Cividini, A. & Gioda, G. (1990). *Géotechnique* **40**, No. 3, 523–526

DISCUSSION

On the variable mesh finite elements analysis of unconfined seepage problems

A. CIVIDINI and G. GIODA (1989). *Géotechnique* **39**, No. 2, 251–267

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The Authors have very effectively presented a series of results for both steady state and transient free-surface problems using the variable mesh finite element technique.

An interesting observation is made regarding the ambiguity of the exit point (point P in Fig. 3), which lies on both the free surface and the seepage face. The Authors present the results of numerical experiments on a rectangular dam in which the point P is fixed, and these lead to ragged free-surface predictions for all cases except the one in which the 'correct' location of P is assumed in advance.

It is claimed that 'standard algorithms' for free-surface analysis may lead to this kind of instability, and then it is demonstrated that the modified technique successfully eliminates the oscillations.

The purpose of this discussion is to present results for the same rectangular dam, but using the published free-surface software of Smith & Griffiths (1988). The mesh and data used in conjunction with the variable mesh program (program 7.1) in that text are given in Fig. 14. The exit point P in this analysis is allowed to vary at each iteration, and is treated no differently to other points on the free surface where the potential is made equal to the elevation above the downstream water level.

The free surface is initially assumed to be horizontal as shown in Fig. 14, and the change in these elevations is monitored from one iteration to the next. Convergence is said to have occurred when the largest change at any node, divided by the upstream potential value (1·25 in this case), does not exceed 0·001.

The algorithm converged in 15 iterations giving the smooth free surface of Fig. 15, which is virtually indistinguishable from that presented by the Authors using their modified approach. In addition, a steady flow rate through the dam of $0{\cdot}7812\,\mathrm{m}^3/\mathrm{s}/\mathrm{m}$ was computed assuming a permeability of 1 m/s.

It appears that the rather trivial problem of steady flow through a rectangular dam is accurately solved by the simple approach described here. As stable results have also been achieved with this method for free-surface problems involving dams with sloping sides, one wonders whether a steady flow problem could be devised for which this method would run into difficulties. There seems to be more than one approach to solving the variable mesh problem, and perhaps some 'standard' methods are better than others.

REFERENCE

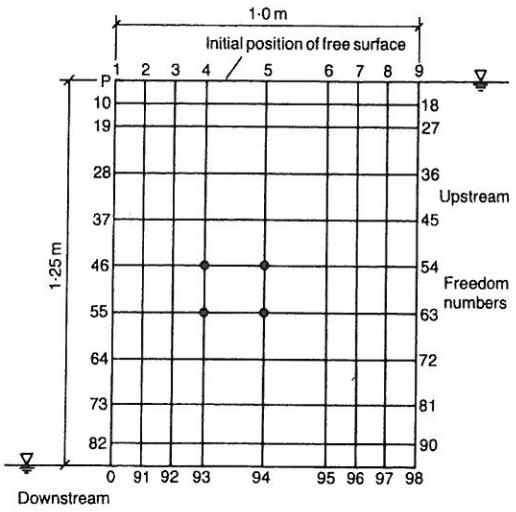
Smith, I. M., & Griffiths, D. V. (1988). *Programming the finite element method* (2nd ed.), pp. 245–249. Chichester: John Wiley & Sons.

Authors' reply

The results shown in the discussion, concerning the steady state flow through a rectangular block dam (Fig. 15), are consistent with those presented in the Paper for the same problem in transient conditions (Fig. 4(a)). In the Paper it is observed that when the nodes move along vertical lines the results obtained with the technique proposed by Taylor *et al.* (1973), without introducing any correction for the position of the seepage point, show only some minor oscillations in the vicinity of the seepage point. These oscillations tend to disappear as the steady state regime is approached (cf. Fig. 4(a)).

Consequently the Authors agree with Dr Griffiths on the fact that no particular provisions seem to be necessary to reach a correct (in engineering terms) finite element solution of steady state or transient unconfined flow problems in two dimensions if the free surface nodes move along vertical lines. However, the situation can be substantially different when dealing with dams with sloping sides.

In this case transient analyses without corrections for the position of the seepage point are affected by marked oscillations of the free surface in the vicinity of the wet boundary (cf. Fig. 5(a)), that might eventually lead to an uncorrect steady state shape of the free surface. In order to show that this problem arises also in steady state analyses, some additional finite element calculations were performed for a trapezoidal dam, using the meshes depicted in Fig. 16.



