PROBABILISTIC SLOPE STABILITY ANALYSIS OF EMBANKMENT DAMS USING RANDOM FINITE ELEMENTS (RFEM)

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Abstract

The computer program Probabilistic Engineered Slopes (PES), coded in FORTRAN.95, provides a repeatable methodology, which allows the user to perform a slope stability analysis on a one- and two-sided sloping structure, using a deterministic or probabilistic approach.

The program PES, in contrast with other deterministic or probabilistic classical slope stability methodologies, is capable of seeking out the critical failure surface without assigning a predefined failure surface geometry. The probabilistic approach of PES applies the Random Finite Element Method (RFEM) by Griffiths and Fenton (1993) [1], taking into account the soil spatial variability and allowing the use of different random fields to characterize the spatial variation of any material type. The methodology is compared against the probabilistic approach proposed with the program SLOPE/W, version 7.14 (Geostudio Group, 2007) [2], and demonstrates its potential for predicting probability of failure (p_f) in nonhomogeneous soil structures for given phreatic conditions and potential postearthquake liquefiable conditions. The p_f results obtained by program PES have proved that underestimating the influence that the soil material variability has on the computation of p_f will lead to unconservative results of probability and underestimate of the risk of slope instability. The program PES has capabilities that could be used by the engineering practice to prioritize intervention activities within a risk context, test the stability conditions of dams during modification phases, and help estimate the probability of failure in cases involving postearthquake liquefaction.

Introduction

Stability analyses are routinely performed to assess the equilibrium conditions of a natural slope. The analysis technique chosen depends on both site conditions and the potential mode of failure, with careful consideration given to the varying strengths, weaknesses, and limitations inherent in each methodology.

The motivation driving this study is closely related to the assessment and mitigation of the hazards caused by the instability processes and the important role that stability analysis of slopes plays in civil engineering applications and design.

For many years, the nature of geotechnical slope stability analysis has been predominantly deterministic, whether performed using design charts or computers.

While much has been investigated on the matter of probabilistic approach to slopes instability, the geotechnical profession has been slow to adopt probabilistic approaches to geotechnical design and risk assessment. However, the need to move toward probabilistic and

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risk-based methods for decisionmaking is today widely recognized by private engineering practices and Federal agencies (e.g., Bureau of Reclamation, U.S. Army Corps of Engineers, Federal Energy Regulatory Commission). Due to the advantages of faster computational analysis and more advance field equipment, this approach is becoming a more valuable alternative for the civil engineering profession.

It is inherent in the deterministic analysis approach that the parameters characterizing the soil materials such as friction angle, cohesion, Young modulus, Poisson ratio, unit weight, and ground water are also treated as deterministic. Intuitively, it can be recognized that, where there are materials more homogeneous than others in nature, there are no perfectly homogeneous natural materials. The deterministic approach, which does not allow any variation in the soil materials properties, clearly introduces a high level of approximation to the analysis and characterization of slope stability. The level of approximation can only be reduced if the natural variation of soil is taken into account, allowing the soil to be characterized by a range of values for each parameter instead of a 'mean' value.

Soil properties measurements usually are taken over a finite volume, which represents a local average of the property with respect to the overall size of the site domain. For this reason, the Local Average Subdivision (LAS) method (Fenton and Vanmarcke, 1990) [3] has been used to generate the random fields in all the investigations presented in this work. The random field model provides a useful tool for generating spatially variable soil properties. A random field is characterized by sets of soil property values, which are randomly generated around their mean value and are mapped onto the finite element mesh creating a two-dimensional (2D) model of variable soil. Each set of property values (e.g., cohesion and friction) characterizes an element within the domain analyzed. The Monte-Carlo method is lastly applied to this model, performing multiple random field realizations. The number of simulation that give FS<1 divided by the total number of simulations represents the probability of failure.

In the computation of slope stability and probability of failure certainly, there are many sources of uncertainty, in addition to those related to soil variability. In current engineering practice, most slope stability analyses, following a deterministic approach or characterized by a one-dimensional (1D) model, do not account for soil variability. The current work will show that, accounting for the influence of soil variability, varying the soil strength parameters and using a 2D model leads to more conservative probability of failure results compared to those computed using classical approaches to geotechnical problems.

Program PES Theory and Characteristics

Program Probabilistic Engineered Slopes (PES)[4] coded in FORTRAN.95 provides a repeatable methodology able to model an embankment structure with a one-sided or two-sided slope, computing a two dimensional (2D) plane strain slope stability analysis of elastic-perfectly plastic soils with a Mohr-Coulomb failure criterion using 8-node quadrilateral elements with reduced integration (four Gaussian-points per element) in the gravity load generation, the stiffness matrix generation and the stress redistribution phases of the algorithm.

In terms of principal stresses and assuming a compression-negative sign convention, the Mohr-Coulomb criterion can be written as shown in equation 1

[4] Program PES is not commercially available at this time. For queries related to the software described in this paper please contact Prof. D.V. Griffiths at Colorado School of Mines.

$$F_{mc} = \frac{\sigma_1' + \sigma_3'}{2}\sin\phi' - \frac{\sigma_1' - \sigma_3'}{2} - c'\cos\phi' \qquad (1)$$

where σ_1' and σ_3' are the major and minor principal effective stresses.

In cases where the soil is characterized by a frictionless component (undrained clays), the Mohr-Coulomb criteria can be simplified into the Tresca criterion substituting $\phi = 0$ in equation 1 and obtaining equation 2.

$$F_t = \frac{\bar{\sigma}(\cos\theta)}{\sqrt{3}} - c_u \qquad (2)$$

The failure function F for both criteria can be interpreted as follows:

F < 0 stresses inside failure envelope (elastic).

F = 0 stresses on failure envelope (yielding).

F > 0 stresses outside failure envelope (yielding and must be redistributed).

The elastic parameter E' and υ' refer to Young's modulus and Poisson's ratio of the soil, respectively. If a value of Poisson's ratio is assumed (typical drained values lie in the range 0.2 < υ' <0.3), the value of Young's modulus can be related to the compressibility of the soil as measured in a one-dimensional (1D) edometer (e.g., Lambe and Whitman 1969) [4] as shown in equation 3,

$$E' = \frac{(1+\upsilon')(1-2\upsilon')}{m_\upsilon(1-\upsilon')} \qquad (3)$$

where m_v is the coefficient of volume compressibility.

In this study, the parameters E' and υ' have the values of E' =2E+06 psf and υ' =0.3, respectively.

The total unit weight γ assigned to the soil is proportional to the nodal self-weight loads generated by gravity. The forces generated by the self-weight of the soil are computed using a gravity procedure that applies a single gravity increment to the slope.

In the program, the application of gravity loading is followed by a systematic reduction in soil strength until failure occurs. This is achieved using a strength reduction factor SRF that is applied to the frictional and cohesive components of strength in the form of equation 4.

$$\phi_f' = \arctan\left(\frac{\tan\phi'}{SRF}\right) \quad \text{and} \quad c_f' = \frac{c'}{SRF} \qquad (4)$$

The factored soil properties ϕ_f' and c_f' are the properties actually used in each trial analysis. When slope failure occurs, as indicated by an inability of the algorithm to find an equilibrium stress field that satisfies the Mohr-Coulomb failure criterion coupled with significantly increasing nodal displacements, the factor of safety is given by equation 5.

$$FS \approx SRF \qquad (5)$$

In the literature, this method is referred to as the “shear strength reduction technique” (e.g., Matsui and San 1992) [5].

