RELIABILITY BASED DESIGN OF SHALLOW FOUNDATIONS SUBJECTED TO COMBINED LOADING WITH APPLICATION TO WIND TURBINE FOUNDATIONS

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Abstract. *This paper examines the application of direct reliability-based design (d-RBD) to shallow foundations subjected to highly eccentric (MVH) loading. Even though the method can account for multiple limit states (ultimate, serviceability, fatigue, etc) as well as design optimization, only the ultimate limit state combined with design optimization is considered in this paper. The d-RBD design approach involves a Monte Carlo process with an "adequate" number of realizations. Loading and soil shear strength parameters are treated as random variables. Design decision parameters such as foundation size and depth are treated as uniformly distributed variables. Each Monte Carlo simulation is evaluated against limit state criteria. The results of the simulation are then used to calculate conditional probabilities of failure and to select the optimal design which meets target reliability indices. The approach is flexible in that it offers a choice of combinations of deterministic and random variables, probability distributions and calculation models, enabling engineers to exercise judgment without the need to re-write the underlying probabilistic models. This paper illustrates the application of the d-RBD procedure to the design of shallow foundations supporting utility-scale wind energy converter structures. Foundation loads for these structures are highly eccentric, consisting mostly of a large moment component. As shown in this paper, the method can be used directly in the design of such foundations. The method can also be used to develop partial material and load factors for design code calibrations.*

Keywords: Reliability based design, combined loading, wind turbine foundations, code calibration

1. INTRODUCTION OF THE d-RBD PROCEDURE

Contemporary design codes, such as Limit State Design (LSD) and Load and Resistance Factor Design (LRFD), use partial factors to assign uncertainty separately to material resistance and loads. This is a great improvement over classical Allowable Stress Design (ASD) where all uncertainty is lumped into one global factor of safety. Partial factor design methods also have the additional advantage of producing designs with known levels of reliability which are consistent for the superstructure and its foundation. In comparison, reliability delivered through ASD methods is inconsistent across different parts of the structure and is often hard to quantify.

Factors used in partial factor design methods, also known as Reliability-Based Design (RBD) methods, have probabilistic underpinnings. They are calculated using First Order Second Moment (FOSM), First Order Reliability Method (FORM), or Monte Carlo Simulation (MCS) techniques, [1]. The factors are published in design codes to simplify and harmonize the design process across markets and industries. A drawback of this simplification is that it leaves little room for extracting benefit from local knowledge or engineering judgment.

The RBD method presented in this paper, Fig. 1, does not use load or material factors. Loads and material parameters affecting the design are modeled as random variables. Design decision parameters, such as those associated with geometry, are modeled as uniformly distributed variables. After defining the limit states and their associated target reliabilities, Monte Carlo Simulations are performed. Each simulation in the MC process involves the generation of the various random input variables and computation of the output quantities of interest. When an "adequate" number of simulations of the chosen computational model have been performed, statistical analysis of the output quantities is performed to identify the optimal realization. The optimal realization is the least cost combination of design decision parameters that meets the reliability requirements of all enforced limit states. In this design method, risk is modeled separately for each load or material parameter and engineering judgment can be applied at any step of the process, e.g. in the choice of the random and deterministic variables, in the choice of their probability distributions and in the selection of the computational models. Because parameters are incorporated as random variables directly, the procedure is termed a direct Reliability-Based Design (d-RBD) procedure. The d-RBD approach is essentially the same as the "Expanded RBD," described in [2], with the exception of the procedure's extension to foundations subjected to highly eccentric combined loading.

This paper describes the application of the d-RBD procedure to the design of shallow foundations subjected to combined Moment-Vertical-Horizontal (MVH) loading. A realistic example of a shallow foundation supporting a wind turbine is used to arrive at an optimal design and to compare results from this procedure to those obtained using state-of-practice methods which involve the use of various international and national-level standards and guidelines [3, 4]. Typically, limit states that are appropriate for this application include ultimate limit states (ULS) under normal and abnormal extreme loads, serviceability limit states (SLS) to check for minimum foundation stiffness, tolerable settlement/tilt, and fatigue limit states (FLS) to verify adequate longevity of the foundation under cyclic loading. In this paper, the d-RBD procedure is applied only to the ULS verifying bearing capacity under extreme loading using a total stress analysis approach ($\varphi_u=0$).

