Influence of highly anisotropic properties on probabilistic slope stability

Influence des propriétés hautement anisotropes sur la stabilité des pentes probabiliste

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ABSTRACT This paper investigates the probability of failure of slopes using the random finite element method (RFEM), which uses elato-plasticity combined with random field theory in a Monte-Carlo framework. The paper demonstrates the role of isotropic spatial variability and goes on to investigate the influence of highly anisotropic random soil properties on probabilistic slope stability predictions. In the limit, infinite spatial correlation lengths are considered in both the vertical and horizontal directions. Parametric studies are presented on some different benchmark slope problems. The influence of infinite spatial correlation length in the horizontal direction is shown to have a significant impact on probabilistic outcomes.

RÉSUMÉ Ce document étudie la probabilité de défaillance des pistes à l'aide de la méthode des éléments finis aléatoire (de RFEM), qui utilise Elato-plasticité combinée avec la théorie des champs aléatoires dans un cadre de Monte-Carlo. L'article démontre le rôle de la variabilité spatiale isotrope et continue d'étudier l'influence de très anisotropes propriétés du sol au hasard sur des prévisions probabilistes de la stabilité des pentes. A la limite, infinite longueurs de corrélation spatiale sont pris en compte dans les deux directions verticale et horizontale. Des études paramétriques sont présentés sur des problèmes différents de la pente de référence. L'influence de l'infini spatial longueur de corrélation dans la direction horizontale est montré pour avoir un impact significatif sur les résultats probabilistes.

1 INTRODUCTION

Probabilistic methods have been used in slope stability analysis since 1970s, and have received considerable attention in the literature. Starting in the early 90's, a new method called the Random Finite Element Method (RFEM), which combines random field theory and the finite element method, was developed for use in probabilistic geotechnical engineering (e.g. Griffiths and Fenton 1993). The method was subsequently applied to several areas of geotechnical engineering including probabilistic slope stability analysis by Griffiths and Fenton 2000, 2004. The Local Average Subdivision method (LAS) proposed by Fenton and Vanmarcke (1990) was used for generating the random fields. It was shown that traditional probabilistic analyses, in which spatial variability is ignored by implicitly assuming perfect correlation, does not necessarily result in a conservative estimates of the probability of failure. Later on, Griffiths et al. (2009) studied the influence of spatial variability of soils more precisely.

One of the advantages of using elastic-plastic finite elements for stability analysis is that the failure mechanism is allowed to find the weakest path through the soil. The ability of the FE approach to model the shape and location of the failure mechanism offers many benefits over traditional methods in which the shape of the failure mechanism is fixed a priori. Slope stability analysis is a good example this, in which commonly used methods such as Bishop's method, require the failure mechanism to be circular.

This paper investigates the influence of highly anisotropic soil properties on probabilistic slope stability analysis. Numerical results show that very (infinite) large spatial correlation length in the horizontal direction, can increase the probability of failure of slopes.

The slope studied in this paper is shown in Figure 1 with consideration of an undrained $\phi_u = 0$, c_u slope. The slope inclination, height, and foundation ratio is given by β, H and D respectively. The saturated unit weight of the soil, γ are held constant, while the shear strength c_u of the slope is assumed to be the random variable. c_u was expressed in dimensionless forms as C_u where $C_u = c_u/(\gamma_{sat} H)$.

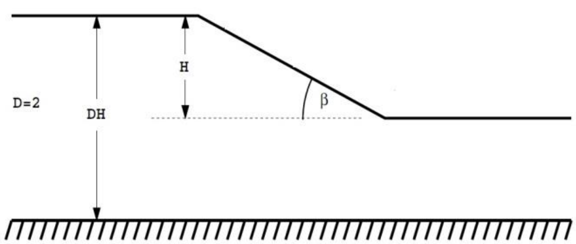

Figure 1. Slope profile.