The reduction of soil strength is followed in the program by the computation of the total body load vectors. A description of generation of the body loads computed in the program can be found in deWolfe (2010) [6], and a detailed description of the algorithm used in the program involving viscoplasticity can be found in Smith and Griffiths (2004) [7].

To model nonhomogeneous slopes, PES allows the user to apply a total of three random fields to characterize the site foundation, the engineered structure, and a potential weak layer for a postliquefaction seismic analysis. Program PES also allows the user to model possible water table conditions at any height through the embankment and/or foundation. The main program PES requires a library, divided into three subsections, created to execute the majority of the basic computations needed in the main program. In addition to a results file providing results values of the computed displacement, deterministic *FS* and relative statistics, and the computed probability of failure (if the probabilistic approach is chosen), program PES generates a PostScript image of the deformed mesh with and without a gray scale, a PostScript image of the nodal displacement vectors, and a PostScript image of the initial mesh representing the problem analyzed.

With regard to the probabilistic analysis computed by program PES, the probability of failure can be calculated using two different approaches. When the program is asked to compute the safety factor (*FS*) for each Monte-Carlo simulation, the probability of failure is described by the proportion of Monte-Carlo simulations with *FS*<1. When the program is asked to compute the probability without determining the exact value of *FS* for each simulation, the probability of failure is described by the proportion of Monte-Carlo slope stability analyses that failed. In this case, the SRF is equal to 1 (no strength reduction is actually applied). In this case, “failure” was said to have occurred if, for any given realization, the algorithm (Mohr-Coulomb failure criterion) was unable to converge within 500 iterations.

The RFEM code enables a random field of shear strength values to be generated and mapped onto the finite elements mesh, taking full account of element size in the local averaging process. In a random field, the value assigned to each cell (or finite elements, in this case) is itself a random variable.The random variables can be correlated to one another by controlling the spatial correlation length, and the cross correlation matrix where the degree of correlation ρ between each property can be expressed in the range of $-1<\rho<1$.

More generally, the correlation coefficient between two random variables *X* and *Y* can be defined by equation 6

$$\rho_{XY} = \frac{COV[X,Y]}{\sigma_x \sigma_y} \tag{6}$$

where *COV* represents the covariance between the two variables *X* and *Y* and their respective standard deviations σ_x and σ_y.

Due to the isotropic approach applied throughout this work, the following simplifications can be made with respect to the mean, standard deviation, and the spatial correlation length: $\mu_x = \mu_y = \mu_c$, $\sigma_x = \sigma_y = \sigma_c$, and $\theta_x = \theta_y = \theta_c$.

Using an exponentially decaying (Markovian) correlation function, equation 6 can be rewritten as in equations 7 and 8.

$$\rho = e^{-\frac{2\tau}{\theta_{\ln c}}} \tag{7}$$

$$\rho = \exp\left\{ -\frac{2}{\theta_{\ln c}} \sqrt{\tau_x^2 + \tau_y^2} \right\} \tag{8}$$

Where ρ is the familiar correlation coefficient, τ is the distance between two points in the random field, and θ_{lnc} represents the spatial correlation length.

The spatial correlation length (θ), also referred to in literature as the “scale of fluctuation,” describes the distance over which the spatially random values will tend to be significantly correlated in the underlying Gaussian field. Mathematically, θ is defined as the area under the following correlation function (e.g., Fenton and Griffiths, 2008 [8] from Vanmarcke, 1983 [9])

$$\theta = \int_{-\infty}^{\infty} \rho(\tau)d\tau = 2\int_{0}^{\infty} \rho(\tau)d\tau \tag{9}$$

where τ represents the distance between two positions in the random field. A large value of θ will imply a smoothly varying field, while a small value will imply a ragged field.

Another important dimensionless statistical parameter involved in this probabilistic approach is the coefficient of variation *v,* which for any soil property can be defined as

$$v = \frac{\sigma}{\mu} \tag{10}$$

where σ is the standard deviation and μ the mean value of the property.

In brief, the analyses involve applying gravity loading and monitoring of stresses at all the Gauss points. The program uses the Mohr-Coulomb failure criterion that, if violated, attempts to redistribute excess stresses to neighboring elements that still have reserves of strength. This is an iterative process that continues until the Mohr-Coulomb criterion and global equilibrium are satisfied at all points within the mesh under quite strict tolerances. Plastic stress redistribution is accomplished using a visco-plastic algorithm with 8-node quadrilateral elements and reduced integration in both the stiffness, and stress redistribution parts of the algorithm. For a given set of input shear strength parameters (mean, standard deviation, and spatial correlation length), Monte-Carlo simulations are performed until the statistics of the output quantities of interest become stable.

A more comprehensive explanation of the random finite elements method, including local averaging approach and discussion on spatial correlation length, can be found in Fenton and Griffiths (2008) [8].

Program PES Applications

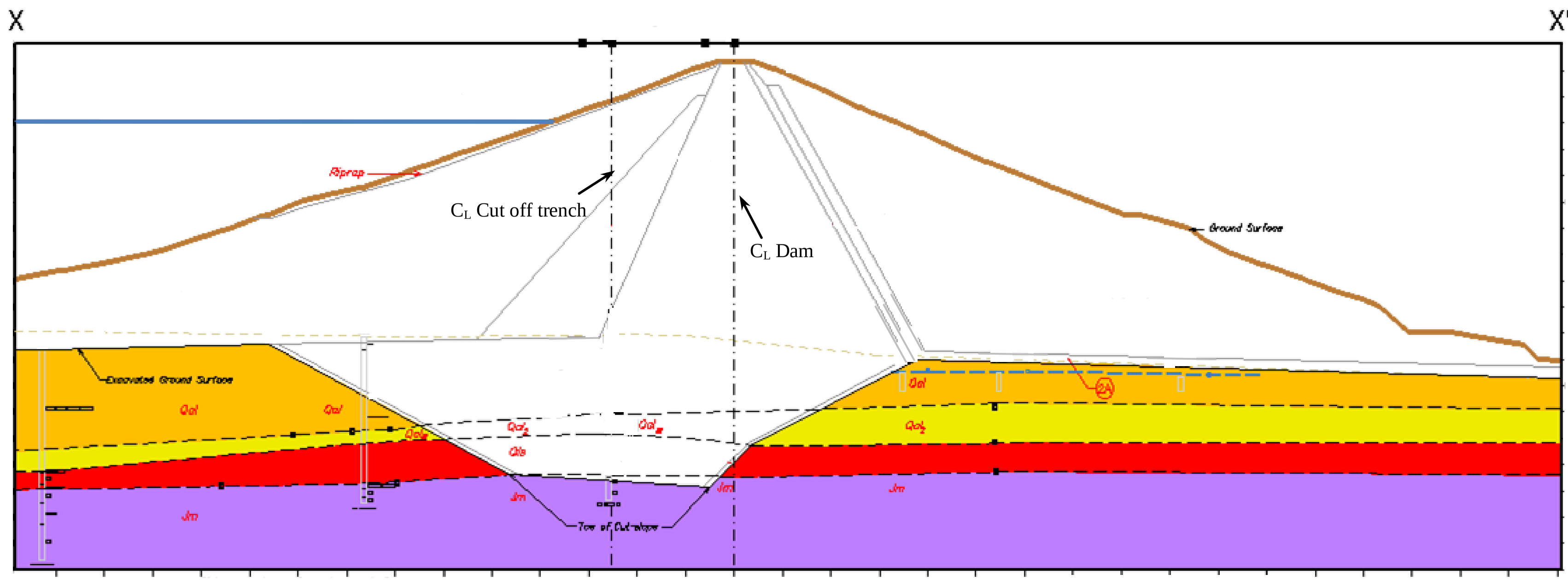
Ridgway Dam Deterministic and Probabilistic Slope Stability Analyses

