Reliability-Based Deep Foundation Design

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Abstract

Modern geotechnical design codes are migrating towards Load and Resistance Factor Design (LRFD) methodologies. The Danish geotechnical code has been based on LRFD for several decades, but more recently the Eurocode and the Australian Standards have turned in this direction. Where the geotechnical system supports a structure, the load factors are generally determined by the structural codes. The geotechnical resistance factors, typically determined by calibration with traditional working stress (or allowable stress) design, have yet to be clearly defined in geotechnical design codes. Research into the reliability of geotechnical systems is needed in order for resistance factors to be determined.

This paper presents the results of a preliminary study into the effect of a soil's spatial variability on the settlement and ultimate load statistics of a pile. The results are used to provide recommendations on approaches to reliability-based deep foundation design at the serviceability and ultimate limit states.

Introduction

Deep foundations, which are typically either piles or drilled shafts, will be hereafter collectively referred to as *piles* for simplicity in this paper. Piles are provided to transfer load to the surrounding soil and/or to a firmer stratum, thereby providing vertical and lateral load bearing capacity to a supported structure. In this paper the random behaviour of a pile subjected to a vertical load and supported by a spatially variable soil is investigated.

The resistance, or bearing capacity, of a pile arises as a combination of side friction, where load is transmitted to the soil through friction along the sides of the pile, and end bearing, where load is transmitted to the soil (or rock) through the tip of the pile. As load is applied to the pile, the pile settles – the total settlement of the pile is due to both deformation of the pile itself and deformation of the surrounding soil and supporting stratum. The surrounding soil is, at least initially, assumed to be perfectly bonded to the pile shaft through friction and/or adhesion so that any displacement of the pile corresponds to an equivalent local displacement of the soil (the soil deformation reduces further away from the pile). In turn, the elastic nature of the soil means that this displacement is resisted by a force which is proportional to the soil's elastic modulus and the magnitude of the displacement. Thus, at least initially, the support imparted by the soil to the pile depends on the elastic properties of the surrounding soil. For example, Vesic (1977) states that the fraction of pile settlement due to deformation of the soil, δ_s, is a constant (dependent on Poisson's ratio and pile geometry) times Q/E_s, where Q is the applied load and E_s is the (effective) soil elastic modulus.

As the load on the pile is increased, the bond between the soil and the pile surface will at some point break down and the pile will both slip through the surrounding soil and plastically fail the soil under the pile tip. At this point, the ultimate bearing capacity of the pile has been reached. The force required to reach the point at which the pile slips through a sandy soil is conveniently captured using a soil-pile interface friction angle, ψ. The frictional resistance per unit area of the pile surface, f, can then be expressed as

$$f = \sigma_n \tan \psi \tag{1}$$

where σ_n is the effective stress exerted by the soil normal to the pile surface. In many cases, $\sigma_n = K\sigma'_o$, where K is the earth pressure coefficient and σ'_o is the effective vertical stress at the depth under consideration. The total ultimate resistance supplied by the soil to an applied pile load is the sum of the end bearing capacity (which can be estimated using the usual bearing capacity equation) and the integral of f over the embedded surface of the pile. For clays with zero friction angle, Vijayvergiya and Focht (1972) suggest that the average of f, denoted with an overbar, can be expressed in the form

$$\bar{f} = \lambda \left(\bar{\sigma}'_o + 2c_u\right) \tag{2}$$

where $\bar{\sigma}'_o$ is the average effective vertical stress over the entire embedment length, c_u is the undrained cohesion, and λ is a correction factor dependent on pile embedment length.

The limit states design of a pile involves checking the design at both the serviceability limit state and the ultimate limit state. The serviceability limit state is a limitation on pile settlement, which in effect involves computing the load beyond which settlements become intolerable. Pile settlement involves consideration of the elastic behaviour of the pile and the elastic (e.g. E_s) and consolidation behaviour of the surrounding soil.

The ultimate limit state involves computing the ultimate load that the pile can carry just prior to failure. Failure is assumed to occur when the pile slips through the soil (we are not considering structural failure of the pile itself) which can be estimated with the aid of Eq's 1 or 2, along with the end bearing capacity equation. The ultimate pile capacity is a function of the soil's cohesion and friction angle parameters.

In this paper, the soil's influence on the pile will be represented by bi-linear springs, as illustrated in Figure 1. The initial sloped portion of the load-displacement curve corresponds to the elastic (E_s) soil behaviour, while the plateau corresponds to the ultimate shear strength of the pile-soil interface which is a function of the soil's friction angle and cohesion. The next section discusses the finite element and random field models used to represent the pile and supporting soil in more detail. In the following section an analysis of the random behaviour of a pile is described and presented. Only the effects of the spatial variability of the soil are investigated, and not, for instance, those due to construction and placement variability. Finally, the results are evaluated and recommendations are made.

