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Computation of Bearing Capacity on Layered Soils

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SYNOPSIS The ultimate bearing capacity of footings resting on soils whose strengths vary with depth has been examined using finite elements. Assuming elasto-plastic behaviour, two classes of problem are considered. Firstly, a problem in which two soils of different strength occur and secondly, a cohesive soil whose strength varies linearly with depth. Where possible, comparisons with analytical and experimental data have been made.

INTRODUCTION

Most bearing capacity prediction methods assume that the soil is homogeneous and can be assigned constant shear strength parameters c' and ϕ'. These methods are usually based on the bearing capacity equation (Terzaghi, 1943) which assumes that the three basic components of bearing resistance, namely cohesion, self-weight and surcharge may be superposed.

It is frequently the case however that the soil under a foundation has variable shear strength properties. If these properties vary in a haphazard way, the designer has little choice but to take some kind of average shear strength value for use in the bearing capacity equation, together with a generous safety factor.

If however, the shear strengths vary in a reproducible manner then analytical solutions which use limit theorems are available. Finite element methods in conjunction with plasticity theory can also be of value in the study of such problems due to the ease with which different properties may be assigned to elements in the mesh.

Two main types of non-homogeneous soil deposit are considered in this paper using finite elements. Two-layer soils are considered in which the shear strength properties of each layer differ significantly from each other, although the properties of each soil are assumed to remain constant. Upper bound solutions assuming simple circular failure surfaces (Button, 1953) and experimental results (Brown and Meyerhof, 1969) are available for the case of undrained clays. More recently, there has been interest shown in two layer problems involving frictional soils (Meyerhof, 1974; Hanna and Meyerhof, 1980). The finite element solutions obtained by the author have also been compared, where possible, with these published solutions.

A brief study using finite elements was also made of the two layer problem under inclined loading. This is a problem of particular interest in offshore construction where large wave forces can occur. Under inclined loading, the influence of soil stratification and anisotropy is thought to become more important than under vertical loads.

Soils in which the cohesion varies linearly as a function of depth have also been considered. This type of soil is particularly relevant when a normally consolidated clay is loaded quickly under undrained conditions.

Available solutions for this problem include simple upper bounds assuming circular or Prandtl-like mechanisms (Raymond, 1967; Salencon, 1974) to "exact" solutions where coincident upper and lower bounds have been found (Davis and Booker, 1973).

MATERIAL PROPERTIES

All the solutions presented here were for plane strain conditions with the soil(s) behaving as elastic-perfectly plastic materials. The Mohr-Coulomb failure criterion was used in conjunction with a non-associated (no plastic volume change) flow rule. As the aim of the calculations was to predict the bearing pressure necessary to cause a general shear failure rather than the settlements prior to failure, the elastic properties operating within the Mohr-Coulomb surface were considered relatively unimportant.

In most cases, the following constant properties were adopted:

$$E = 2 \times 10^5 \text{ kN/m}^2 \qquad \nu = 0.35 \qquad (1)$$

Although some attempts were made to relate E to the undrained strength C_u, little resulting influence on collapse stresses was observed.

METHOD OF ANALYSIS

Plasticity was introduced using the viscoplastic technique which is described elsewhere (Zienkiewicz and Cormeau, 1972,1974) and has been shown (Humpheson, 1976; Griffiths, 1980, 1981) to be an efficient and versatile way of solving plasticity problems in geomechanics. The method, which falls into the "initial strain" family of solution techniques iterates using equivalent elastic solutions until any stresses that originally violated yield, return to the failure surface within quite strict tolerances. Equilibrium and continuity are also satisfied in the normal way using a displacement finite element formulation.

Eight-node quadrilateral, isoparametric elements were used throughout, with "reduced" (2-point) Gaussian quadrature in both the stiffness and the relaxation phases of the calculation.