Ridgway Dam is located 1 mile north of Ridgway, Colorado, on the Uncompahgre River located just above the confluence of Dallas Creek in Ouray County, Colorado. Construction of the dam started in 1978 and was completed in 1987. Ridgway Reservoir has a capacity of 84,591 acre-feet (top of water conservation). The dam is a compacted zoned, earthfill structure that has a structural height of approximately 330 feet, a hydraulic height of approximately 206 feet, a crest width of 30 feet, and a crest length of 2,430 feet. The upstream face has a 3H:1V slope from the crest down to a 20 foot-wide berm at elevation 6,790, and a 3.5H:1V slope down to the foundation, as shown in figure.1. The upstream face is protected by a 3-foot-thick layer of riprap. The downstream face has a 2.5H:1V slope down to elevation 6,800, and a 3H:1V slope down to the foundation, as shown in figure 1. The downstream face is characterized by selected silt, sand, gravel, and cobbles to 12-inch size and has 6-inch-thick seeded topsoil. A cutoff trench was excavated through valley surficial deposits into bedrock with a maximum depth of 115 feet, and is for the most part located along the dam centerline as shown in figure 1. The base of the trench at maximum section is approximately 160 feet wide with side slopes 1½:1. Beyond the limit of the cutoff trench, the embankment materials were placed directly on alluvial material. An aerial view of Ridgway Dam is showed in figure 2. Ridgway Dam is located in a glacial valley close to the outwash source. The Mancos Shale Formation is found in the upper mesas, the Dakota Formation on the upper left abutment, and the Morrison Formation on the abutments and the valley. The Morrison Formation consists of sandstones, siltstones, and mudstones. Surficial materials remaining under the dam consist primarily of Quaternary alluvium (Qal), with lesser amounts of buried Quaternary Landslide deposits (Qls).The alluvium includes stream fill, low level terraces, and flood plain deposits and consists mainly of stream deposited rounded to well rounded gravels, cobbles, and boulders with some sand and minor amounts of silt and clay. Figure 1 shows a geologic cross section along the maximum section of the dam. More information on the geology and engineering properties of the embankment and the foundations at the site is available in Technical Memorandum No. RD-8312-6 by the Bureau of Reclamation (2003) [10]. The case history of Ridgway Dam was selected because past studies of the site conducted by the Bureau of Reclamation presented possible postearthquake liquefiable conditions in the foundation.

The first dynamic analysis conducted on Ridgway Dam was performed in 1981 and indicated that induced deformations of up to a magnitude 7.0 earthquake will not be sufficient to cause an overtopping failure of the dam. According to the 1999 Comprehensive Facility Review conducted on the dam by the Bureau of Reclamation, the foundation material was considered to contain potentially liquefiable materials. Based on this observation, a slope stability assessment of the dam was conducted in 2003 to estimate the actual displacements that might occur as a result of a seismic event. The analysis conducted in 2003 considered Ridgway Dam as a structure sensitive to potential seismic hazard and indicated that there is a continuous or interconnected zone of liquefiable materials in the foundation to potentially cause a significant downstream slope failure during a seismic event. It also was determined that a minimum of 350 to 400 feet, upstream to downstream, liquefiable material would be required to cause a liquefaction related failure at the dam.

The geometry and material assumptions used in the 2003 analysis were the same as those used in the 1981 dynamic analysis, except that the zone 1 strength characterized by a friction angle of 25 degrees in the 2003 analysis was not reduced by 20 percent as it was in the 1981 analysis. The 2003 analysis only used failure surfaces passing through the downstream foundation because the cutoff trench beneath the dam is located upstream of the centerline and would increase the stability of failure surfaces passing through the upstream foundation. In 2008, a slope stability analysis performed by the Bureau of Reclamation [11] modeling possible postliquefaction conditions showed a FS= 1.09 (figure 3). This analysis was conducted using the software SLOPE/W version 7.11 using Spencer’s method and Janbu’s method. The soil properties used in the 2008 postliquefaction study were the same as those used in the 1981 dynamic analysis. In spring 2009, the civil engineering firm URS was contracted by the Bureau of Reclamation to review the results from the site investigation performed by Reclamation in recent years and to develop recommended strengths to be used in a new dynamic deformation analysis [12].

While the overall dimensions of the Ridgway Dam model in the current analyses are the same as those used in the 2008 slope stability analysis, the model complexity has been simplified. To reduce the computational aspect of program PES, the filters have been removed from the embankment structures, and the cutoff trench also has been removed because the failure surfaces analyzed in the 2008 slope stability analysis appear to be independent of the presence of the cutoff trench.



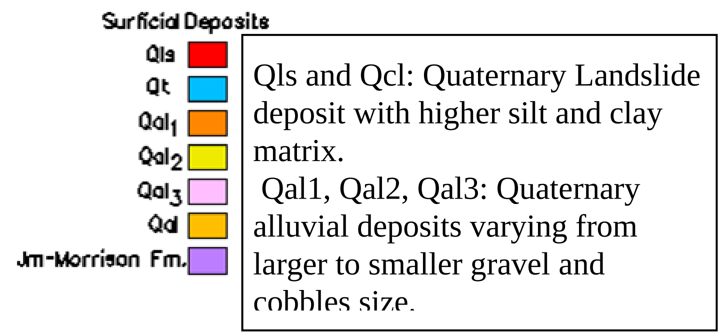
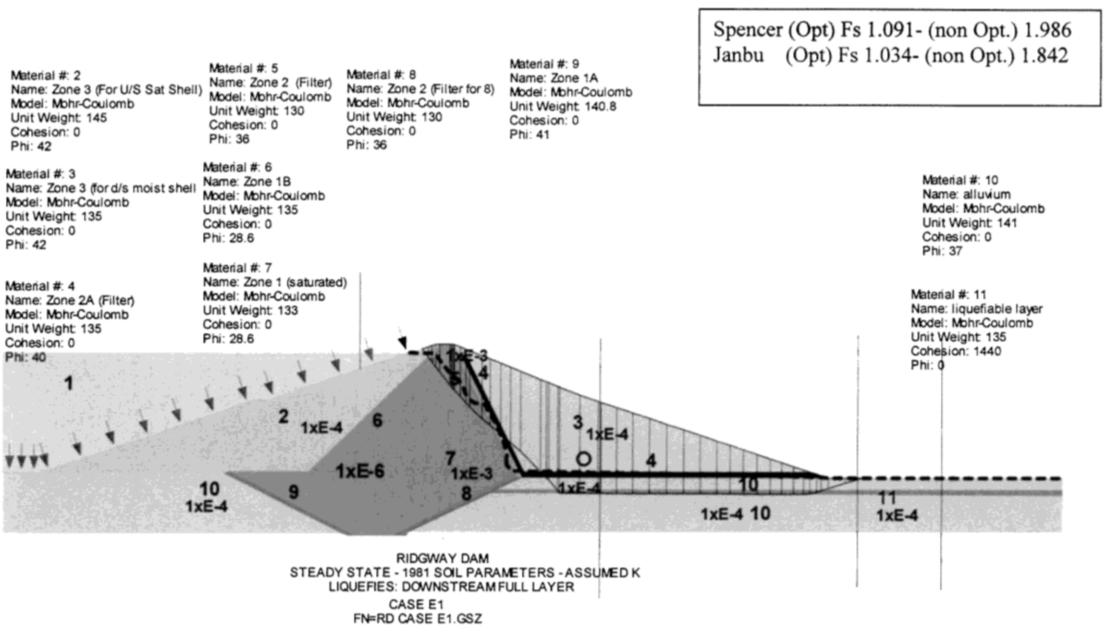