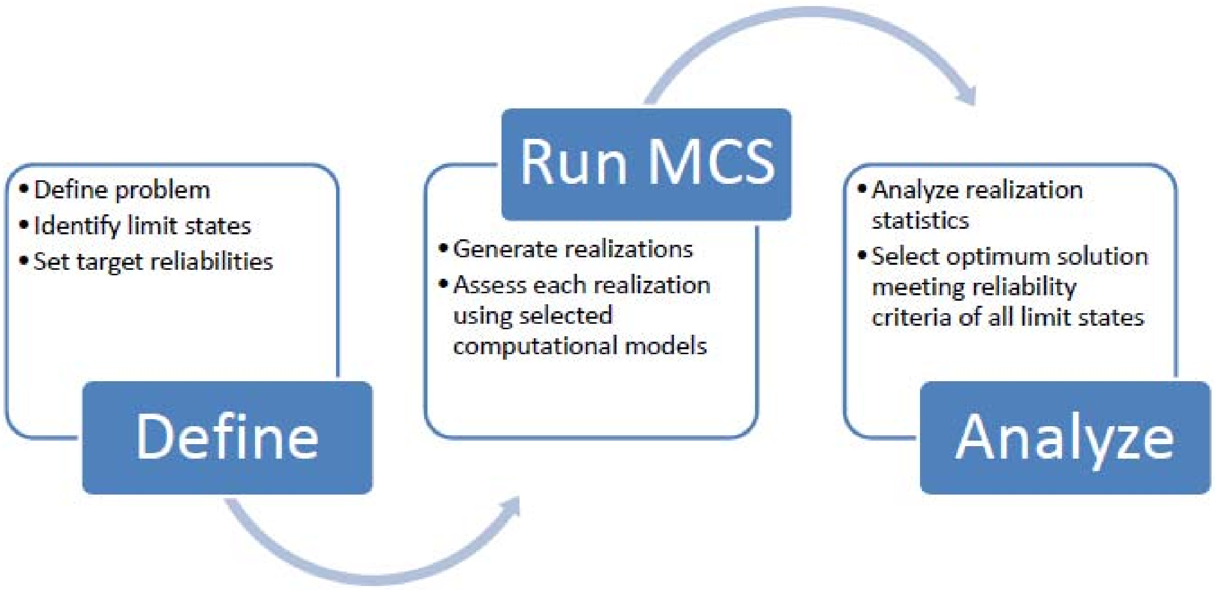

Figure 1- Reliability Based Design Procedure using MCS.

2. APPLICATION OF THE d-RBD PROCEDURE

As illustrated in Fig. 1, the d-RBD approach is a non-iterative MCS process that yields an optimal design meeting the specified reliability requirements. In this section, the three steps identified in Fig. 1 are described in more detail and the specifics of the example problem are provided. The example problem consists of an octagon-shaped spread foundation used to support a utility-scale wind turbine, Fig. 2. The foundation system relies on gravity to resist overturning and it must be sized to meet multiple limit states (ultimate, serviceability and fatigue). Typical foundation width for common contemporary turbine sizes ranges from 12 to over 25 meters and the foundation volume can exceed 500 m^3.

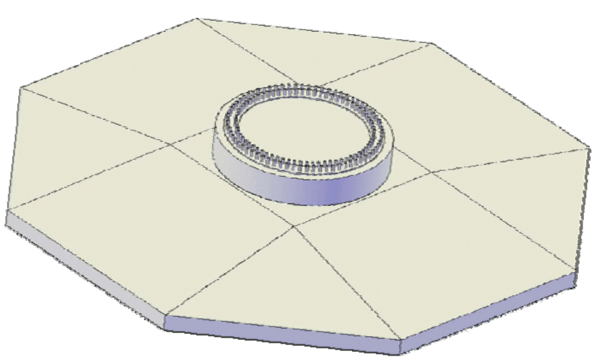

Figure 2- Wind Turbine Gravity Base Foundation.

2.1 Problem Definition

Problem definition and setup involves identifying the limit states of interest, selecting their associated target reliabilities, selecting the computational models for each limit state, identifying the random variables and selecting their probability distributions. For the example problem, a wind turbine foundation is to be designed for bearing capacity ULS (one limit state). The target probability of failure is selected at $p_T = 0.001$ which is equivalent to a target reliability index of about $\beta=3.0$. The selected bearing capacity computational model is for a total stress analysis ($\varphi_u=0$) where the ultimate bearing capacity can be calculated using the Brinch-Hansen equation:

$$q_{ult} = 5.14 s_u (1 + s_c + d_c) \tag{1}$$

where s_u is the undrained shear strength which is treated as a normally distributed random variable, and s_c and d_c are correction factors that are functions of the foundation depth and effective dimensions, [5].