For most of the analyses presented here, loads were applied to the soil in the form of prescribed vertical displacements at the nodes representing the base of the footing. For smooth footings ($\delta = 0$), the displaced nodes were free to move laterally but the rough case ($\delta \geqslant \phi$) was simulated by allowing no lateral movement of the displaced nodes.

In all cases, the bearing pressure mobilised by a given vertical displacement was obtained by averaging the vertical stress component occurring in the first row of integrating points below the displaced nodes.

For the analyses of inclined loading on layered soils, load control was used by applying vertical and horizontal forces in a fixed ratio at the footing nodes.

HOMOGENEOUS SOILS

Before embarking on finite element predictions of the bearing capacity of non-homogeneous soils, it must first be established that the method can confidently predict the bearing capacity of homogeneous soils.

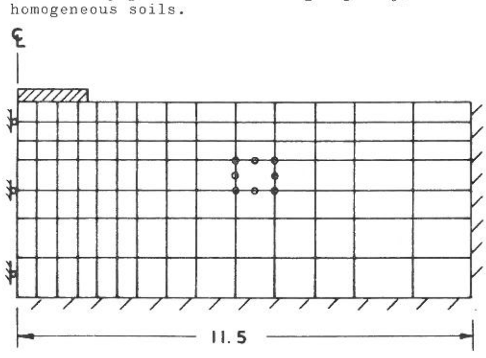

Fig.1 Mesh used for Bearing Capacity Computations

Such studies have been performed by the author for a wide range of soil types. Using the mesh of Fig.1 and considering both rough and smooth footings, the results of Fig.2 were obtained.

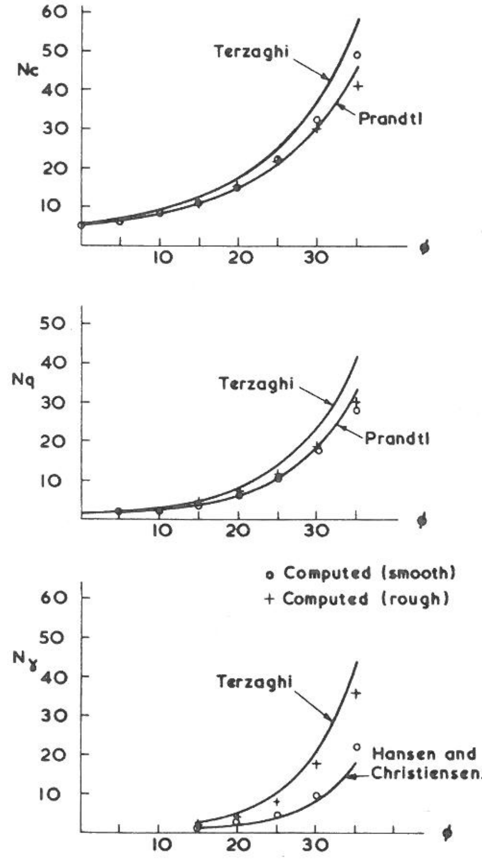

Fig.2 Computed and Theoretical Bearing Capacity on Homogeneous Soils

The analyses were split into three groups corresponding to each of the bearing capacity factors. Excellent agreement was obtained with "exact" solutions for N_c and N_q and reasonable agreement with well known approximate solutions for N_γ .

Although all three classes of problem showed a marked increase in computer time as the friction angle was increased, this effect was considerably worse in the prediction of N_γ . It was found that 35° represented the largest friction angle for which reasonable N_γ predictions could be obtained using the raw viscoplastic algorithm.

The slow convergence observed in cohesionless soils with high friction angles was thought to be a result of the complex state of stress beneath the footing and the rapidly varying strength with depth. No such problems were observed with high friction angles in simpler boundary value problems such as earth pressure analyses (Griffiths, 1980).