Figure 1: Geologic cross section along the maximum section of Ridgway Dam

Figure 2: Aerial view of Ridgway Dam.



Original Analysis from Roger Torres 2008

Figure 3: Representation of the 2008 deterministic postliquefaction analysis model.

Even though the filters have been removed, the phreatic surface in the model is the same as the one used in the 2008 analysis and represents the top of active conservation capacity. This assumption was made in the attempt to model a condition more similar to what the site is experiencing. Generally, the model portrays an embankment section characterized by a core material (Zone 1) and a shell material (Zone 3) placed in the upstream and downstream outer embankment sections as shown in figure 4. The foundation section is characterized by a homogeneous alluvium material approximately 100 feet thick and a potentially liquefiable layer, located under the downstream side of the dam about 20 feet below the surface. The liquefiable layer is assumed to be about 5 feet thick and 1,010 feet in length from the location where it would theoretically intercept the cutoff trench to the right boundary of the model.

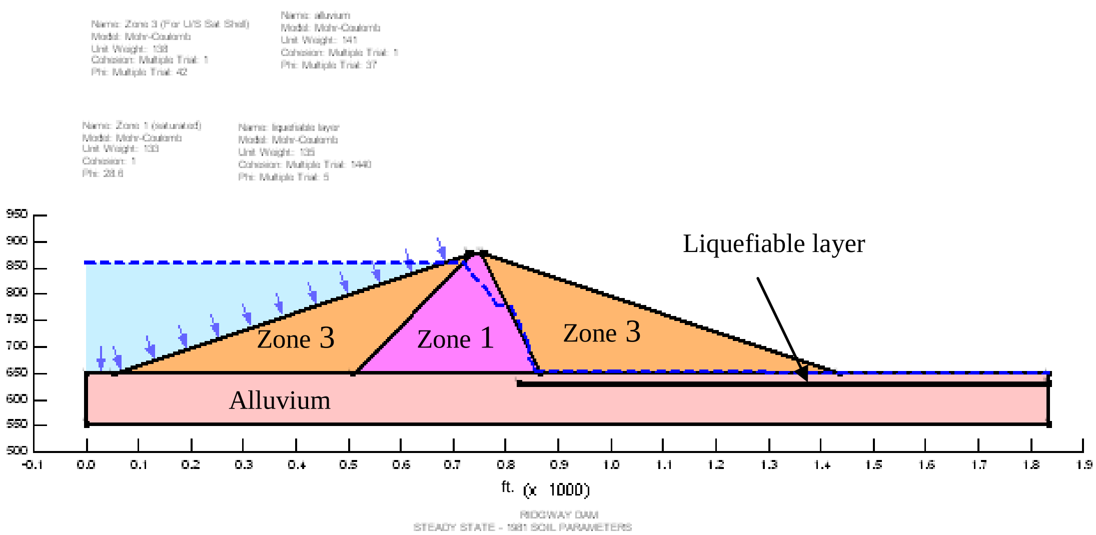


Figure 4: Simplified deterministic and probabilistic model used for Ridgway Dam.

Deterministic pre and postliquefaction conditions analyses have been conducted for this case history using program PES. The soil properties used in the deterministic analyses to characterize the embankment zones, foundation, and liquefiable layer in the foundation are taken from the URS study [12] conducted in spring 2009 and are summarized in table 1.

Table 1: Deterministic soil properties used in the Ridgway Dam pre and post-liquefaction analyses

	Material	Unit weight (pcf)	φ' (º)	c' (psf)
Postliquefaction conditions	Embankment core	133	28.6	1
	Embankment shell	138	42	1
	Foundation	141	37	1
	Quaternary alluvium	135	5	1440
Preliquefaction conditions	Embankment core	133	28.6	1
	Embankment shell	138	42	1
	Foundation	141	37	1
	Quaternary alluvium	Layer removed in the pre-liquefaction analysis		

Subsequently, the postliquefaction deterministic model is run using the probabilistic capability offered by the program PES. The soil properties as probabilistic variables and their statistical parameters used during the probabilistic analysis are summarized in table 2.

The probabilistic analysis associates one random field with the embankment and one with the foundation, and the liquefiable layer is described by the foundation random field, which is modified to address the new values describing the liquefiable material. Because the core material can be considered an engineered material with very little variability, this analysis will consider it to be deterministic with fixed properties.

In this probabilistic model, only the strength parameters of friction and cohesion are analyzed in a probabilistic approach; the other parameters—dilation angle, unit weight, Young's modulus, and Poisson's ratio—are analyzed following a deterministic approach.

To address the level of uncertainty incorporated into the mean values describing the properties, the same probabilistic model is run one time with a higher Coefficient of Variation (*v*) and one time with a lower *v*. The *v* values used in each analysis for all material types are summarized in table 3

Table 2: Probabilistic soil properties used in the Ridgway postliquefaction analyses

Material	μ	σ characterize by lower *v*	σ characterize by higher *v*	Distribution Type
Embankment core ϕ' (º)	28.6	NA	NA	deterministic
Embankment core c' (psf)	1	NA	NA	deterministic
Embankment shell ϕ' (º)	42	6.3	25.2	Lognormal
Embankment shell c' (psf)	1	0.15	0.6	Lognormal
Foundation ϕ' (º)	37	11.1	22.2	Lognormal
Foundation c' (psf)	1	0.3	0.6	Lognormal
Quaternary alluvium ϕ' (º)	5	1.5	3.0	Lognormal
Quaternary alluvium c' (psf)	1440	432	864	Lognormal

Table 3: *v* values characterizing Ridgway probabilistic runs.

Material	lower *v*	higher *v*
Embankment shell ϕ'(º) and c'(psf)	0.15	0.3
Foundation ϕ'(º) and c'(psf)	0.3	0.6
Quaternary alluvium ϕ'(º) and c'(psf)	0.3	0.6

The *v* values characterizing the probabilistic analyses were chosen evaluating suggested values available in the literature for similar soil material (e.g., Lee et al., 1983 [13], Phoon and Kulhawy, 1999 [14]).