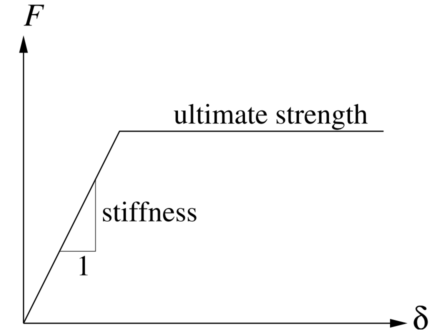

Figure 1. Bi-linear load (F) vs. displacement (δ) curve for soil springs.

The Random Finite Element Model

The pile itself is divided into a series of elements, as illustrated in Figure 2. Each element has cross-sectional area, A, (assumed constant) and elastic modulus, E_p, which can vary randomly along the pile. The stiffness assigned to the i^{th} element is the geometric average of the product AE_p over the element domain.

As indicated in Figure 1, the i^{th} soil spring is characterized by two parameters; its initial stiffness, S_i, and its ultimate strength, U_i. The determination of these two parameters from the soil's elastic modulus, friction angle, and cohesion properties is discussed conceptually as follows;

1) The initial spring stiffness, S_i, is a function of the soil's spatially variable elastic modulus, E_s. Since the strain induced in the surrounding soil due to displacement of the pile is complex, not least because the strain decreases non-linearly with distance from the pile, the effective elastic modulus of the soil as seen by the pile at any point along the pile is currently unknown. The nature of the relationship between E_s and S_i remains a topic for further research. In this paper, the spring stiffness contribution per unit length of the pile, $S(z)$, will be simulated directly as a lognormally distributed one-dimensional random process.
2) The ultimate strength of each spring is somewhat more easily specified, so long as the pile-soil interface adhesion, friction angle, and normal stress are known. Assuming that soil properties vary only with depth, z, the ultimate strength per unit pile length at depth z, will have the general form (in the event that both adhesion and friction act simultaneously)

$$U(z) = p\Big[\alpha c_u(z) + \sigma_n(z)\tan\psi(z)\Big]$$

where $\alpha c_u(z)$ is the adhesion at depth z (see Das, 2000, pg. 519, for estimates of the adhesion factor, α), p is the pile perimeter length, $\sigma_n(z)$ is the normal effective soil stress at depth z, and $\psi(z)$ is the interface friction angle at depth z. The normal stress is often taken as $K\sigma'_o$, where K is the earth pressure coefficient. Rather than simulate c_u and $\tan\psi$ and introduce the empirical and uncertain factors α and K, both of which could also be spatially variable, the ultimate strength per unit length, $U(z)$, will also be simulated directly as a lognormally distributed one-dimensional random process.

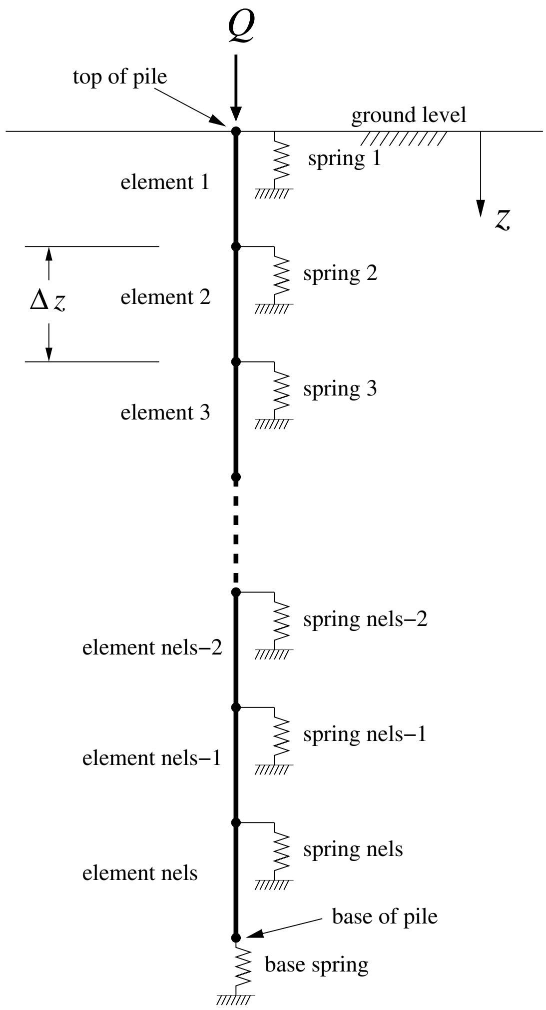

Figure 2. Finite element representation of the pile-soil system.

The random finite element model (RFEM) thus consists of a sequence of pile elements joined by nodes, a sequence of spring elements attached to the nodes (see Figure 2), and three *independent* 1-D random processes described as follows;

- $S(z)$ and $U(z)$ are the spring stiffness and strength contributions from the soil per unit length along the pile, and
- $E_p(z)$ is the elastic modulus of the pile.

