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Piled raft foundation analysis by finite elements

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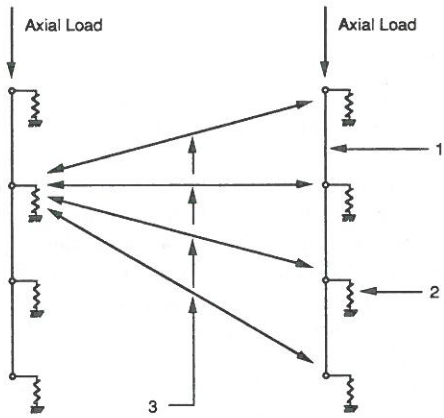
ABSTRACT: A program for the analysis of pile-raft foundation systems has been developed. The piles are modelled with rod elements and the raft by 'thin' plate elements. The three kinds of interactions, namely pile-soil-pile, pile-soil-raft and raft-soil-raft are accounted for using the elastic theory of Mindlin. Results have been validated against elastic solutions for a range of pile/soil/raft stiffness properties.

1 INTRODUCTION

Heavy structures are regularly supported on piled raft foundations, yet relatively little is known about the complex interactions that occur between the raft, piles and soil. Current design methods are based on approximate analysis of the interaction between the pile group and the raft (Randolph 1983, Fleming *et al* 1985). Recent experience by the authors on practical problems of this type, have highlighted the need for a more systematic approach to the analysis of these soil/structure interactions. The result of work by Clancy (1990) has led to the development of a general purpose finite element method for computing deflections and moments in a piled raft foundation. The program allows the stiffness of the raft, the soil and the piles to be varied together with the pile group spacing and loading. This paper describes some of the main features of the method and presents results for a 3 by 3 pile group.

2 PILE GROUP

A solution to the problem of pile-soil-pile interaction in the analysis of pile groups has been suggested by Chow (1986,1987), and a computer coding of this method has been presented by Smith and Griffiths (1988). Figure 1 shows the main points of the analysis, namely the pile stiffness, the soil stiffness in the form of t-z springs, and the interactions between the piles through the soil.



1 One-dimensional pile element

2 Ground resistance at each node represented by non-linear 'T-Z' springs

3 Pile-soil-pile interaction effects calculated between pairs of nodes using Mindlin's equation

Figure 1. Representation of piles and soil.

Chow's work is based on the popular method of single pile analysis where one-dimensional beamcolumn finite elements are used to model the pile. The soil response is modelled by a discrete spring at each node, based on load transfer curves suggested by Seed and Reese (1957), (referred to as *t-z* or *p-y* curves, depending on whether axial or lateral response is being considered).

In the present work, only linear *t-z* response has been considered, with the gradient of the *t-z* curve linked to the elastic soil modulus using the relationships proposed by Randolph and Wroth (1978).

To model a pile group, Mindlin's (1936) elastic continuum solution, for displacements due to a point load, is used to provide interaction effects between pile nodes.

Although only axially-loaded vertical piles have been considered here, the method can be extended to look at more general pile groups with lateral loading and raked piles, and is capable of modelling a soil in which the stiffness increases linearly with depth.

Unlike the boundary element method, consideration of non- homogeneous soil and soil nonlinearity do not increase the size of the system of equations to be solved.

3 PLATE ELEMENT

To model the raft, a four-node quadrilateral isoparametric plate bending finite element (Hughes, 1987; Huang, 1989) was first considered. The element had three freedoms at each node: a transverse displacement and two rotations. Although in this application a 'thin' plate was being considered, the element was based on 'thick' plate theory such that transverse shear strains were accounted for.

It had been demonstrated that under full integration the element would give excessively stiff solutions in thin plate applications. This 'shear locking' behaviour can be overcome by using reduced integration on the shear term in the stiffness matrix.

The plate bending finite elements were attached to the piles so that vertical freedoms were common at the connected nodes. To test the adequacy of this arrangement, the raft was given a very high stiffness. Thus when a uniformly distributed load was applied, a uniform displacement of the raft was expected. However, this wasn't observed because the reduced integration technique had allowed formation of two spurious zero-energy modes. Under other circumstances, e.g. a simply- supported or fixed boundary condition, these modes would be unable to form (Clancy and Griffiths 1991).

A second, triangular isoparametric element, also based on thick plate theory (Zienkiewicz and Lefebvre, 1988) was considered. Again, reduced integration was employed to avoid shear locking, but the use of 'bubble functions' prevented the formation of spurious zero-energy modes. However, it was found that a large number of degrees of freedom were required to ensure convergence to a solution.