Another critical value in the analysis is the spatial correlation length used to determine the soil spatial variability. The set of isotropic values chosen to investigate the spatial correlation length θ for all probabilistic runs is reported in table 4.

Table 4: Isotropic θ values characterizing Ridgway spatial variation of soil.

θ=	4 ft
θ=	25 ft
θ=	60 ft
θ=	100 ft
θ=	200 ft
θ=	300 ft
θ=	500 ft
θ=	2,000 ft

All the probabilistic analyses are run using 1,000 Monte-Carlo simulations. It has been observed during this investigation that the probabilistic model representing Ridgway Dam associated with 1,000 Monte-Carlo simulations returns a probability that can vary up to 3 percent (%) as showed in figure 5, which represent a repeatable reproducibility. During all probabilistic and deterministic analyses, all soil properties are considered uncorrelated between each other.

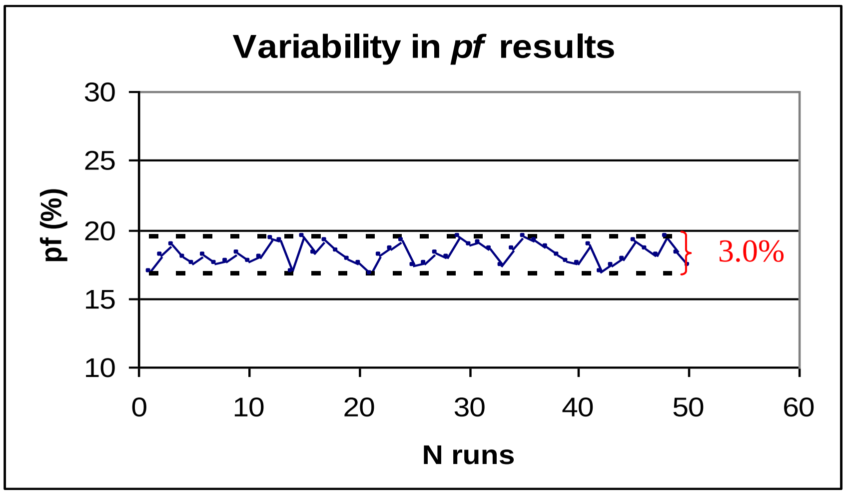


Figure 5. Variability in p_f results using 1,000 Monte-Carlo simulations. To recognize how much the p_f computed by the Ridgway model could vary in a probabilistic setting, the same data file was run 50 times.

The results of the probabilistic analyses, as well as the comparison with the results generated by the program Slope\W version 7.14, are described in the following section.

Programs PES and SLOPE/W: Deterministic and Probabilistic Slope Stability Results Comparison.

The SLOPE\W result from the deterministic preliquefaction model according to Spencer's Method returns a FS=2.49 (figure 6) while the result run using program PES shows a FS=2.31 (figure 7).

The deterministic postliquefaction model was initially run with a liquefiable layer entirely frictionless; and this assumption, as shown in figures 8 and 9, led to very low FS for both programs (PES FS=0.58, SLOPE/W FS=0.6).

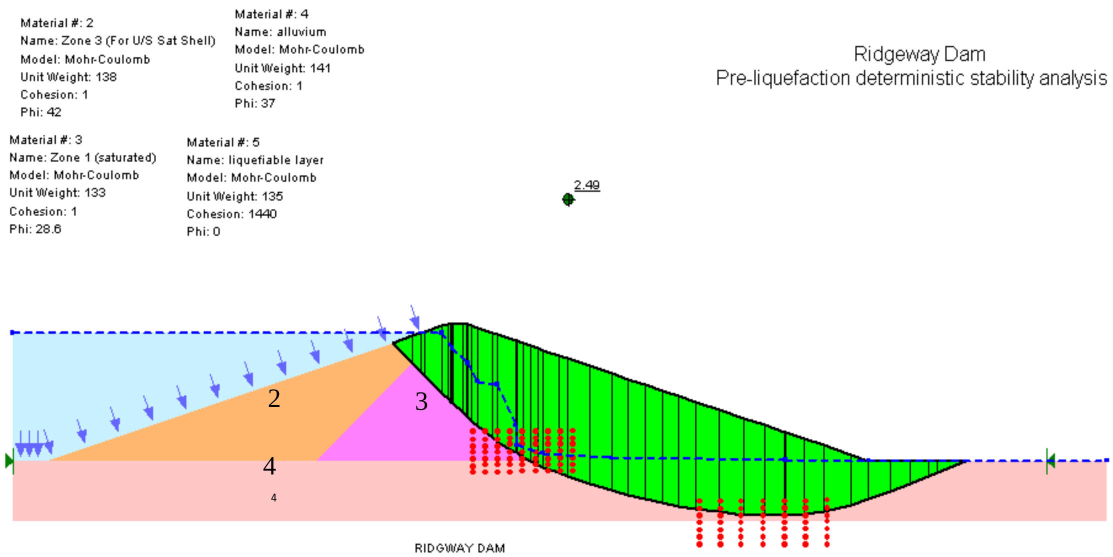


Figure 6: Graphic representation according to Spencer's Method of the SLOPE/W results describing the deterministic pre-liquefaction conditions at Ridgway Dam.

Since it would not be very meaningful to run the probabilistic approach with such low FS, a second deterministic postliquefaction analysis was run with a liquefiable layer characterized by a friction angle equal to 5 degrees (º).

This second analysis computed by PES returned a value of FS=0.98 (figure 10) while the SLOPE/W result on the same model according to Spencer's Method returned a FS=1.07 (figure 11).

In the probabilistic analysis computed by PES, the deterministic variables are characterized by the same values used in the postliquefaction deterministic analysis, and the probabilistic values are described by the statistical parameters summarized in the previous section. For the Ridgway probabilistic analysis computed using SLOPE/W, the failure surface associated with the FS of 1.07 (figure 11) was chosen as critical one to test with the probabilistic approach offered by SLOPE/W. The soil properties statistical parameters and soil spatial variation parameters used in this analysis are the same as those used in the analysis run with program PES and are summarized in tables 2, 3, and 4.

Tables 5 and 6, respectively, summarize the results for the Ridgway case history from the SLOPE/W analyses and the analyses run with PES. Figure 12 shows a direct comparison of the results from the two programs for both lower and higher *v*.

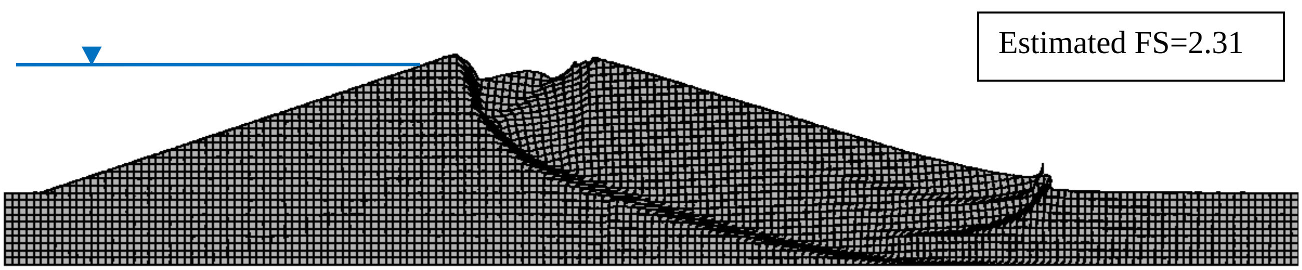


Figure 7: Figure showing displacement associated with the deterministic pre-liquefaction conditions at Ridgway Dam.