Design decision parameters are treated as uniformly distributed variables to be selected randomly from discrete values covering reasonable ranges. In the wind turbine foundation example, foundation width, B, and depth, D, are such design decisions. For the octagonal foundation, width is the distance between flats. The selected range of foundation width is from 15 to 24 meters (10 discrete values). The selected range of foundation depth is from 1.8 to 2.6 meters at 0.2 meter interval (5 discrete values). Selected random variables are the undrained shear strength, s_u, and applied MVH loading. Normal distributions, summarized in Table 1, are assumed. Remaining parameters such as unit weights and depth of design water table are assumed to be deterministic, even though they can easily be treated as random variables if justified.

Table 1. Random & Deterministic Variables

Parameter	Distribution	Distribution Parameters
Undrained shear strength, s_u (kPa)	Normal	mean=170, std=40
Load – Moment (kN.m)	Normal	mean=79400, std=10000
Load – Vertical (kN)	Normal	mean=3600, std=400
Load – Horizontal (kN)	Normal	mean=890, std=250
Foundation width (m)	Uniform	15 to 24
Foundation depth (m)	Uniform	1.8, 2.0, 2.2, 2.4, 2.6
Groundwater depth (m)	Deterministic	0.0
Saturated unit weight (kN/m^3)	Deterministic	21.5

2.2 Monte Carlo Simulation

Monte Carlo Simulation consists of generating a large number of realizations (or simulations), n_{sim}, and evaluating each realization using the selected computation models to decide if limit states are violated. The total number of failure, n_F, is determined. For each combination of design decision parameters, the number of violations (failures) is counted. An acceptable combination of design decision parameters is a combination that has an acceptable conditional failure probability; namely, its conditional failure probability must be less than the target failure probability, p_T:

$$p(Failure|B,D) \leq p_T \tag{2}$$

The conditional probability *p(Failure|B,D)* is calculated using Bayes' Theorem as:

$$p(Failure|B,D) = \frac{p(B,D|Failure)}{p(B,D)} p_F \tag{3}$$

where:

- p_F is the probability of failure for the entire MCS run: $p_F = n_F/n_{sim}$ where n_F is the total number of failures.
- *p(B,D|Failure)* is the conditional joint probability of *B* and *D* given failure: $p(B,D|Failure) = n_{fBD}/n_F$ where n_{fBD} is the number of failures for combination *B-D*.
- *p(B,D)* is the probability of discrete, uniformly distributed design decision parameters, in this case, $p(B,D) = 1/(n_B * n_D)$ where n_B and n_D are the numbers of discrete *B* and *D* values.

Combining the above definitions into Eqn. 3, the n_F term drops out and conditional probability *p(Failure|B,D)* can be calculated using:

$$p(Failure|B,D) = \frac{n_{fBD}}{n_{sim}} n_B * n_D \tag{4}$$

The accuracy of the MCS results depends on the number of realizations. A rule of thumb suggested is [6] is that the minimum number of realizations is 10 times the reciprocal of the target probability:

$$NR_{min} = \frac{10 * n_B * n_D}{p_T} \tag{5}$$

In the case of the foundation design example, the suggested minimum number of realizations is 10*5*10/0.001 = 500000 realizations. A 64-bit laptop computer with Intel ® Core ™ i5 CPU running at 2.53 GHz performs this MCS in a few minutes using calculation software MathCAD Prime 1.0, [7].

2.3 Analysis of MCS Results

Figures 3 and 4 show the conditional probability of failure, computed per Eqn. 4, for all combinations of design decision parameters *B* and *D*. Figure 3 illustrates the effect of foundation width and Fig. 4 illustrates the effect of depth. As expected, foundation width has the greatest impact, especially for combined loading where the effective area shrinks very quickly with increased eccentricity.