It is assumed that the elastic modulus of the pile is a 1-D stationary lognormally distributed random process characterized by the mean pile stiffness, μ_{AE_p}, standard deviation, σ_{AE_p}, and correlation length $\theta_{\ln E_p}$, where A is the pile cross-sectional area. Note that for simplicity, it is assumed that all three random processes have the same correlation lengths and all have the same correlation function (Markovian). While it may make sense for the correlation lengths associated with $S(z)$ and $U(z)$ to be similar, there is no reason that the correlation length of $E_p(z)$ should be the same as that in the soil. Keeping them the same merely simplifies the study, while still allowing the study to assess whether a "worst case" correlation length exists for the deep foundation problem.

The elastic modulus assigned to each pile element will be some sort of average of $E_p(z)$ over the element length and in this paper the geometric average will be used;

$$E_{p_i} = \exp\left\{ \frac{1}{\Delta z} \int_{z_i}^{z_i+\Delta z} \ln E_p(z)\, dz \right\} \tag{3}$$

where z_i is the depth to the top of the i^{th} element. The geometric average is dominated by low stiffness values, which is appropriate for elastic deformation. It is to be noted that for a pile idealized using an elastic modulus varying only along the pile length, the true "effective" pile stiffness is the harmonic average

$$E_H = \left[\frac{1}{\Delta z} \int_{z_i}^{z_i+\Delta z} \frac{1}{E_p(z)}\, dz \right]^{-1}$$

which is even more strongly dominated by low stiffness values than the geometric average. However, the following justification can be argued about the use of the geometric average rather than the harmonic average over each element;

1) if the elements are approximately square (i.e. $\Delta z \simeq D$), and the pile's true 3-D elastic modulus field is approximately isotropic (i.e. not strongly layered) then the effective elastic modulus of the element will be (at least closely approximated by) a geometric average. See, e.g., Fenton and Griffiths (2002 and 2005), where this result was found for a soil block, which is a similar stochastic settlement problem to the pile element "block".
2) if the pile is subdivided into a reasonable number of elements along its length (say, ten or more), then the overall response of the pile tends towards a harmonic average in any case, since the finite element analysis will yield the exact "harmonic" result.

We are left now with the determination of the spring stiffness and strength values, S_i and U_i, from the 1-D random processes, $S(z)$ and $U(z)$. Note that the spring parameters, S_i and U_i, have units of stiffness (kN/m) and strength (kN), respectively, while $S(z)$ and $U(z)$ are the soil's contribution to the spring stiffness and strength *per unit length along the pile*. That is, $S(z)$ has units of kN/m/m and $U(z)$ has units of kN/m.

To determine the spring parameters, S_i and U_i, from the continuously varying $S(z)$ and $U(z)$ we need to think about the nature of the continuously varying processes and how they actually contribute to S_i and U_i. In the following we will discuss this only for the stiffness contribution, S, the strength issue is entirely analogous and can be determined simply by substituting S with U in the following.

We will first subdivide each element into two equal parts, as shown in Figure 3, each of length $\Delta h = \Delta z/2$. The top of each subdivided cell will be at $t_j = (j-1)\Delta h$ for $j = 1, 2, \ldots, 2(nels) + 1$, where $nels$ is the number of elements. This subdivision is done so that the tributary lengths for each spring can be more easily defined: the stiffness for spring 1 is accumulated from the soil stiffness contribution, $S(z)$, over the top cell from $z = t_1 = 0$ to $z = t_2 = \Delta h$. The stiffness for spring 2 is accumulated from the cell above spring 2 as well as from the cell below spring 2, i.e. from $z = t_2 = \Delta h$ to $z = t_3 = 2\Delta h$ and from $z = t_3 = 2\Delta h$ to $z = t_4 = 3\Delta h$, and so on.

