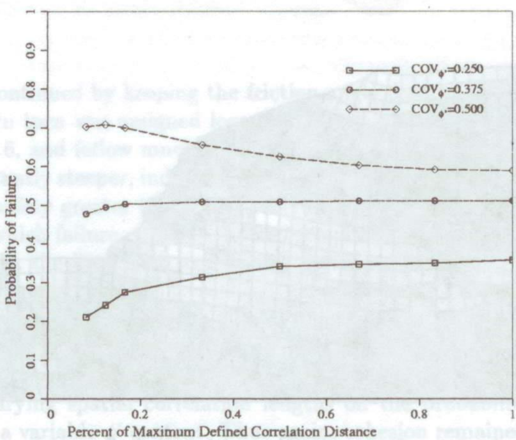

Figure 9: Percent of Maximum Correlation Distance vs Probability of Failure for a slope with FS = 1.13 based on the mean (variable ϕ', deterministic c').

Conclusion

The paper has shown that the variability of the friction angle in a soil profile has a greater effect on slope stability than the cohesion. The deterministic study correctly indicated that the influence of the friction angle would have a greater effect on the stability calculation than the influence of the cohesion. In the probabilistic study with a variable friction angle, failures were noted up to a deterministic factor of safety of 1.3, while with variable cohesion, failures occurred up to a factor of safety of 1.16. Thus the variability of the friction angle caused failures not only at a broader range of deterministic factors of safety but at higher values as well.

The fact that failures occured at factors of safety well above 1.0 requires some discussion also. In certain cases, mean values gave unacceptably high probabilities of failure for factor of safety values which would normally be considered acceptable. In the majority of cases in geotechnical design, mean values of variables such as cohesion and friction angle are calculated from field and lab data. Experienced engineers seldom use these mean strengths as accurate representations of the strength of the soil mass, but instead reduce them further before they are used in calculations of safety. The need for this factoring has been shown in this paper where significant probabilities of failure occurred up to a factor of safety of 1.3 based on mean values. It is important to note that although no failures at factors of safety greater than 1.3 were observed, it has been shown that there is always some probability of failure regardless of the design factor of safety (e.g. Cheung and Tang 2000, Mostyn and Li 1993).

The spatial correlation lengths highly influenced the probability of failure at values approximately up to the width of the slope, after which their effect lessened. Although this may be a result of the slope dimensions, there may also be a limit where the slope will fail regardless of whether there is an additional increase in the spatial correlation length. In many instances, such as in Figure 7, toe circular failure occurred within 10 meters of the surface of the slope, thus it may be inconsequential if elements deeper in the slope are highly correlated to ones within 10 meters of the surface. These issues and others are currently being further investigated by the authors.

Acknowledgement

The results in this paper form part of a much broader study into the influence of soil spatial variability on slope stability in geotechnical engineering. The writers wish to gratefully acknowledge the support of NSF Grant No. CMS-9877189.

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