The shear strength parameter of the soil C_u is treated as random variable, characterized statistically by lognormal distributions; in other words, the logarithms of the properties are normally distributed. The lognormal distribution is one of many possible choices (e.g. Fenton and Griffiths, 2008) that has been advocated and used by several other investigators as a reasonable model for variable soil properties (e.g. Massih et al., 2008). Lognormal distributions guarantee that the random variable will never have negative values.

The lognormal distribution is defined by a mean μ and a standard deviation σ. The probability density function of C_u is given by Equation 1.

$$f(C_u) = \frac{1}{C_u\,\sigma_{\ln C_u}\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{\ln C_u - \mu_{\ln C_u}}{\sigma_{\ln C_u}}\right)^2\right\} \quad (1)$$

The mean and standard deviation can conveniently be combined in terms of the dimensionless coefficient of variation defined as:

$$V_C = \frac{\sigma_{C_u}}{\mu_{C_u}} \quad (2)$$

2 RANDOM FINITE ELEMENT METHOD (RFEM)

The RFEM implementation used in this study combines elastic-plastic finite element analysis with random field theory in slope stability analysis. The methodology has been described in details elsewhere (e.g. Fenton and Griffiths, 2008).

The RFEM is used in conjunction with Monte Carlo simulations in which the stability analysis is repeated until the probabilities relating to output quantities of interest become statistically reproducible. In the case of slope stability analysis, the probability of failure is defined by dividing the number of realizations in which the slope failed by the total number of realizations.

2.1 Spatial correlation

Generally, the mean and standard deviation of a variable are well understood by engineers. However, the spatial correlation length θ of a random property is less well understood. This property, called the "scale of fluctuation" or "spatial correlation length", has units of length, and represents the distance over which the soil or rock property in question is reasonably well-correlated to its neighbors. In this paper, a "Markovian" correlation function is used where the spatial correlation is assumed to decay exponentially with distance (Vanmarcke 1984).

$$\rho = e^{-2|\tau|/\theta} \quad (3)$$

In Eq (3) which is for an isotropic material, τ is the absolute distance between any two points in the random field, and ρ is the correlation coefficient between properties assigned to two points in the random field separated by τ.

Since the actual undrained shear strength field is lognormally distributed, its logarithm yields an underlying normal distributed (or Gaussian) field. The spatial correlation length is measured with respect to this underlying field, that is, with respect to $\ln C_u$. In particular, the spatial correlation length ($\theta_{\ln Cu}$) describes the distance over which the spatially random values tend to be significantly correlated in the underlying Gaussian field. Thus, a large value of $\theta_{\ln Cu}$ implies a smoothly varying field, while a small value will imply a ragged field. In this study, the spatial correlation length has been non-dimensionalized by dividing it by the height of the slope H and will be expressed in the form:

$$\Theta_{C_u} = \frac{\theta_{\ln C_u}}{H} \tag{4}$$

The influence of θ on a wide range of geotechnical systems has been assessed through parametric studies (e.g. Griffiths and Fenton 2004, Griffiths et al. 2009, Huang et al. 2010, Kasama and Whittle 2011, Al-Bitar and Soubra 2013) and has been shown to have a significant influence on probabilistic quantities under considerations.

3 RFEM RESULTS

The results of the RFEM analysis for the undrained slope are presented in this section. Using this method, random fields are generated and assigned to each element. Gravity loads are then applied, and if the algorithm could not converge within a specific number of iterations, failure is said to have occurred. Lack of convergence means that no stress redistribution could be found that is simultaneously able to satisfy both the Mohr Coulomb failure criterion and global equilibrium. The analysis is repeated numerous times, using Monte Carlo simulations, using the same mean, standard deviation, and spatial correlation length of soil properties. The spatial distribution of properties, however, varies from one realization to the next. Following a "sufficient" number of realizations, the probability of failure p_f is estimated by dividing the number of failures by the total number of simulations.

