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Estimation of seepage through spatially random soil by equivalent rectangles

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ABSTRACT: A method of estimating the statistics of the flow rate through a complex two-dimensional seepage domain with spatially random soil permeability by utilising a simplified boundary value problem has been studied. Random field theory for the generation of soil permeability properties with a fixed mean, standard deviation and spatial correlation structure, have been combined with finite element methods to perform 'Monte Carlo' simulations of both the original problem and the simplified problem. The results of parametric studies to gauge the effectiveness of the simplified problem to model the behaviour of the full domain for two example problems are presented.

1 INTRODUCTION

The assessment of seepage beneath water retaining structures is of great significance for both the serviceability and stability of the design. The estimation of the seepage quantity, exit gradients and for structures like dams, the uplift forces are classically obtained through carefully drawn flow nets (Casagrande, 1940; Cedergren 1967; Verruijt, 1970) or by the 'Method of Fragments' (Pavlovsky, 1933; Harr, 1962; Griffiths, 1984). In these methods, anisotropy and stratification can be taken into account but not the spatial variability of the soil. Previous studies that incorporate random field theory (Vanmarcke, 1984) and the finite element solution of Laplace's equation for steady state seepage have shown the effects of a stochastic soil permeability on the statistics of the required quantities for a number of boundary conditions (Smith and Freeze, 1979 Pts. 1 and 2; Fenton and Griffiths, 1993; Griffiths and Fenton, 1993; Paice, 1993; Griffiths et al, 1994).

The work contained in this paper presents a method of estimating the seepage quantity statistics of the flow through a complex two-dimensional stochastic soil domain by studying only the flow between two parallel plates, refered to as the 'equivalent rectangle' for the rest of this paper. This approach leads to a simpler boundary value problem to be solved and therefore often a large saving in computation time, ultimately allowing the presentation of results in the form of charts. This time saving allows the design to be optimised to a greater degree before carrying out a complete boundary value analysis of the problem if the exit gradient and uplift is required. The current method is applicable only to isotropic random fields where the covariance structure is independent of translation and rotation. This restriction is a consequence of the alteration of the boundary value that is being solved. Covariance structures that do not display independence of translation and rotation will yield a stochastic system that bears no resemblance to the original problem.

The key quantity required for the presented method is the shape factor, §, which is equal to the number of flow channels divided by the number of equipotential drops of a well drawn flow net. This shape factor is equivalent to a "finger print" of the steady state flow conditions through the soil domain and may be obtained either through the construction of a flow net by hand or through a deterministic finite element analysis of the complete boundary value problem. Once this shape factor has been obtained an 'equivalent rectangle' boundary value problem may be set up and the flow problem solved.

2 BRIEF DESCRIPTION OF FINITE ELEMENT AND RANDOM FIELD MODELs

The finite element program used for the solution of Laplace's equation for the boundary value problems presented in this paper is similar to that published in the text by Smith and Griffiths (1988). In all the analyses a uniform mesh of 4-node quadrilateral elements were used. The element conductivity matricies were computed explicitly and formed into the global conductivity matrix using a 'skyline' storage approach to optimise both the speed of the computations and the storage requirements.

Field measurements of permeability have indicated an approximately lognormal distribution (see e.g. Hoeksema and Kitanidis 1985, and Sudicky 1986). The same distribution has therefore been adopted for the simulations generated in this paper.

Essentially, the permeability field is obtained through the transformation

$$k_i = \exp\{\mu_{\ln k} + \sigma_{\ln k}\, g_i\} \tag{1}$$

in which k_i is the permeability assigned to the i^{th} element, g_i is the local average of a standard Gaussian random field, g, over the domain of the i^{th} element, and $\mu_{\ln k}$ and $\sigma_{\ln k}$ are the mean and standard deviation of the logarithm of k (obtained from the 'target' mean and standard deviation μ_k and σ_k).