A further consideration when the soil was cohesionless was that no solution could ever be obtained if a uniform stress was applied. This always resulted in failure at the footing edge due to the very low confining pressures existing there. Displacement control, simulating a rigid footing, was particularly helpful in this case giving a vertical stress distribution at failure beneath the footing which was a maximum under the centreline tending to zero at the edge. Load control may be used to apply stresses to a cohesionless soil provided some stress distribution (eg. linear) is assumed that varies from a maximum at the centreline to zero at the footing edge.

NON-HOMOGENEOUS SOILS

The previous section, and the results of Fig.2, showed that the viscoplastic algorithm was confidently able to predict the bearing capacity of homogeneous soils. A great advantage of Finite Elements is the ease with which non-homogeneity can be introduced. Using the same computer program, several types of non-homogeneity were attempted.

Two Layer Problems

(a) Undrained clays

The first problem to be tackled was one involving a rough footing sitting on a thin layer of clay of strength C_t overlying a deep layer of another clay of different strength C_b (Fig.3).

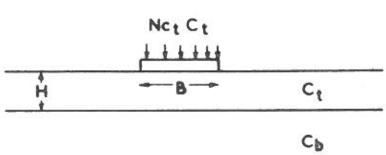

Fig.3 Two-Layer Undrained Clays

Both the clays were assumed to be undrained. If the ratio $C_b/C_t \gg 1$ then this type of foundation may be applicable when soft post-glacial clays overly older, stiffer glacial till deposits. If $C_b/C_t \ll 1$ then the problem is one of a footing resting on a stiff surface crust which may have been caused by dessiccation or weathering.

This two-layer problem has received the attention of several workers over the years. Button (1953), assuming circular mechanisms of failure, produced upper bound solutions to the problem, and Brown and Meyerhof (1969) performed experimental studies of the two-layer problem. Their results do not differ greatly and are presented in the form of dimensionless charts giving N_{c_t} where

$$q_{ULT} = C_t N_{c_t} \quad (2)$$

as a function of H/B and C_b/C_t .

The interested reader is referred to these papers, but the particular case of H/B = 0.5 was chosen for comparison with the author's finite element results for a range of C_b/C_t ratios in Fig.4. Generally good agreement was found for other H/B ratios.

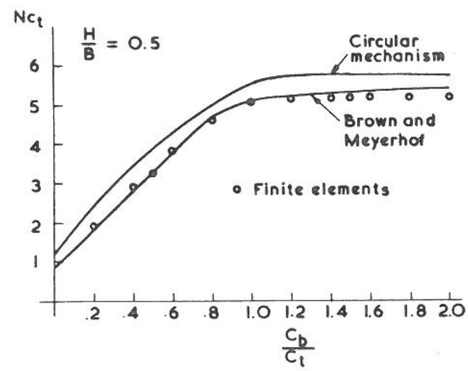

Fig.4 Finite Element Solution for Two Cohesive Layers

One of the chief questions when studying the bearing capacity of a two-layer system is, "How thick must the upper layer be in order that the lower layer makes no difference?". Results indicate that much more interaction between the layers occurs if the upper layer is the stronger of the two. Brown and Meyerhof recorded a reduction in bearing capacity up to a depth ratio of $H/B \simeq 2.5$ provided the lower layer had a shear strength of no more than 20% of that of the upper. The interaction between layers was considerably lessened if the lower layer was the stronger, the strength of the lower layer being immaterial for $H/B > 0.7$. The lower layer in this case acted like a firm base with no direct contribution to shear resistance. This seemed consistent with the Prandtl mechanism for rough footings on $\phi = 0$ materials which has a depth ratio $H/B = 0.707$.

An advantage of using finite element methods for obtaining bearing capacity is that in addition to giving an adequate stress field at failure, they also give a good indication of the collapse mechanism without any assumptions needing to be made as to the form of the mechanism in advance. To emphasise this, displacement vectors at failure were plotted for a smooth footing with two extreme values of C_b/C_t. In Fig.5a the upper layer was considerably weaker than the lower, and the mechanism confined itself almost exclusively to the upper soil. The displacements beneath the footing moved sideways to avoid the stronger material below. Footing roughness could have a considerable effect in this case by causing the mechanism to go deeper.