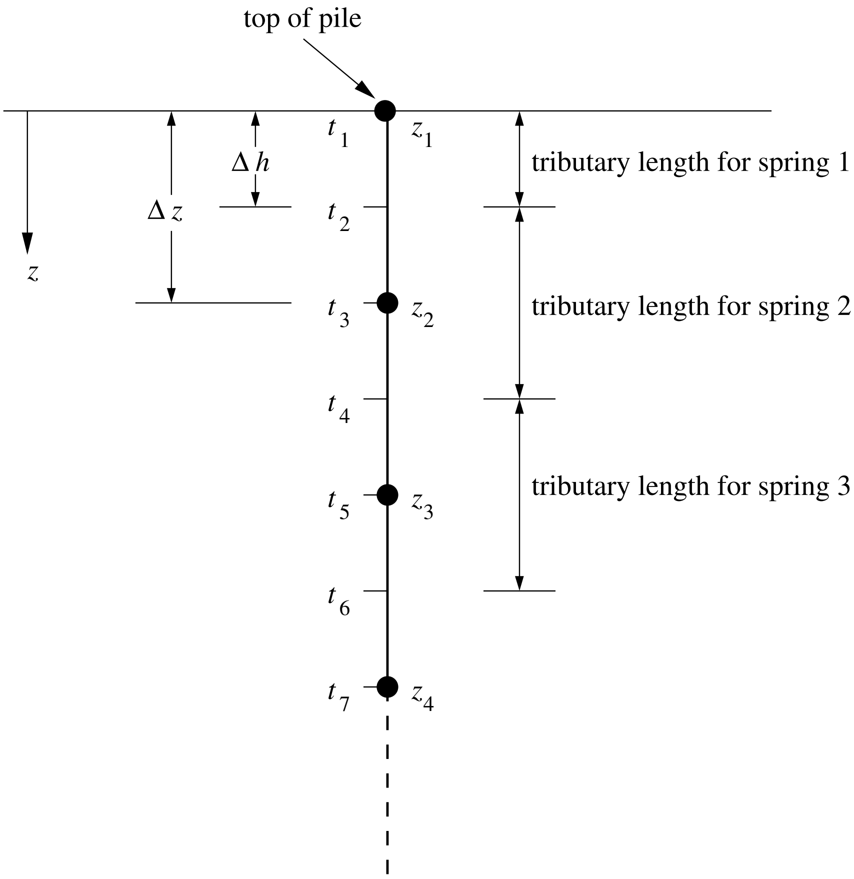

Figure 3. Subdivisions used to compute geometric averages.

If the stiffness contributions, $S(z)$, at each point z are independent (i.e., white noise), and if the pile stiffness is significantly larger than the soil stiffness, then S_i should be an arithmetic sum of $S(z)$ over the spring's tributary length,

$$S_i = \int_{z_i-\Delta z/2}^{z_i+\Delta z/2} S(z)\, dz$$

In other words, S_i should be an arithmetic average of $S(z)$ over the tributary length multiplied by the tributary length. However, $S(z)$ is not a white noise process – a low stiffness region close to the pile will depress the stiffness contribution over a length of pile which will probably be significantly larger than the low strength region itself. Thus, it makes sense to assume that S_i should be at least somewhat dominated by low stiffness regions in the surrounding soil.

In this paper, a compromise shall be made: S_i will be an arithmetic sum of the two geometric averages over the i^{th} spring's tributary areas (in the case of the top and bottom springs, only one tributary area is involved). The result is less strongly low stiffness dominated than a pure geometric average, as might be expected by this sort of a problem where the strain imposed on the soil is relatively constant over the element lengths (i.e. the constant strain results in at least some arithmetic averaging). The exact nature of the required average is left for future research.

If the mean of $S(z)$ is allowed to vary linearly with depth, z, then

$$\mu_S = \mathrm{E}\left[S(z)\right] = a + bz$$

If the stiffness per unit length at the top and bottom of the pile are s_{top} and s_{bot}, respectively, and we measure z downwards from the top of the pile, then

$$\begin{aligned} a &= s_{top} \\ b &= (s_{bot} - s_{top})/L \end{aligned}$$

where L is the pile length.

It is assumed that $S(z)$ is lognormally distributed. It thus has parameters

$$\begin{aligned} \mu_{\ln S} &= \ln(a + bz) - \tfrac{1}{2}\sigma^2_{\ln S} \\ \sigma^2_{\ln S} &= \ln(1 + V_S^2) \end{aligned}$$

where V_S is the coefficient of variation of $S(z)$. It will be assumed that V_S is constant with depth, so that $\sigma_{\ln S}$ is also constant with depth. $S(z)$ can now be expressed in terms of the underlying mean zero, unit variance, normally distributed 1-D random process, $G(z)$,

$$\begin{aligned} S(z) &= \exp\left\{\mu_{\ln S} + \sigma_{\ln S} G(z)\right\} \\ &= \exp\left\{\ln(a + bz) - \tfrac{1}{2}\sigma^2_{\ln S} + \sigma_{\ln S} G(z)\right\} \end{aligned}$$

In other words

$$\ln S(z) = \ln(a + bz) - \tfrac{1}{2}\sigma^2_{\ln S} + \sigma_{\ln S} G(z).$$

Now let S_{G_j} be the geometric average of the soil spring stiffness contribution, $S(z)$, over the j^{th} cell, that is over a length of the pile from t_j to $t_j + \Delta h$, $j = 1, 2, \ldots, 2(nels)$,

$$\begin{aligned} S_{G_j} &= \exp\left\{ \frac{1}{\Delta h} \int_{t_j}^{t_j + \Delta h} \ln S(z)\, dz \right\} \\ &= \exp\left\{ \frac{1}{\Delta h} \int_{t_j}^{t_j + \Delta h} \left[\ln(a + bz) - \tfrac{1}{2}\sigma^2_{\ln S} + \sigma_{\ln S} G(z)\right] dz \right\} \\ &= \exp\left\{ \frac{1}{\Delta h} \int_{t_j}^{t_j + \Delta h} \ln(a + bz)\, dz - \tfrac{1}{2}\sigma^2_{\ln S} + \sigma_{\ln S} G_j \right\} \end{aligned}$$

where G_j is the *arithmetic* average of $G(z)$ from $z = t_j$ to $z = t_j + \Delta h$;

$$G_j = \frac{1}{\Delta h} \int_{t_j}^{t_j + \Delta h} G(z)\, dz$$

Now define

$$m_j = \frac{1}{\Delta h}\int_{t_j}^{t_j+\Delta h} \ln(a+bz)\,dz - \tfrac{1}{2}\sigma^2_{\ln S}$$
$$= \frac{1}{b\Delta h}\left[a_1\ln(a_1) - a_2\ln(a_2)\right] - 1 - \tfrac{1}{2}\sigma^2_{\ln S}$$