The LAS technique (Fenton 1990, Fenton and Vanmarcke 1990) generates realizations of the local averages g_i which are derived from the random field g having zero mean, unit variance, and a spatial correlation controlled by the scale of fluctuation, θ_k. As the scale of fluctuation goes to infinity, g_i becomes equal to g_j for all elements i and j – that is the field of permeabilities tends to become uniform on each realization. At the other extreme, as the scale of fluctuation goes to zero, g_i and g_j become independent for all $i \neq j$ – the soil permeability changes rapidly from point to point.

In the two dimensional analyses presented in this paper, the scales of fluctuation in the vertical and horizontal directions are taken to be equal (isotropic) to allow the mapping of the more complex boundary value problem to that of the 'equivalent rectangle'. The 2-d model used herein implies that the out-of-plane scale of fluctuation is infinite – soil properties are constant in this direction – which is equivalent to specifying that the streamlines remain in the plane of the analysis.

3 EQUIVALENT RECTANGLE METHODOLOGY

The general methodology of the 'equivalent rectangle' method will be demonstrated by taking the case of the steady state seepage under a single sheet pile wall penetrating into a confined soil medium. The full problem and 'equivalent rectangle' are shown in Figures 1 and 2 respectively.

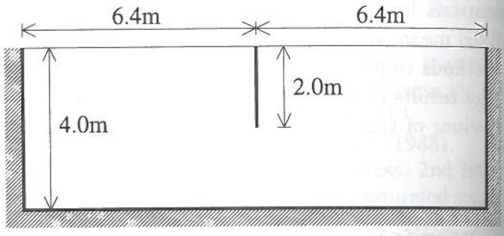

Figure 1: Full boundary value problem - not to scale

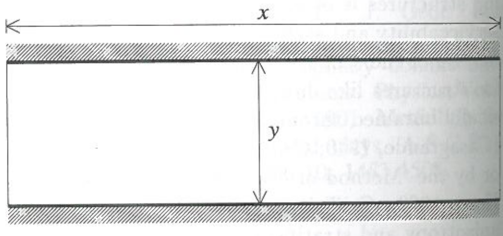

Figure 2: 'Equivalent rectangle' boundary value problem

1. Solve the complete boundary value problem deterministically to obtain the shape factor, §, that corresponds with the current design.
2. The minimum flow restricting dimension is selected from the complete problem which will then become the dimension of the entry and exit seepage faces of the 'equivalent rectangle'.
3. Since deterministically the shape factor of the flow through the equivalent rectangule is

equal to the ratio of the dimension of the entry and exit seepage faces to the impermeable boundary, the dimension of the impermeable boundaries can be calculated simply through the following equation:

$$x = \frac{y}{\S} \tag{2}$$

4. Perform a 'Monte Carlo' simulation of the flow through the 'equivalent rectangle' to obtain the statistics of the flow rate (mean and standard deviation).

5. Subject to the reliability of the current design in the context of the flow rate, the procedure may return to the first step and modify the boundary value problem to produce a more reliable design.

For a uniform permeability field there is no restriction on the dimension of the seepage faces in step 2. Under stochastic conditions a restriction has been placed so that the height of the 'equivalent rectangle' should be equal to that of the minimum restricting dimension of the full problem. This restriction imposes the same 'blockage' effect that the scale of fluctuation has on the full problem when the fluid passes through the minimum resticting dimension.

4 EXAMPLE PROBLEMS

For both examples presented, each 'Monte Carlo' simulation consisted of 5000 realisations of the permeability field based on the set of input statistics $(\mu_k, \sigma_k, \theta_k)$ for both the full problem and the 'equivalent rectangle'. In all the analyses presented, the input mean permeability has been set to $\mu_k = 1.0 \times 10^{-5}$ m/s. For convenience the flow rate statistics have been presented in a non-dimensional form by representing it in terms of a normalised flow rate $\bar{Q}$ where:

$$\bar{Q} = Q/(H\mu_k) \tag{3}$$

and H is the total head loss across the boundary value problem, typically set to unity.