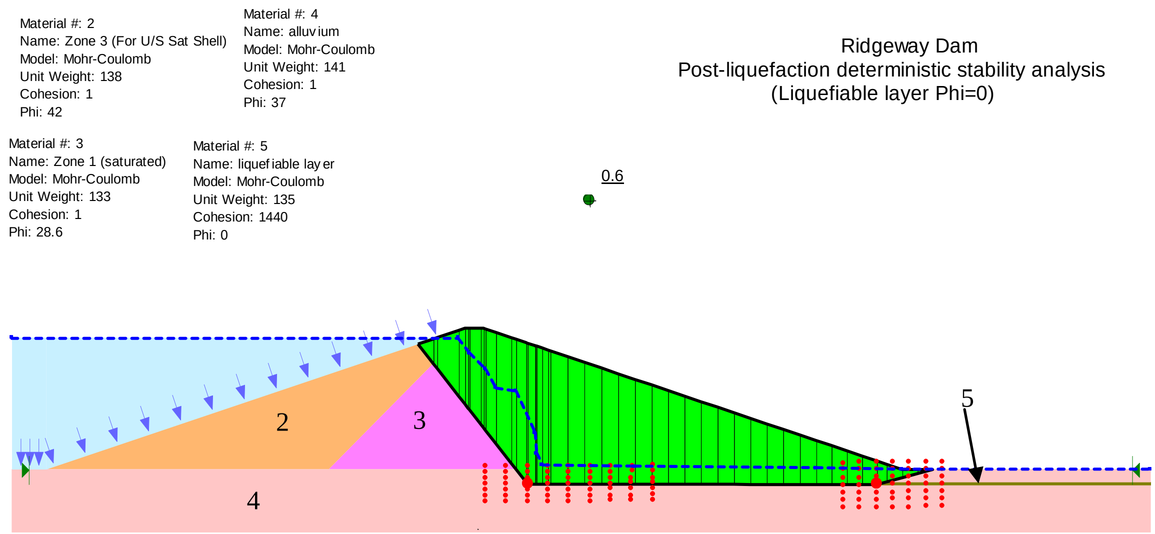


Figure 8: Graphic representation according to Spencer's Method of the SLOPE/W results describing the deterministic postliquefaction conditions at Ridgway Dam when the liquefiable layer is assumed to be frictionless.

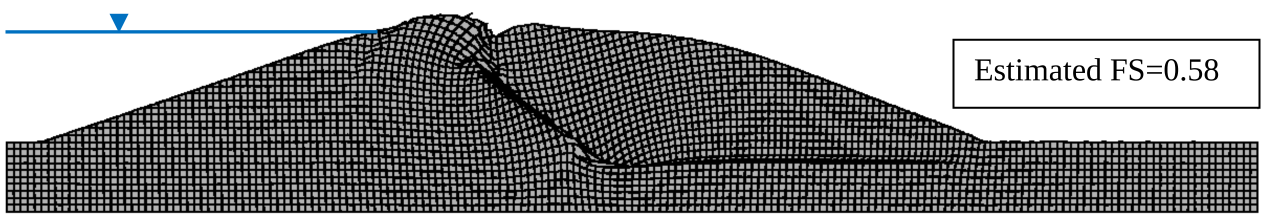


Figure 9: Figure showing displacement associated with the deterministic postliquefaction conditions at Ridgway Dam when the liquefiable layer is assumed to be frictionless.

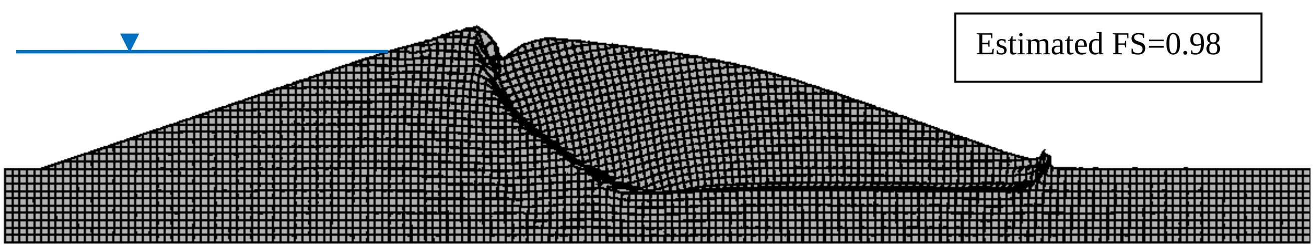


Figure 10: Figure showing displacement associated with the deterministic post-liquefaction conditions at Ridgway Dam when the liquefiable layer is characterized by a friction angle of 5º.

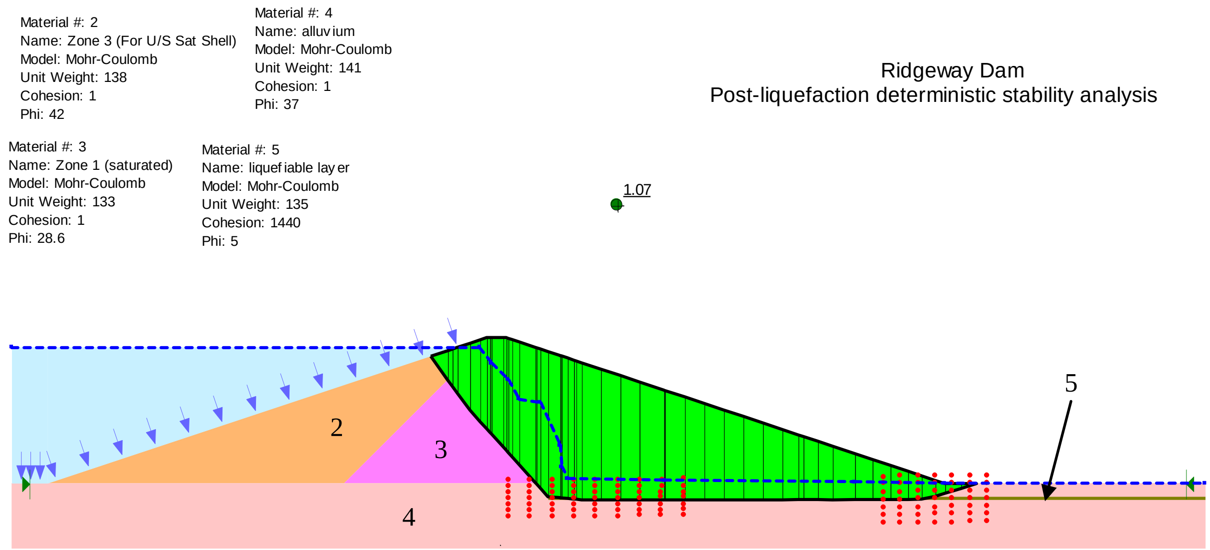


Figure 11: Graphic representation according to Spencer's Method of the SLOPE/W results describing the deterministic post-liquefaction conditions at Ridgway Dam when the liquefiable layer is characterized by a friction angle of 5º

The results showed in figure 12 outlines fundamental differences between the two programs. A detailed effort has been made during this study to comprehend the differences among the two programs; but while for the program PES a full version of the program's code is available, for the program SLOPE/W, the author of this research has to solely rely upon the program manual, published by Geostudio, which does not provide detailed information on the program code.