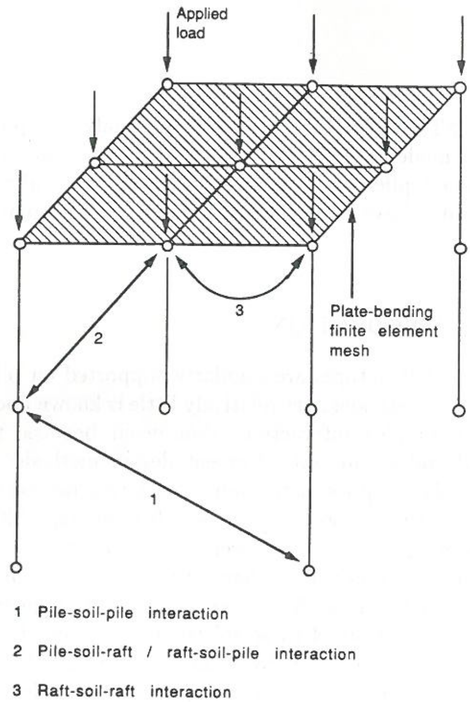


Figure 2. Full pile group analysis including raft-soil-pile interaction

The third element considered was a four-node rectangle (Smith and Griffiths, 1988) and, unlike the others, non- isoparametric which meant that it was no longer possible to model a general raft geometry. However, the element was based on 'thin' plate theory so that no shear locking was encountered, even under full integration. It was also found that fewer freedoms were required for convergence because of a fourth 'twist' freedom at each node.

Results of a number of plate bending tests showed that the non-isoparametric element was far superior where the geometry of the problem allowed its use. In cases where the raft can't be exactly modelled by rectangular elements, it might be adequate to use the element in a piecewise approximation.

4 RAFT-SOIL-PILE INTERACTION

Having decided on the most suitable plate-bending finite element to model a pile raft, and having attached it to the piles via common vertical freedoms, all that remained was to take account of raft-soil-pile interaction.

Hain and Lee (1978) had also used finite elements to model a raft, in conjunction with a boundary element pile group analysis. Following their method, it was assumed that a uniform pressure area existed around each raft node. This allowed the elastic solutions of Giroud (1968) to be used to provide a soil 'spring' stiffness at each raft node. Mindlin's equation was again used to calculate three new sets of interaction: raft-soil-raft interaction; raft-soil-pile interaction; and pile-soil-raft interaction (figure 2).

Before analysing the complete raft-soil-pile system, this method of raft analysis was checked against analytical solutions given in the Sydney University data sheets. Figure 3 shows a typical set of results for a raft of length L and breadth B supported only by soil, with a point load P acting along the centreline a distance sL from one end. The bending stiffness of the raft is proportional to the Young's modulus E_r, and the cube of the thickness t. Following Hain and Lee (1978), a raft-soil stiffness ratio K_{rs} is introduced given by:

$$\begin{aligned} K_{rs} &= \text{raft-soil stiffness} \\ &= 4E_r Bt^3(1-\nu_s^2)/3\pi E_s L^4 \end{aligned}$$

where E_s and ν_s are respectively the Young's modulus and Poisson's ratio for the soil.

The results in figure 3 are for the case of $L/B = 10$, $K_{rs} = 0.001$ (a very flexible raft) and $s = 0.1$ and 0.5. The centreline displacement w is plotted normalised as $wLE_s/P(1-\nu_s^2)$. Bending moments in the plane of the centreline are plotted normalised as M/PL.

The finite element analysis shows very good agreement with the analytical results in this case. Equally good results were obtained for all raft stiffnesses.

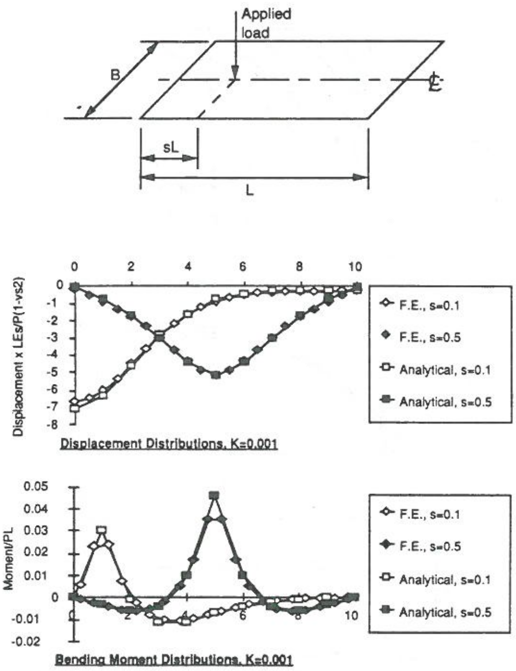


Figure 3. Raft footing: Displacements and bending moments.