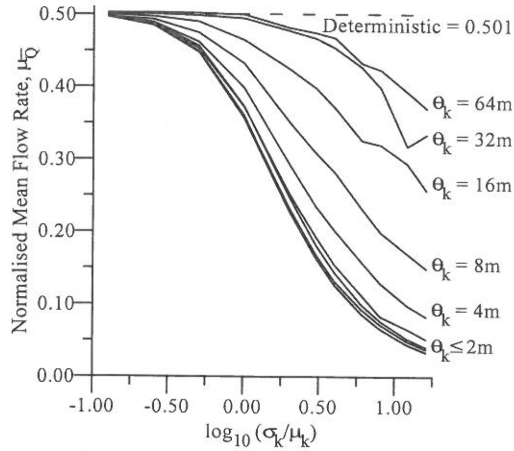

Figure 3: Influence of random permeability on mean flow rate, $\mu_{\bar{Q}}$

Briefly concentrating on the full problem, Figure 3 indicates a consistent fall in the expected value of the flow rate, $\mu_{\bar{Q}}$, from its deterministic value of $\bar{Q} \approx 0.5$ for increasing input permeability coefficient of variation σ_k/μ_k. For smaller values of the scale of fluctuation θ_k the reduction is more substantial than for higher scales of fluctuation. The expected flow rate is clearly tending towards the deterministic result for the higher scales of fluctuation which is expected for a strongly correlated permeability field.

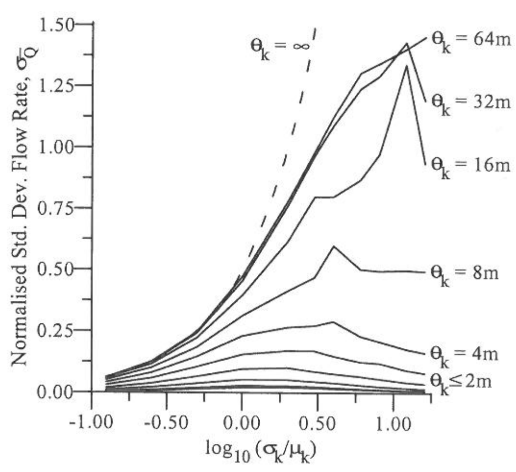

Figure 4: Influence of random permeability on standard deviation of flow rate, $\sigma_{\bar{Q}}$

Figure 4 shows the standard deviation of the normalised flow rate, $\sigma_{\bar{Q}}$, for the full problem. For small θ_k little variation in $\bar{Q}$ was observed but as θ_k increased then the flow rate became more variable, tending towards the limiting value indicated by $\theta_k = \infty$ given by equation (4).

$$\sigma_{\bar{Q}} = \frac{\sigma_k}{\mu_k}\bar{Q}_{det} = \frac{\sigma_k}{\mu_k}\S \qquad (4)$$

The maximum point observed in the plot of $\sigma_{\bar{Q}}$ vs. $\log_{10}(\sigma_k/\mu_k)$ is an interesting result and appears to occur at a higher value of σ_k/μ_k for increasing θ_k. These results are consistent with those previously published by Griffiths *et al*, 1994. Two examples are presented with representative results to gauge the effectiveness of the 'equivalent rectangle' method.

4.1 Symmetrical Single Sheet Pile Wall

For this problem the wall penetrates halfway into a confined soil medium with the same boundary conditions shown in Section 3. From inspection of the problem, the minimum restricting dimension is equal to the distance vertically underneath the wall which in this case is equal to 2.0m. Therefore the height of the 'equivalent rectangle', y, is equal to 2.0m. Solution of the deterministic full boundary value problem leads to a shape factor of $\S = 0.5$ which allows the calculation of the length of the equivalent rectangle, x, to be 4.0m.