Table 5: Results from the Ridgway probabilistic analyses run with the program SLOPE/W

LOW *v*		**HIGH *v***	
(θ) ft	**p_f%**	**(θ) ft**	**p_f%**
4	2.71	4	16.45
10	2.41	10	16.76
15	2.34	15	16.16
20	2.6	20	16.37
25	3.27	25	18.43
30	5.26	30	20.16
35	5.16	35	20.7
40	4.75	40	20.18
50	6.27	50	24.08
100	12.76	100	31.11
200	18.91	200	36.53
350	26.93	350	42.45
500	29.81	500	43.74
600	29.96	600	43.87
800	30.78	800	43.94
1000	30.94	1000	43.95
2000	31.75	2000	45.82

Table 6: Results from the Ridgway probabilistic analyses run with the program PES

LOW *v*		**HIGH *v***	
(θ) ft	**p_f%**	**(θ) ft**	**p_f%**
4	99.9	4	100
25	96	25	100
60	92	60	100
100	89.6	100	100
200	80.9	200	100
300	78.5	300	99.6
500	74.1	500	98.1

The p_f results trend between program PES and the program SLOPE/W, shown in figure 12, corroborated the results obtained in the probabilistic validation presented in deWolfe 2010. The results presented in figure 12 confirm that the probability of failure computed by SLOPE/W is unconservative with respect to the probability of failure estimated by program PES.

Figure 12 shows that, for high values of spatial correlation, the p_f results from both programs will vary little, which is expected because high values of spatial correlation correspond to a virtually homogeneous soil material at each simulation. Lower values of spatial correlation instead emphasize a very different trend between the two programs.

The trend showed in figure 12, by SLOPE/W results, that associates lower p_f to a highly spatially variable soil (low spatial correlation) and a higher p_f with a more homogeneous soil (high spatial correlation). On the other hand, program PES shows results that associate higher p_f with more variable soils and lower p_f to a more homogeneous soil. As mentioned in the program SLOPE/W manual, the program does not apply any reduction to the standard deviation or the mean values of a random property unless the length between two sections, ΔZ , is equal to or greater than the scale of fluctuation or spatial variation length.

In the specific case of the model representing Ridgway Dam, the average distance between two slices is approximately 4 feet; therefore, no reduction was ever applied to the standard deviation or the mean values of a random property through all analyses. In general, in the case of a deterministic FS>1, a random field characterized by a reduced mean and variance values will lead to a higher probability of failure, and that could explain why the SLOPE/W results are consistently unconservative with respect to the results computed by program PES. Instability in the results produced by program PES can be observed when the spatial correlation length value is equal to or smaller than the element size. In general, a case where the element size is greater than the spatial correlation length do not represent a very meaningful model, when instead, if many elements are able to define the variability inside the spatial correlation length, this can be considered a representative model. Nevertheless, this situation does not apply to the specific analysis of Ridgway Dam.

Without a doubt, it is quite difficult to determine the correct value of a soil variability, and this parameter represents a key component of this probabilistic analysis. Only expert engineering judgment supported by exploration can truly lead to the understanding of what that meaningful range of soil variability is for a specific material. The results computed by the program PES and shown in figure 12 clearly emphasize that not accounting properly for soil variability will lead to unconservative results of p_f or nonconvergence and underestimate the probability of slope instability. It needs to be remembered that the high probability of failure computed by program PES associated with Ridgway Dam is strictly dependent on the liquefaction of a continuous layer located at the downstream toe of the dam. Even though the presence of potentially liquefiable material has been corroborated by field testing in the area, the absolute continuity of the potentially liquefiable layer still remains highly uncertain. Furthermore, based on the blow counts values describing the strength of the alluvial fine grain material characterizing the potentially liquefiable layer, liquefaction can occur only for an event associated with a high seismic return period, such as the 10,000- and 50,000-year return period characterized by an acceleration value of 0.26 g and 0.51g, respectively. The probability of such an event occurring in this area is highly unlikely. For further information on the seismicity of the area, the reader is referred to the seismotectonic report produced by the Bureau of Reclamation in 2009 (Bureau of Reclamation, 2009 [15]).