A complete piled raft analysis has been undertaken for a 3 by 3 square pile group (see figure 4). the piles have length l, diameter d, Young's modulus E_p and are spaced at distance s. The raft is square with an overhang of $s/2$ beyond the outer piles, giving $L = B = 3s$. the problem may be defined in terms of a series of dimensionless groups:

pile length	L/d	25
pile spacing	s/d	5
pile/soil stiffness	$K_{ps} = E_p/E_s$	$10^2 - 10^5$
raft aspect ratio	L/B	1
raft/soil stiffness	K_{rs}	$0.01 - 10$

In figure 4, results are presented for the case of a uniform pressure load q, in terms of:

a) Load sharing between pile group and raft.
b) Average displacement w_{av}, normalised as $w_{av}E_s/qB(1-\nu_s^2)$.
c) Differential displacement $w_{mid-edge} - w_{centre}$, normalised as $(w_{mid-edge} - w_{centre})/w_{av}$.
d) Maximum bending moment in raft M_{max} normalised as M_{max}/qLB^2.

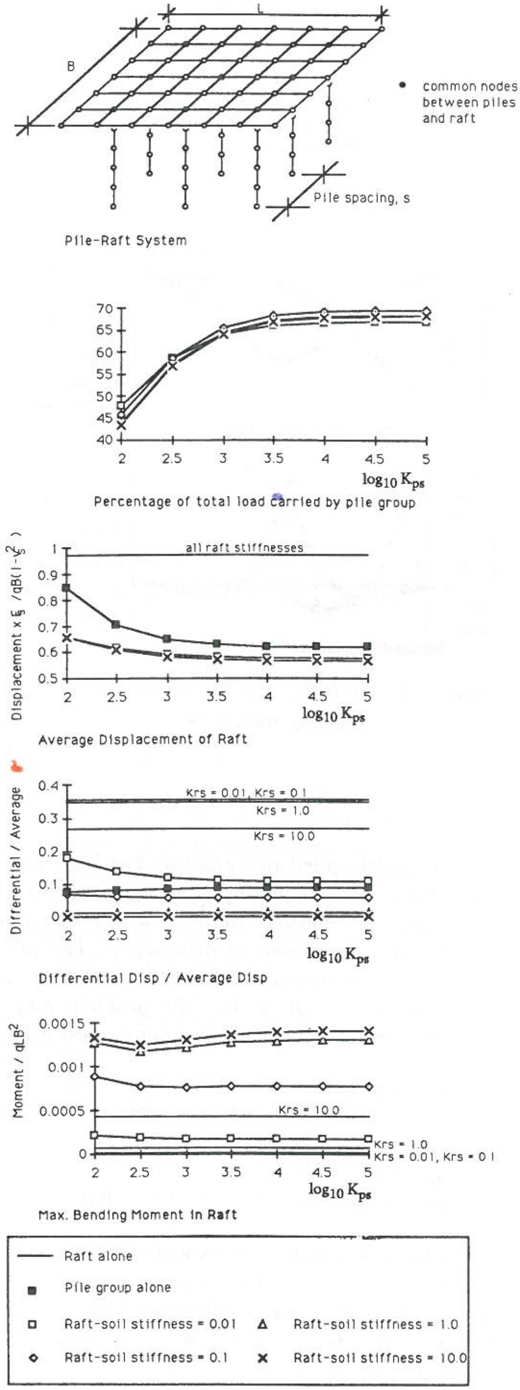


Figure 4. Effects of pile and raft stiffness on displacements and bending moments

In addition to the piled raft, results are shown for the free- standing pile group and for the raft alone. It can be seen that the load sharing and the average displacement of the raft are dependent mainly on the pile-soil stiffness ratio. The differential displacements and bending moments are dependent on the raft-soil stiffness ratio.