A typical finite element mesh used for this problem is shown in Figure 2. The majority of the elements are 8-node square except the elements adjacent to the slope which are degenerated into triangles. The slope model has 910 total elements which results in 910 random variables for the slope in each simulation. 4000 simulations were used to ensure the reproducibility of the model.

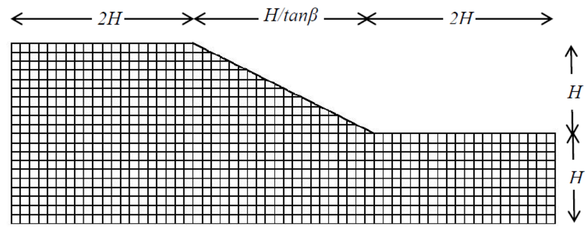

Figure 2. Typical mesh used for the RFEM slope stability analysis.

In this study, H and β are equal to 1 and 26.6° respectively and the μ_{Cu} of the soil has been chosen to be 0.25. The slope has been modeled with 4 different coefficients of variation equal to 0.25, 05, 1, and 2. Θ_{Cu} has varied from 0.01 to 10 for each slope. Figure 3 shows the Probability of Failure p_f versus non-dimensionless spatial correlation length Θ_{Cu} for these slopes. The spatial correlation lengths in the both X and Y directions are equal. Figure 3 clearly indicates two branches, with the Probability of Failure tending to unity or zero for higher and lower values of V_C, respectively.

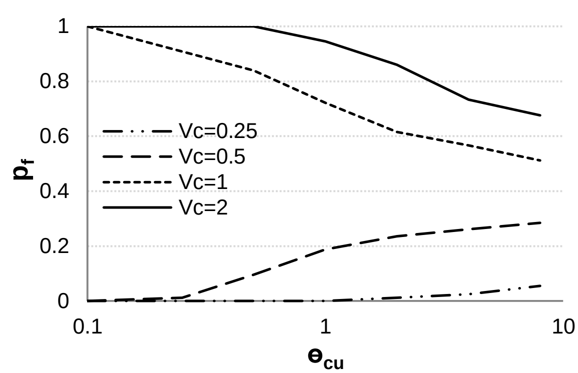

Figure 3. Probability of failure vs. spatial correlation length.

Six slopes identical as the previous one with Vc = 0.5 and μ_{Cu} = 0.25 has been analyzed while Θ_{Cu} was chosen to be different in horizontal and vertical directions with consideration of infinite spatial correlation length. Table 1 indicates the input values of Θ_{Cu} and corresponding p_f for each simulation. Θ_x and Θ_y represent the spatial correlation lengths of C_u in horizontal and vertical directions respectively.

Table 1. Input values of Θ_{Cu} and p_f for the slope models.

Slope No.	Θ_x	Θ_y	p_f
1	1	1	0.185
2	2	2	0.221
3	Infinite	1	0.278
4	Infinite	2	0.296
5	1	Infinite	0.108
6	2	Infinite	0.194

The result of RFEM presented in Table 1 shows that the slopes with infinite Θ_x have the maximum probability of failure while slopes modeled with an infinite Θ_y led to lower values for p_f.

Figures 4 and 5 illustrate sample random field realizations and failure of the slope with the infinite Θ_x and infinite Θ_y respectively. The figures depict the variation in lnC_u and have been scaled in such a way that the dark and light regions depict “strong” and “weak” soils, respectively. Black represents the strongest element and white is the weakest in a particular realization.

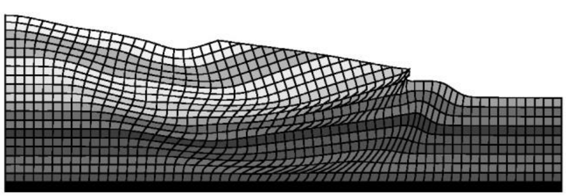

Figure 4. Failure mechanism for the slope with Θ_y =2 and Θ_x= infinite.