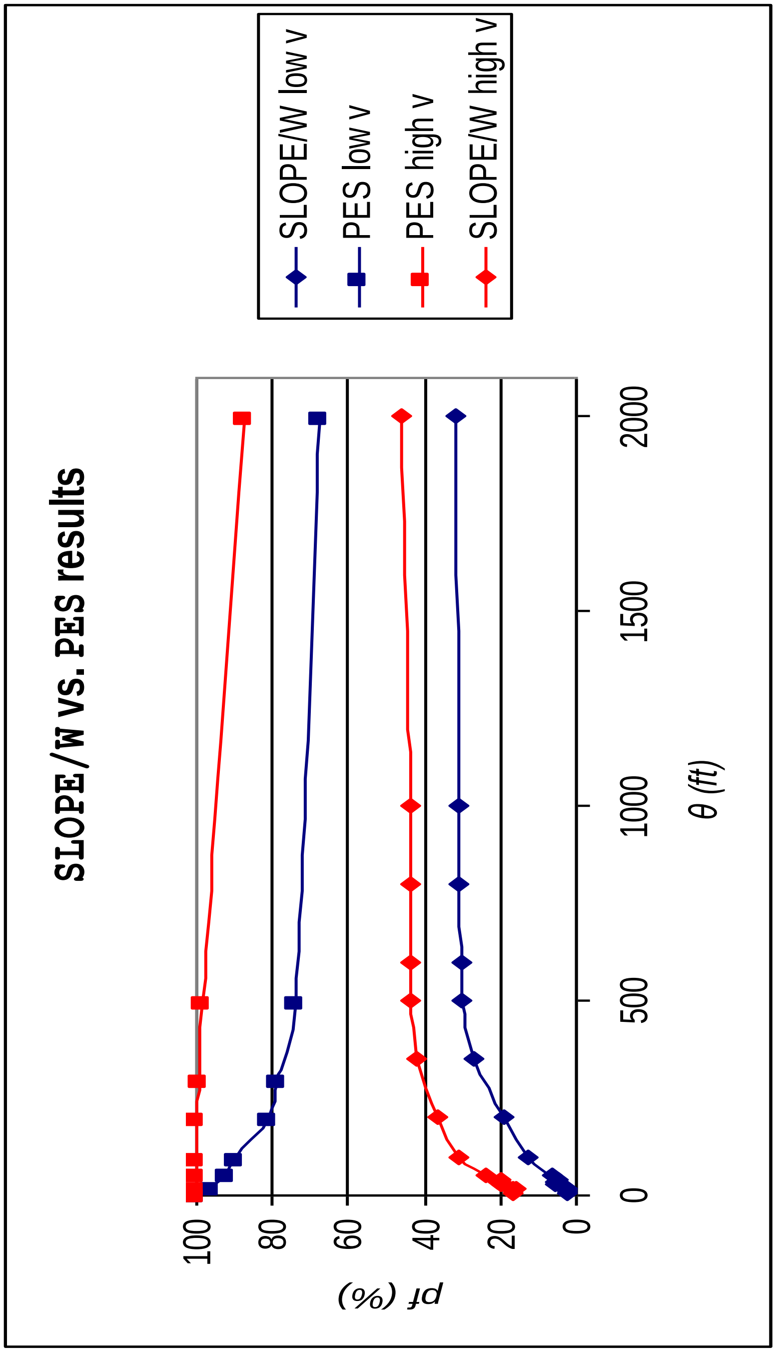


Figure 12: Comparison of the results from programs PES and SLOPE/W for both lower and higher *v*. PES results are based on the deterministic FS of 0.98 and the SLOPE/W results on the deterministic FS of 1.07.To gain a better prospective on the comparison between the element size and the spatial correlation length in this model, it is important to remember that a single square element size is equal to 5 feet, and the total dimensions of the problem are approximately, 325 feet in height and 1,840 feet in length.

Concluding Remarks

Program PES provides a repeatable methodology able to improve the confidence associated with the computation of probability of slope instability, which is a key component of risk assessment for an engineering structure.

The probabilistic approach used in program PES applies a combination of the random field technique and the finite element method.

At the core of the RFEM approach is the capability of accounting for spatially random shear strength parameters and spatial correlation. This methodology combines a nonlinear elasto-plastic finite element analysis with random field theory generated using the Local Average Subdivision Method (Griffiths and Fenton, 2004 [16]). More specifically, the spatially variable soil properties are correlated through the parameter spatial correlation length or scale of fluctuation (θ), which indicates the distance within which the values of a property show a relatively strong correlation and the parameter correlation coefficient (ρ). The main advantage of the RFEM over traditional probabilistic slope stability techniques is that it enables slope failure to develop naturally by "seeking out" the most critical mechanism.

The methodology utilized in program PES is compared against the probabilistic approach proposed by the program SLOPE/W version 7.14 and demonstrates its potential for predicting probability of failure in a nonhomogeneous soil structure characterized by phreatic conditions and a possible liquefiable layer. While the results computed from the deterministic analyses using programs PES and SLOPE/W show a very close agreement, the results from the probabilistic analyses from the two programs are generally in disagreement, and the SLOPE/W results consistently show lower values of p_f than obtained using program PES.

In the author's opinion, the difference in p_f computed by the two programs can be explained by the following three observations:

1. Both programs, PES and SLOPE/W, produce results of deterministic FS, *pf* , mean and standard deviation of FS, but it is important to remember that, for both probabilistic and deterministic analyses, program SLOPE/W represents a 1D model of the soil property correlations along the potential failure surface, while PES characterizes the soil property correlations using a 2D model. In the probabilistic approach, the program PES investigates the soil variability through the spatial correlation length over the entire foundation and embankment zones while SLOPE/W investigates the soil variability only along the line characterizing the critical slip surface.
2. Another major difference between the two programs is that SLOPE/W will perform the probabilistic analysis on a failure surface found using traditional slope stability methods (Jambu, Spencer, Bishop, etc.) that require a subdivision of the slope into columns, while the program PES, based on a strength reduction, allows the modeled slope to fail naturally by "seeking out" the path of least resistance of each Monte-Carlo simulation. In the author's opinion, the number of columns initially selected by the user in program SLOPE/W not only influences the precision of the deterministic FS but also influences the computation of the probability of failure.
3. Another component that may lead to the low values of probability by SLOPE/W, especially at lower values of the spatial correlation length (θ), is the difference in the way local averaging is implemented in the two programs.

The establishment of a robust methodology provided by this research will not only allow testing of the stability conditions of dams during modification phases but will also help estimate the probability of failure in cases involving post-earthquake liquefaction. Although, in the current study, interest was concentrated on a classical two-sided embankment geometries, the methodology can be applied to a wide range of geotechnical engineering problems, taking into account the soil spatial variability and its capability of “seeking out” the critical failure surface without assigning a pre-defined failure surface geometry.

The current work has proven that not accounting for spatial variability can lead to unconservative results with respect to more classical approaches computing probability of failure in geotechnical problems.

References

[1] Griffiths, D.V., and Fenton, G.A. (1993). "Seepage beneath water retaining structures founded on spatially random soil," *Géotechnique*, 43(6), pp. 77–587.

[2] Geostudio Group, 2007. "SLOPE/W 2007 software for Slope Stability Analysis," *Geo-SLOPE International*.

[3] Fenton, G.A. and Vanmarcke, E. H. (1990). "Simulation of random fields via local average subdivision." *J Eng Mech, ASCE,* Vol. 116, No. 8, pp. 1733–1749.

[4] Lambe, T.W., Whitman R.V., (1969). *Soil Mechanics,* Wiley, New York*,* pp. 553.

[5] Matsui, T. and San, K-C., (1992). "Finite element slope stability analysis by shear strength reduction technique". *Soils and Foundations*, Vol. 32, No. 1, pp. 59–70.

[6] deWolfe, G.F. (2010). "Probabilistic and Deterministic Slope Stability Analysis by Finite Element. Unpublished dissertation. Colorado School of Mines, Department of Engineering.

[7] Smith I.M. and Griffiths D.V., (2004). *Programming the Finite Element Method*, 4th edition, Wiley & Sons*,* West Sussex, England.

[8] Fenton, G.A. and Griffiths, D.V. (2008). *Probabilistic Methods in Geotechnical Engineering*, Springer Wien. New York.

[9] Vanmarcke E.H., (1983). *Random Fields: Analysis and Synthesis*,. MIT Press, Cambridge, Mass.

[10] Bureau of Reclamation (2003), *Technical Memorandum No. RD-8312-6 Ridgway Dam Issue Evaluation report of Findings*, Bureau of Reclamation, Internal publication, Denver, Colorado, September 2003.

[11] Bureau of Reclamation (2009), *Evaluation of Seismic and Hydrologic Risk at Ridgway Dam, Dallas Creek Project, Colorado, Upper Colorado Region,* Bureau of Reclamation, Internal publication, Denver, Colorado, October 2009.

[12] URS Corporation, Ridgway *Dam Deformation Analysis*, Final Report, July 17, 2009.

[13] Lee, I.K., White W., and Ingles O.G., *Geotechnical Engineering*, Pitman, London, 1983.

[14] Phoon, K.-K., and Kulhawy, F.H., (1999). "Characterization of geotechnical variability," *Canadian Geotechnical Journal*, 36, pp.612–624.

[15] Bureau of Reclamation (2009), *Acceleration Time Histories and Probabilistic Seismic Hazard Analysis for Ridgway Dam,* Seismotectonic report 2009–06. Bureau of Reclamation, Internal publication, Denver, Colorado, July 2009.

[16] Griffiths, D.V. and Fenton, G.A., (2004). "Probabilistic slope stability analysis by finite elements," *J Geotech Geoenviron*, Vol.130, No.5, pp.507–518.