where

$$a_1 = a + b(t_j + \Delta h)$$
$$a_2 = a + bt_j$$

If $b = 0$, i.e. the soil stiffness contribution is constant with depth, then m_j simplifies to

$$m_j = \ln(s_{top}) - \tfrac{1}{2}\sigma^2_{\ln S}$$

Using m_j, the geometric average becomes

$$S_{G_j} = \exp\left\{m_j + \sigma_{\ln S}G_j\right\}$$

Notice that m_j is the arithmetic average of $\mu_{\ln S}$ over the distance from $z = t_j$ to $z = t_j + \Delta z$.

The contribution to the spring stiffness is now $\Delta h S_{G_j}$. In particular, the top spring has contributing soil stiffness from $z = 0$ to $z = \Delta h$, so that $S_1 = \Delta h S_{G_1}$. Similarly, the next spring down has contributions from the soil from $z = \Delta h$ to $z = 2\Delta h$ as well from $z = 2\Delta h$ to $z = 3\Delta h$, so that

$$S_2 = \Delta h\left[S_{G_2} + S_{G_3}\right]$$

and so on.

The finite element analysis is displacement controlled. In other words, the load corresponding to the prescribed maximum tolerable serviceability settlement, δ_{max}, is determined by imposing a displacement of δ_{max} at the pile top. Because of the non-linearity of the springs, the finite element analysis involves an iteration to converge on the set of admissible spring forces which yield the prescribed settlement at the top of the pile. The relative max-error convergence tolerance is set to a very small value of 0.00001.

The pile capacity corresponding to the ultimate limit state is computed simply as the sum of the U_i values over all of the springs.

Monte Carlo Estimation of Pile Capacity

To assess the probabilistic behaviour of deep foundations, a series of Monte Carlo simulations, with 2000 realizations each, were performed and the distribution of the serviceability and ultimate limit state loads were estimated. The serviceability limit state was defined as being a settlement of $\delta_{max} = 25$ mm. Because the maximum tolerable settlement cannot easily be expressed in dimensionless form, the entire analysis will be performed for a particular case study; namely a pile of length 10 m is divided into 30 elements with $\mu_{AE_p} = 1000$ kN, $\sigma_{AE_p} = 100$ kN, $\mu_S = 100$ kN/m/m, and $\mu_U = 10$ kN/m.

The base of the pile is assumed to rest on a slightly firmer stratum, so the base spring has mean stiffness 200 kN/m and mean strength 20 kN (note that this is in addition to the soil contribution arising from the lowermost half-element). Coefficients of variation of spring stiffness and strength, V_S and V_U, taken to be equal, ranged from 0.1 to 0.5. Correlation lengths, $\theta_{\ln S}$, $\theta_{\ln E_s}$, and $\theta_{\ln U}$, all taken to be equal and referred to collectively simply as θ, ranged from 0.1 m to 100.0 m. The spring stiffness and strength parameters were assumed to be mutually independent, as well as being independent of the pile elastic modulus.