The conditional probability for each *B-D* combination is compared to the target probability, p_T. Those combinations meeting the reliability requirement (i.e., probability of failure less than the target probability) are acceptable designs. The foundation with smallest volume of concrete is selected as the optimal solution. Note that a different criterion can be used in this selection. For example, at rocky sites where excavation is costly, the foundation with the smallest width or depth may be selected from the pool of acceptable solutions. In the example problem, the optimal foundation obtained with an MCS of 5 million realizations (10 times the suggested minimum), is 20 m wide and 2.2 m deep. The volume of concrete in this optimal foundation is 357 m^3. For this example problem, the d-RBD procedure produces a more economical design than that obtained by state-of-practice methods using a variety of non-calibrated codes.

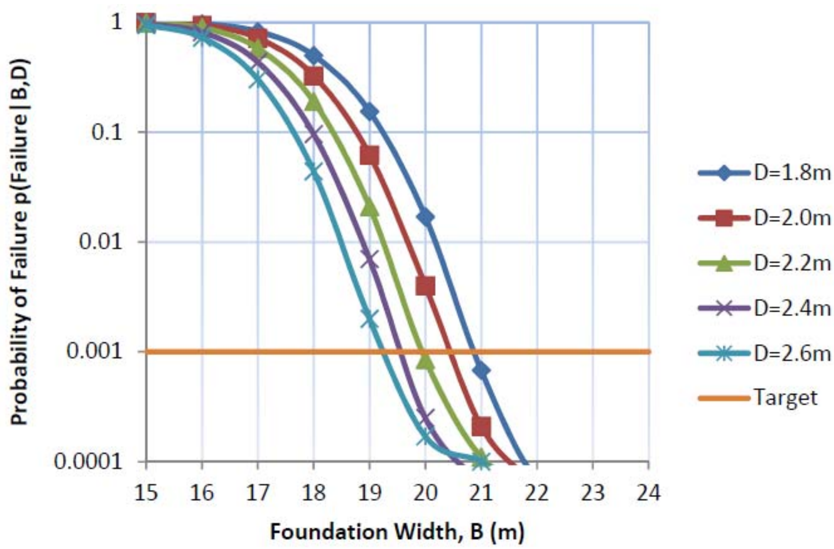

Figure 3- Conditional Probability of Failure as a Function of *B*.

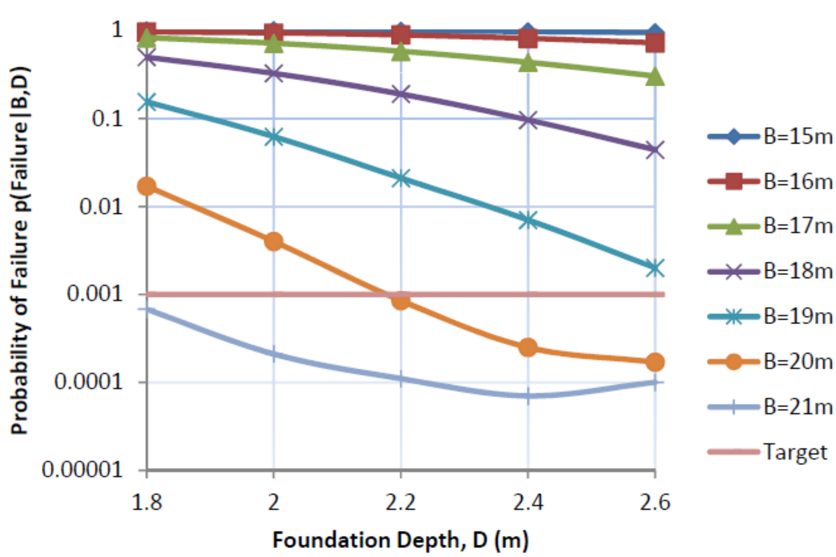

Figure 4- Conditional Probability of Failure as a Function of *D*.

3. CONCLUDING REMARKS

This paper presents a direct reliability-based design procedure (d-RBD) which can be used in a broad range of engineering design applications. The method is flexible and frees the designer to focus on the development of better problem inputs. With further guidance and educational emphasis on random variables and risk assessment, the method has potential as an improved alternative to partial factor methods outlined in design codes. The method has been illustrated in this paper for a total stress ($\varphi_u=0$) ULS bearing capacity design of a wind turbine shallow foundation subjected to combined loading. In addition to its potential as a design tool, the method should prove to be a suitable vehicle for calibrating multiple codes used currently in wind turbine structural and foundation design.

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