Parameter restrictions IKV ≥ 1078
ILOADS ≥ 98
INF ≥ 99
INO ≥ 20
INX ≥ 9

Mesh data	NXE	NYE	N	IW	NN	NR	NGP	ITS			
	8	10	98	10	99	1	2	20			
Element data	PERMX		PERMY								
	1·0		1·0								
Width data	0·0	0·1	0·2	0·3	0·5	0·7	0·8	0·9	1·0		
Initial free surface	1·25	1·25	1·25	1·25	1·25	1·25	1·25	1·25	1·25		
Node freedom data	91	0									
Fixed freedom data	IFIX	(NO	(I),	I=1,	IFIX)						
	20	9	10	18	19	27	28	36	37	45	46
		54	55	63	64	72	73	81	82	90	98

Fig. 14. Initial mesh and data for free-surface analysis

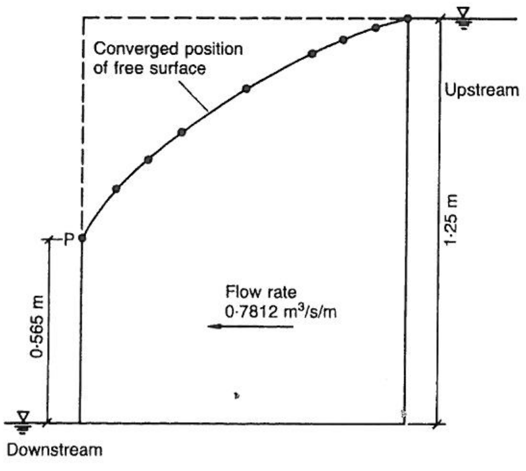


Fig. 15. Computed free surface at convergence

A first series of analyses was based on the approach suggested in the discussion (i.e. the seepage point is treated as an impervious node during the iterations and then its elevation is made equal to the evaluated hydraulic head). The iterative process terminates when the difference between the nodal elevation and the computed hydraulic head, divided by the current nodal elevation, is less than 0·1%. The results of these calculations are presented in Fig. 17. Some non-negligible oscillation of the free surface is shown. As observed in the Paper, similar oscillations were obtained also by other authors (e.g. Taylor *et al.*, 1973; Taylor & Brown, 1967) who adopted various provisions to eliminate them. In particular, the use of thin elements close to the seepage points was suggested. This suggestion is consistent with results in Fig. 17 showing a reduction of the oscillations with decreasing size