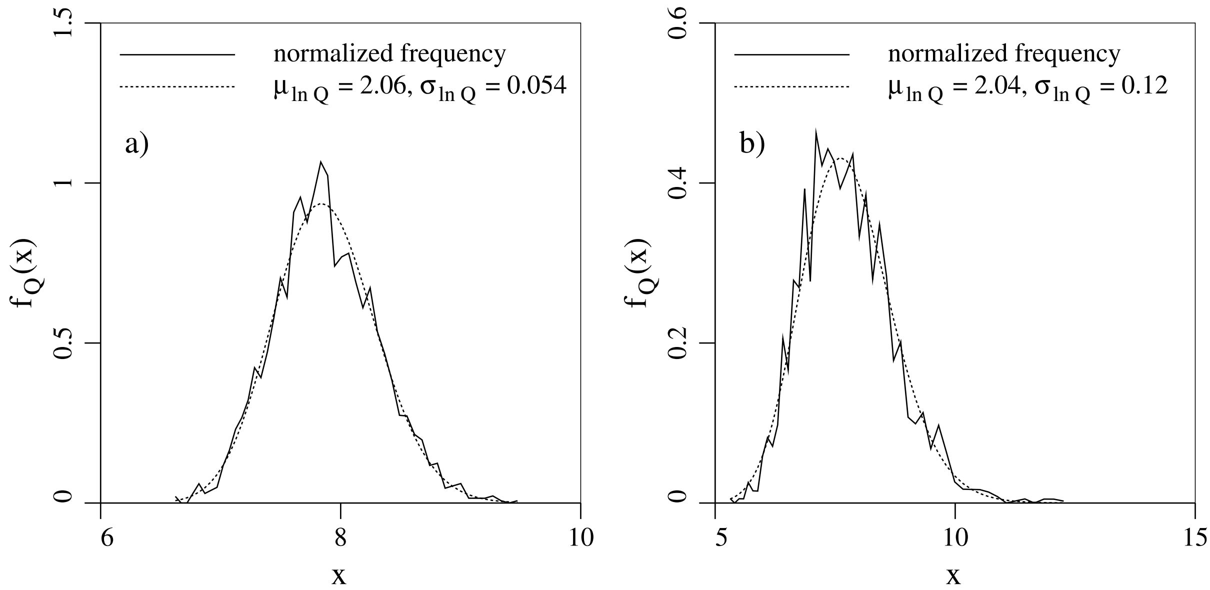

Figure 4. Estimated and fitted lognormal distributions of serviceability limit state loads, Q for a) V = 0.2 and θ = 1 m (p-value = 0.84) and b) V = 0.5 and θ = 1.0 m (p-value = 0.00065).

The first task is to determine the nature of the distribution of the serviceability and ultimate pile loads. Figure 4 shows one of the best (on the left) and worst (on the right) fits of a lognormal distribution to the serviceability pile load histogram with chi-square goodness-of-fit p-values of 0.84 and 0.0006, respectively (the null hypothesis being that the serviceability load follows a lognormal distribution). The right-hand plot would result in the lognormal hypothesis being rejected for any significance level in excess of 0.06%. Nevertheless, a visual inspection of the plot suggests that the lognormal distribution is quite *reasonable* – in fact it is hard to see why one fit is so much 'better' than the other. It is well known, however, that when the number of simulations is large, goodness-of-fit tests tend to be very sensitive to small discrepancies in the fit, particularly in the tails.

Figure 5 shows similar results for the ultimate pile capacities. In both figures, the lognormal distribution appears to be a very reasonable fitted, despite the very low p-value of Figure 5(b).

If the pile capacities at both the serviceability and ultimate limit states are lognormally distributed, then the computation of the probability that the actual pile capacity,

Q, is less than the design capacity, Q_{des}, proceeds as follows,

$$\mathrm{P}\left[Q < Q_{des}\right] = \Phi\left(\frac{\ln Q_{des} - \mu_{\ln Q}}{\sigma_{\ln Q}}\right)$$

where Φ is the standard normal cumulative distribution function. For this computation we need only know the mean and standard deviation of $\ln Q$. Figure 6 shows the estimated mean and variance of $\ln Q$ for the serviceability limit state, i.e. those loads, Q, which produce the maximum tolerable pile settlement which in this case is 25 mm. The estimate of $\mu_{\ln Q}$ is denoted $m_{\ln Q}$ while the estimate of $\sigma_{\ln Q}$ is denoted $s_{\ln Q}$. Similarly, Figure 7 shows the estimated mean and standard deviation of $\ln Q$ at the ultimate limit state, i.e. at the point where the pile reaches failure by starting to slip through the soil.

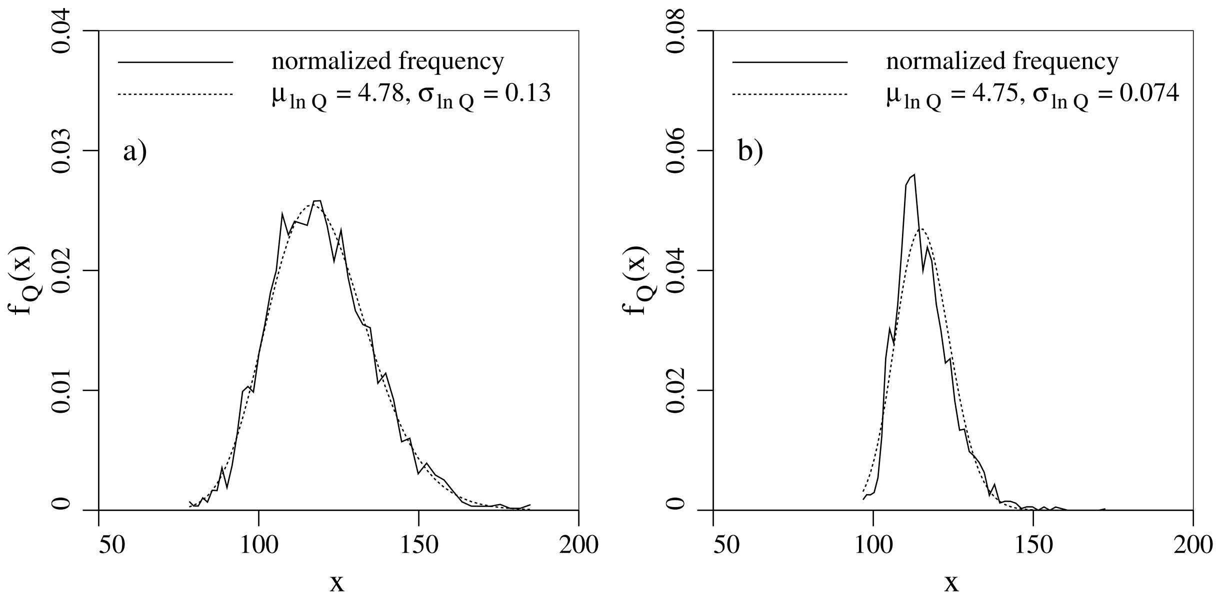

Figure 5. Estimated and fitted lognormal distributions of ultimate limit state loads, Q for a) $V = 0.2$ and $\theta = 10$ m (p-value = 0.94) and b) $V = 0.4$ and $\theta = 0.1$ m (p-value = 8×10^{-11}).