The strengths of the two soils were then reversed, with the stronger soil above, and the mechanism at failure was totally different (Fig.5b). The majority of movement in this case occurred in the weaker layer below. The displacements beneath the footing this time moved vertically in order to take the shortest route possible through the stronger material. Footing roughness would be expected to have little influence in this case.

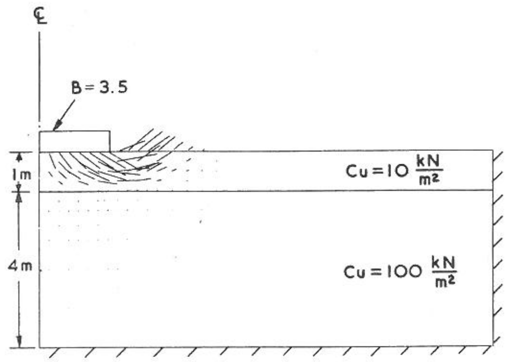

Fig.5a Displacement Vectors for Weak on Strong

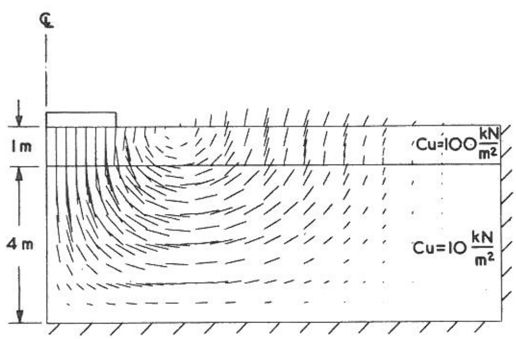

Fig.5b Displacement Vectors for Strong on Weak

(b) Sand overlying clay

The next type of problem considered was that of a thin sand layer overlying a deep bed of clay. This combination of material is encountered in certain parts of the North Sea.

The problem has received some attention in the literature and Meyerhof (1974) considered the cases of a loose sand on stiff clay and a dense sand on soft clay. A semi-empirical theory was derived which was compared favourably with some model test results. As with the layered clay problem, the case of a dense sand overlying a weak clay proved more interesting in that a "punching shear coefficient" was required which depended on the relative bearing capacities of the two soils. The assumption was that the footing pushed a sand mass of an approximately truncated pyramid shape into the soft clay below. The case of loose sand over stiff clay was approached by using the familiar bearing capacity equation, but with modified factors. As was observed with the layered clays, if the bearing capacity of the lower soil was considerably greater than the upper, the simplifying assumption was made that the lower soil acted as a rigid base.

There is no question that if the upper layer is very thin, the bearing capacity is governed by the lower soil, and as the upper layer thickness increases relative to the foundation width, the bearing capacity tends to that of the upper soil. Myslivec and Kysela (1978) proposed a simple linear variation between the two bearing capacity values, but allowed for the "punching" effect when a thin layer of stronger material overlay weak soil.

The earlier work of Meyerhof (1974) was formalised for the case of dense sand overlying weak clay by Hanna and Meyerhof (1980) in the form of design charts which have been used as a basis for comparison with finite element results obtained by the author.

It was a simple matter to give the upper rows of elements in the mesh (Fig.1) a friction angle and zero cohesion, but as was found in the N_γ analysis of homogeneous soils, this had the effect of slowing convergence in the numerical process. Also, in spite of the very different deformation characteristics of cohesionless soil and undrained clay, a constant Young's modulus was applied to both soils. This was justified on the grounds that, as ultimate conditions only were of interest in the present work and not the displacements prior to failure, the deformation properties assigned to the soil were secondary to their strength parameters.