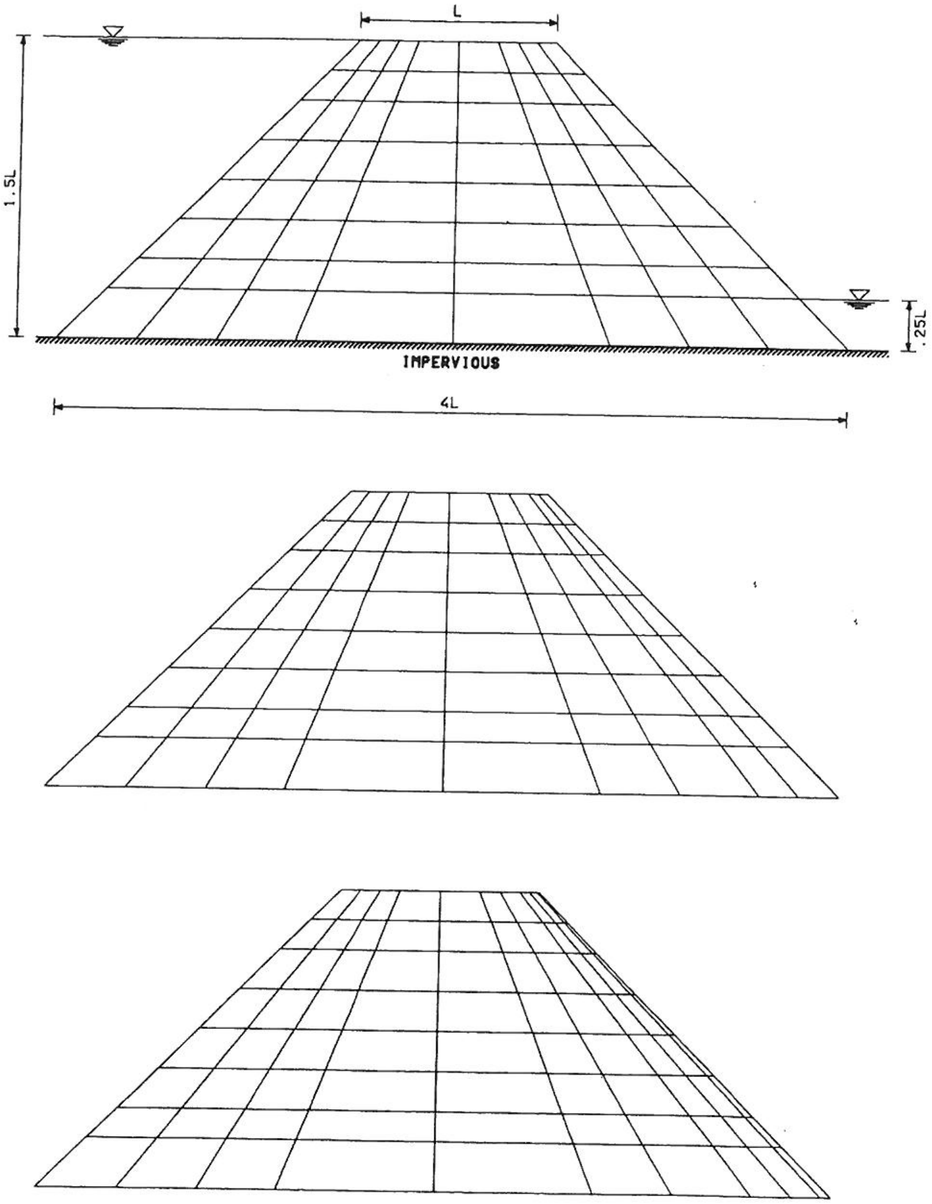


Fig. 16. Steady state seepage flow through trapezoidal dam; finite element meshes with decreasing size of elements facing wet boundary

of the elements facing the wet boundary. An excessive reduction of the element size may lead to stability problems due to ill conditioning of their flow matrices.

The results obtained using the procedure proposed in the Paper are presented in Fig. 18. In this case the influence of the element size is barely appreciable and a reasonable approximation of the elevation of the seepage point is reached even with a rather coarse mesh. The proposed technique, based on the minimization of a suitable error function, requires more iterations than that adopted in the discussion.

On the basis of the above observations it seems reasonable to conclude that so-called standard variable mesh techniques for unconfined seepage analysis might lead to oscillations of the numerically evaluated free surface. In order to eliminate

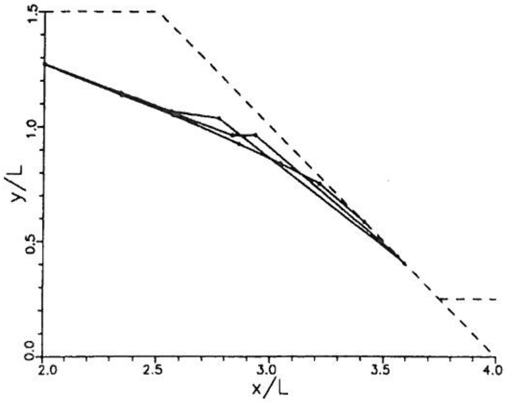


Fig. 17. Shapes of the numerically-evaluated free surface obtained without applying correction for position of seepage point

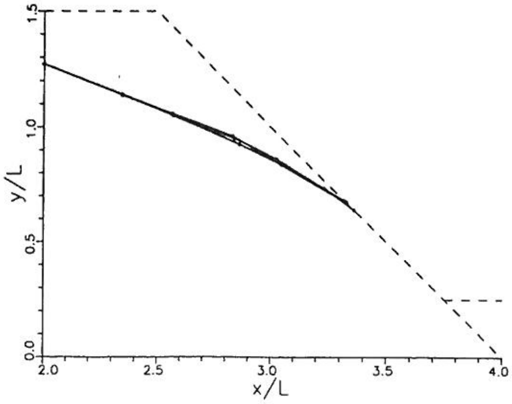


Fig. 18. Shapes of numerically-evaluated free surface obtained applying correction for position of seepage point

them one can either refine the mesh in the vicinity of the wet boundary (this would involve a trial and error procedure for the choice of the optimal mesh) or adopt one of the provisions suggested in the literature, perhaps considering among them also the one proposed in the Paper.