Discussion and Conclusions

Aside from the changes in the magnitudes of the means and standard deviations, the statistical behaviour of the maximum loads at serviceability and ultimate limit states are very similar. First of all the mean loads are little affected by both the coefficient of variation (V) and the correlation length (θ) – note that the vertical axes for the left plots in Figures 6 and 7 are over a fairly narrow range. The mean in Q and the mean in $\ln Q$ show similar behaviour. There are only slight reductions in the mean for increasing V. This suggests that the pile is more strongly controlled by arithmetic averaging of the soil strengths, which is perhaps not surprising if the pile is much stiffer than the surrounding soil. In fact, it could be argued that some of the reduction in mean with V is due to the fact that geometric averaging was done over the half element lengths.

In other words, it is possible that only arithmetic averaging should be done in this pile model. This needs further study.

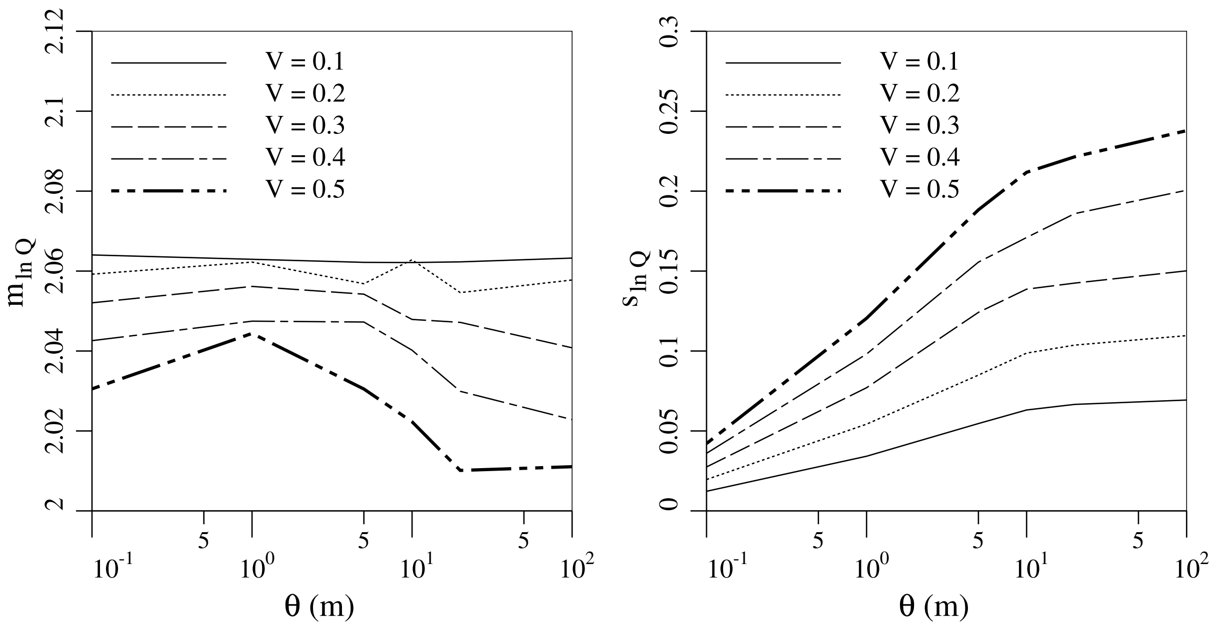

Figure 6. Estimated mean, $m_{\ln Q}$, and standard deviation, $s_{\ln Q}$ of the maximum load, Q, at the serviceability limit state.

The load standard deviation (for both Q and $\ln Q$) increases monotonically for increasing coefficient of variation, as expected (i.e. as the soil becomes increasingly variable, one expects its ability to support the pile would also become increasingly variable). This behaviour was also noted by Phoon et al. (1990). The standard deviation approaches zero as the correlation length goes to zero, which is also to be expected due to local averaging (geometric or otherwise). At the opposite extreme as $\theta \to \infty$, the standard deviation approaches that predicted if the soil is treated as a single lognormally distributed random variable (with an independent base spring variable). For example, when $\theta \to \infty$, $\sigma_{\ln Q}$ is expected to approach 0.4 for the ultimate limit state with V = 0.5. It is apparent in the right plot of Figure 7 that the uppermost curve is approaching 0.4, as predicted.