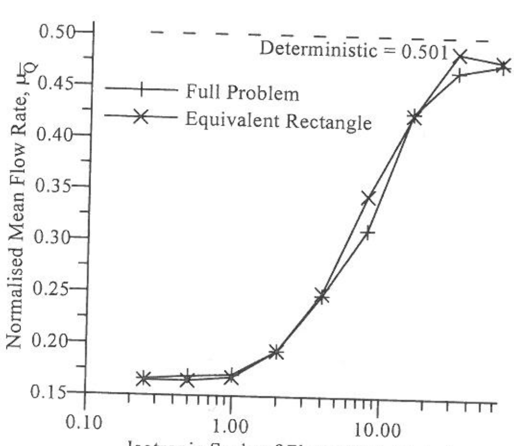

Figure 5: Comparision of $\mu_{\bar{Q}}$ obtained from full problem and 'equivalent rectangle' for $\sigma_k/\mu_k = 3.0$

Figure 5 shows the expected value of the mean flow rate, $\mu_{\bar{Q}}$, for both the full problem and the 'equivalent rectangle' for $\sigma_k/\mu_k = 3.0$. This value of σ_k/μ_k is equal to the recommended upper limit suggested by Lee *et al*, 1983. This high coefficient of variation represents the worst correlation between the results of the full and 'equivalent rectangle' analyses with the 'equivalent rectangle' predicting a value of $\mu_{\bar{Q}}$ equal to a maximum of 1.10 times that given by the full problem at $\theta_k = 8.0$m.

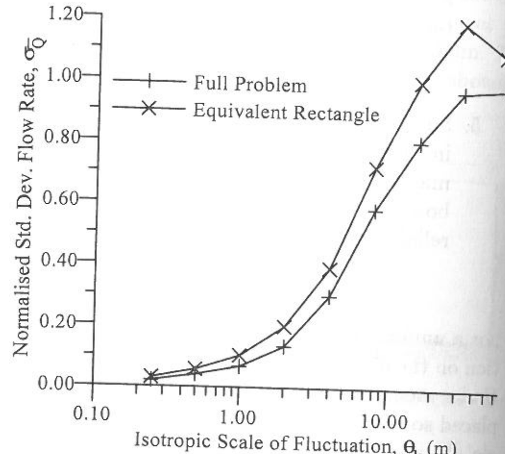

Figure 6: Comparision of $\sigma_{\bar{Q}}$ obtained from full problem and 'equivalent rectangle' for $\sigma_k/\mu_k = 3.0$

Figure 6 shows the standard deviations of the normalised flow rate for the same input permeability statistics presented in Figure 5. Agreement between the values of $\sigma_{\bar{Q}}$ predicted through the analysis of the full and 'equivalent rectangle' boundary value problems is worse than for $\mu_{\bar{Q}}$ with the 'equivalent rectangle' predicting a value of $\sigma_{\bar{Q}}$ equal to a maximum of 1.52 times that given by the full problem at a value of $\theta_k = 1.0$m. The sporadic result obtained using the 'equivalent rectangle' for a value of $\theta_k = 32.0$m appears to be a induced by the high scale of fluctuation and the use of more realisations would improve this prediction.

Visual inspection of Figure 5 and 6 show that although the predicted values of the statistics of the normalised flow rate are not perfect the 'equivalent rectangle' displays reasonable agreement for increasing scale of fluctuation and are close over a large range of θ_k.

4.2 Unsymmetrical Single Sheet Pile Wall

For this problem the wall penetrates 30% into the confined soil medium but the dimensions either side of the wall are unequal. This boundary value problem is equivalent to a double-walled cofferdam. For this case the minimum restricting dimension is equal to the distance from the left of the wall to the impermeable boundary, 1.6m. Solving for the shape factor gives § = 0.514 and therefore the length of the 'equivalent rectangle' is equal to 3.113m.