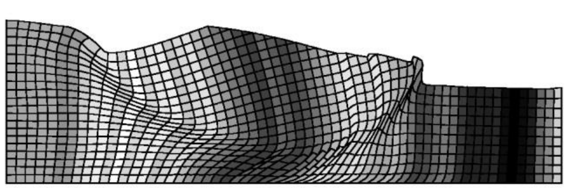

Figure 5. Failure mechanism for the slope with Θ_x =2 and Θ_y = infinite.

As it can be seen in Figures 4 and 5, slopes have different failure mechanism for two cases. The slope with an infinite spatial correlation length in horizontal direction tends to have a more linear failure, while the other slope has a circular failure profile. A reason for this could be that in the slopes with infinite Θ_x, failure can find a weak layer through the soil in the horizontal direction and create a more linear failure path. This also could be the reason for having a higher probability of failure comparing to the slopes with infinite Θ_y.

Additional RFEM models of the slopes with infinite Θ_x were developed to study the effect of spatial correlation length and coefficient of variation more precisely. The reason for choosing the infinite values for Θ_x is that it resulted in higher probability of failure for the slope. In other word, slopes with infinite Θ_x seem to be more critical than the slopes with infinite Θ_y.

Figure 6 shows the result of these analyses and compares them with previous results provided in Figure 3 when $\Theta_{Cu} = \Theta_x = \Theta_x$.

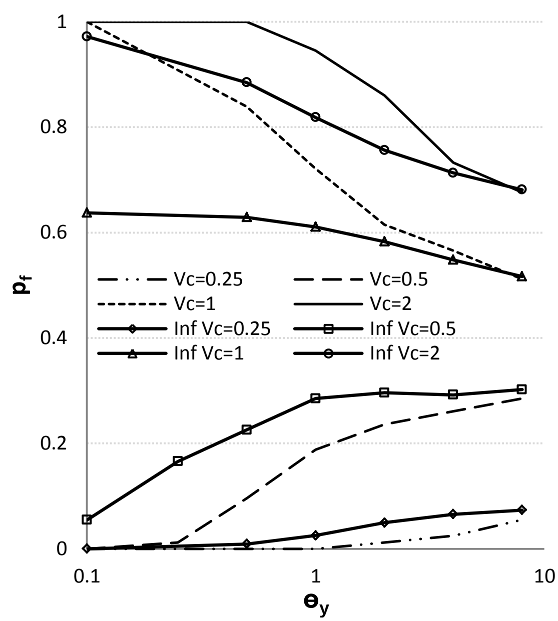

Figure 6. Probability of failure vs. Θ_y while Θ_x is constant and equal to infinity for “Inf Vc” and $\Theta_y = \Theta_y$ for “Vc” cases.

Based on Figure 6 it can be resulted that for smaller values of V_c, considering the infinite values for Θ_x results in higher probability of failure comparing to the cases with identical spatial correlation lengths in both vertical and horizontal directions. On the other hand, for $V_c \geq 1$ the slopes with infinite Θ_x result in lower probability of failure. However, based on Duncan (2000) the typical V_c values for undrained shear strength lie in the range of 0.13-0.5.

4 CONCLUSIONS

This paper has investigated the probability of failure for undrained slopes using the RFEM. The RFEM combines the FEM with Monte Carlo simulation in which spatial variability is properly taken into account. The RFEM enables the failure mechanism to seek out a weakest path through heterogeneous soil which can lead to higher probabilities of failure than might be predicted spatial variability is ignored. The influence of the coefficient of variation V_c and spatial correlation length Θ_{Cu}, on the probability of failure p_f was studied. Infinite spatial correlation length was also modeled in the both horizontal and vertical directions, and it was shown that slopes with infinite spatial correlation lengths lead to higher p_f values for the $V_c \leq 1$ in which the coefficient of variation for undrained shear strength of most of the soils lies in this range. Soils usually have bigger Θ in horizontal direction and a good example of the slopes with infinite horizontal spatial correlation length could be the slopes with layered soils. The finding of this study can be used in such slopes.

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