The mean shows somewhat of a maximum at correlation lengths of 1 to 10 m for $V > 0.1$. If the design load, Q_{des}, is less than the limit state load, Q, then this maximum means that the nominal factor of safety, FS, reaches a maximum for values of θ around L/2. The reason for this maximum is currently being investigated more carefully. However, since the mean only changes slightly while the standard deviation increases significantly with increasing correlation length, the probability of design failure, i.e. the probability that the actual pile capacity Q is less than the design capacity Q_{des}, will show a general increase with correlation length (assuming that $\ln Q_{des} < \mu_{\ln Q}$) to a limiting value when $\theta \to \infty$. In other words, from a reliability-based design point of view, the worst case correlation length is when $\theta \to \infty$ and the soil acts as a single random variable.

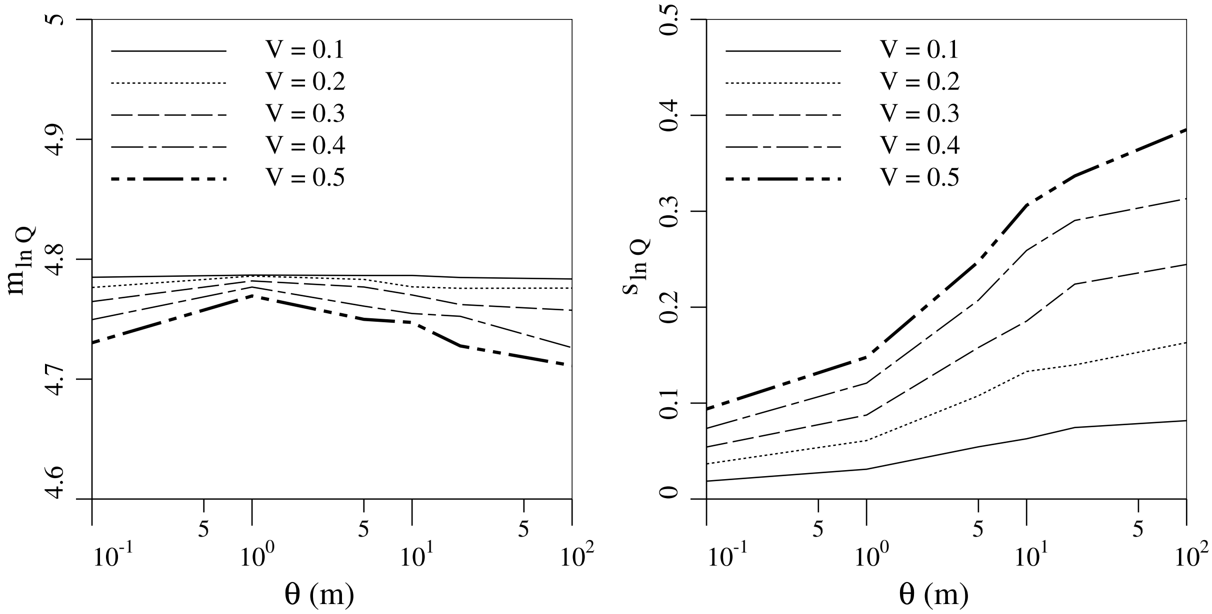

Figure 7. Estimated mean, $m_{\ln Q}$, and standard deviation, $s_{\ln Q}$ of the maximum load, Q, at the ultimate limit state.

This observation makes sense since variance reduction only occurs if independent random variables are averaged. That is, if the soil acts as a single random variable, then the variance remains unreduced and the failure probability is maximized. The implication of this "worst case" is that reliability-based pile design can conservatively ignor spatial variation in soil properties so long as end-bearing is not a major component of the pile capacity (bearing capacity is significantly affected by spatial variability – see, e.g., Fenton and Griffiths, 2003). For piles that depend mostly on skin friction, then, the reliability-based design at both serviceability and ultimate limit states can proceed using single random variables to represent the soil's elastic behaviour (serviceability) and shear strength (ultimate).

References

Das, B.M. (2000). *Fundamentals of Geotechnical Engineering*, Brooks/Cole, Pacific Grove, California.

Fenton, G.A. and Griffiths, D.V. (2002). "Probabilistic foundation settlement on spatially random soil," *ASCE J. Geotech. Geoenv. Eng.*, **128**(5), 381–390.

Fenton, G.A. and Griffiths, D.V. (2003). "Bearing capacity prediction of spatially random $c - \phi$ soils," *Can. Geotech. J.*, **40**(1), 54–65.

Fenton, G.A. and Griffiths, D.V. (2005). "Three-dimension probabilistic foundation settlement," *ASCE J. Geotech. Geoenv. Eng.*, **131**(2), 232–239.

Phoon, K.K., Quek, S.T., Chow, Y.K., and Lee, S.L. (1990). "Reliability analysis of pile settlement," *ASCE J. Geotech. Geoenv. Eng.*, **116**(11), 1717–1735.

Vesic, A.S. (1977). *Design of Pile Foundations*, in *National Cooperative Highway Research Program Synthesis of Practice No. 42*, Transportation Research Board, Washington, D.C..

Vijayvergiya, V.N. and Focht, J.A. (1972). "A new way to predict capacity of piles in clay," *Fourth Offshore Technology Conference*, Houston, Paper 1718.