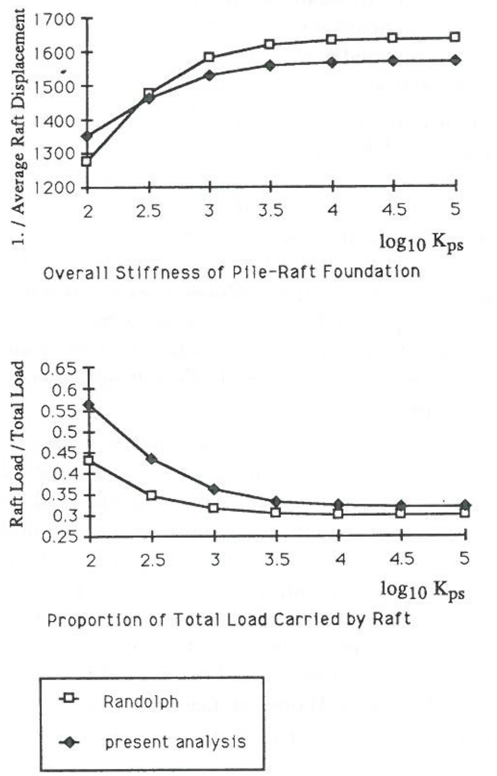


Figure 5. Comparison between the present analysis and the approximate method of Randolph.

The results from the analyses with $K_{rs} = 10$ were compared with the approximate method of piled raft analysis proposed by Randolph (1983). Figure 5 shows how the overall stiffness of the foundation and the load distribution between piles and raft vary as the stiffness of the piles is increased.

Although the results of Randolph's analysis show the correct trends across the range, the values of the results differ by up to 6% in the case of foundation stiffness, and up to 30% in the case of load distribution. the use of a finer raft mesh in the present analysis brings the difference in load distribution up to 45%.

5 CONCLUSIONS

A method of analysis for pile-raft systems has been developed using finite elements. The three kinds of interactions, namely pile-soil-pile, pile-soil-raft and raft-soil-raft are accounted for using the elastic theory of Mindlin.

In isolation, both the pile group analysis and the raft analysis have been demonstrated to give results consistent with elastic theory.

Example runs on a 3 by 3 rafted pile group give expected trends in both displacements and load distributions, as the stiffnesses of the piles and raft are varied. Comparison of results with an approximate method by Randolph (1983) show similar trends, although values differ.

Further work will be performed in order to validate the program against field performance, but the early indications are that the method represents an improved yet efficient method of analysis for piled raft foundations.

REFERENCES

[1] Y.K. Chow. Analysis of vertically loaded pile groups. *International Journal for Numerical and Analytical Methods in Geomechanics*, 10:59–72, 1986.

[2] Y.K. Chow. Three dimensional analysis of pile groups. *Proceedings of the American Society of Civil Engineers, Geotechnical Engineering Division*, 113(GT6):637–651, 1987.

[3] P. Clancy. Computer analysis of pile groups. Master's thesis, Department of Engineering, University of Manchester, 1990.

[4] P. Clancy and D.V. Griffiths. A spurious zero-energy mode in the numerical analysis of piled raft foundations. 1991. Under preparation for publication.

[5] W.G.K. Fleming, A.J. Weltman, M.F. Randolph, and W.K. Elson. *Piling engineering.* Surrey University Press, 1985.

[6] J.-P. Giroud. Settlement of a linearly loaded rectangular area. *Journal of the Soil Mechanics and Foundation Engineering Division, ASCE*, 94(SM4):813–832, 1968.

[7] S.J. Hain and I.K. Lee. The analysis of flexible raft-pile systems. *Géotechnique*, 28(1):65–83, 1978.

[8] H-C. Huang. *Static and dynamic analysis of plates and shells.* Springer-Verlag, 1989.

[9] T.J.R. Hughes. *The finite element method.* Prentice-Hall, Englewood Cliffs, New Jersey, 1987.

[10] R.D. Mindlin. Force at a point in the interior of a semi-infinite solid. *Physics*, 7:195–202, 1936.

[11] M.F. Randolph. Design of piled raft foundations. In *Proc. Int. Symp. on Rec. Develop. in Lab. and Field tests and Anal. of Geotech. Probs.* 1983.

[12] M.F. Randolph and C.P. Wroth. Analysis of deformation of vertically loaded piles. *Proceedings of the American Society of Civil Engineers, Geotechnical Engineering Division*, 104(GT12):1465–1488, 1978.

[13] H.B. Seed and L.C. Reese. The action of soft clay along friction piles. *Transactions of the A.S.C.E.*, 122:731–754, 1957.

[14] I.M. Smith and D.V. Griffiths. *Programming the Finite Element Method.* John Wiley and Sons, Chichester, New York, 2nd edition, 1988.

[15] O.C. Zienkiewicz and D. Lefebvre. A robust triangular plate bending element of the Reissner-Mindlin type. *International Journal for Numerical Methods in Engineering*, 26:1169–1184, 1988.