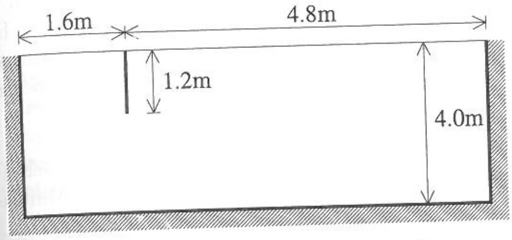

Figure 7: Full boundary value problem - not to scale

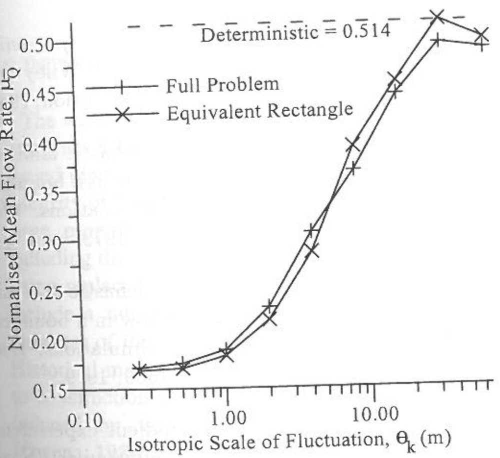

Figure 8: Comparision of $\mu_{\bar{Q}}$ obtained from full problem and 'equivalent rectangle' for $\sigma_k/\mu_k = 3.0$

Figure 8 shows the expected value of the mean flow rate, $\mu_{\bar{Q}}$, for both the full problem and the 'equivalent rectangle' for $\sigma_k/\mu_k = 3.0$. As for the previous example, this value of σ_k/μ_k corresponds to the worst agreement between the values predicted by the full problem and the 'equivalent rectangle'. With these input permeability statistics the 'equivalent rectangle' predicting a value of $\mu_{\bar{Q}}$ equal to a

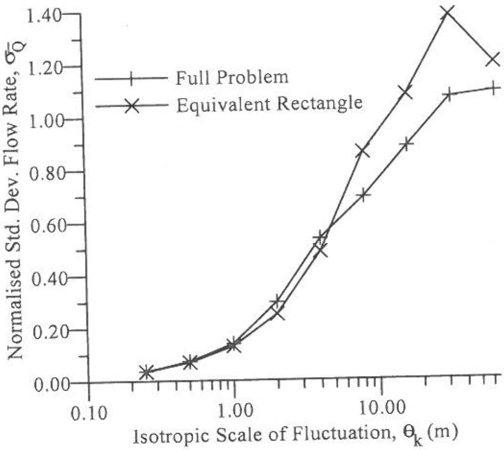

Figure 9: Comparision of $\sigma_{\bar{Q}}$ obtained from full problem and 'equivalent rectangle' for $\sigma_k/\mu_k = 3.0$

maximum of 1.06 times that given by the full problem at $\theta_k = 8.0$m.

Figure 9 shows the standard deviations of the normalised flow rate. On the whole, the agreement between the full problem and the 'equivalent rectangle' is better than that observed for the symmetrical boundary value problem with the 'equivalent rectangle' predicting a value of $\sigma_{\bar{Q}}$ equal to a maximum of 1.24 times that given by the full problem at a value of $\theta_k = 8.0$m.

5 DISCUSSION AND CONCLUDING REMARKS

The paper has presented a method of estimating the statistics of the flow rate through a complex two-dimensional soil medium with spatially variable permeability by the solution of only the flow between two parallel plates. This method results in considerably faster analyses (for example, the problem presented in section 4.1 typically ran 12 times faster for the 'equivalent rectangle' based on the same degree of element discretisation) with acceptable accuracy in most cases.

Overall the 'equivalent rectangle' modelled the mean flow rate better than the standard deviation and the improvement of these standard deviations is the subject of further work. Improvement may be obtained through the recognition that the full problem demonstrates a behaviour which tends to follow the geometric mean permeability (Paice, 1993; Griffiths *et al*, 1994) while the parallel plates problem tends to behave like a series of blocks of permeability due to the value of the shape factor, §, being less than 1.0 for the majority of seepage problems. The parallel plate problem therefore exhibits a behaviour that is closer to the harmonic mean permeability for lower §.

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