Results obtained for the bearing capacity of a relatively thin layer of sand of $\phi' = 40^\circ$ overlying undrained clay of variable cohesion are shown in Fig.6. The finite element results were in good agreement with those of Hanna and Meyerhof (1980), but more noteworthy perhaps was how little effect the thin sand layer had on the ultimate bearing value. The line corresponding to a sand thickness of zero gives the theoretical bearing capacity of an undrained clay (gradient 5.14). A 1m thick layer of sand, which on its own would have a bearing capacity many times

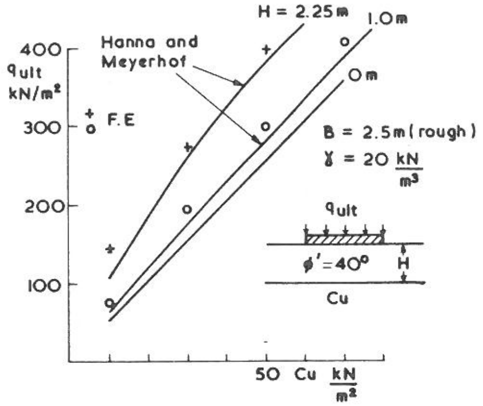

Fig.6 q_{ULT} for Dense Sand over Weak Clay

greater than the clay, only increased the bearing capacity of the two-layer system by a small percentage. The relative influence of the sand got even less as the clay strength increased.

For thicker sand layers, the bearing capacity was increased but this required more computing effort.

The small influence of a thin sand layer was not unexpected, because in spite of a high friction angle, the normal confining pressures near ground level are relatively low.

(c) Inclined loading on layered soil

A brief study was made of the effects of inclined loading on a surface footing. Firstly, a new mesh was designed (Fig.7) because the inclined loading problem has no line of symmetry and a full footing width must be considered.

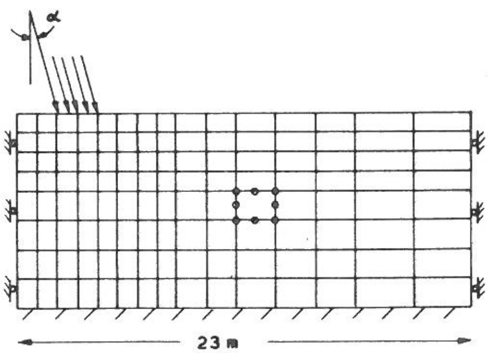

Fig.7 Mesh used for Inclined Loading

Previous finite element work on $c'-\phi'$ soils under inclined loading (Zienkiewicz et al, 1978) indicated that available theories such as Hansen (1970) were conservative by up to 100%. Work by the author (Fig.8) with an homogeneous, undrained clay agreed fairly well with theoretical results

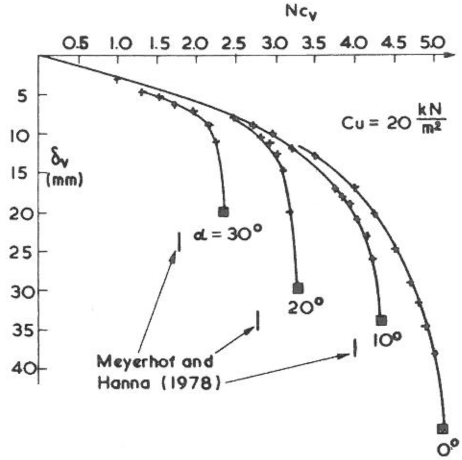

Fig.8 Inclined Loads on Homogeneous Soil

quoted by Meyerhof and Hanna (1978), but results computed for layered clays indicated considerably greater bearing resistance than suggested by the theory of those authors (Fig.9). Further investigation is required to resolve this discrepancy.

In both Figs.8 and 9, N_{c_v} refers to the vertical component of bearing resistance divided by the soil cohesion immediately below the footing.

Strength Varying with Depth

It is frequently the case that the undrained shear strength C_u of normally consolidated clays increases with depth, the ratio C_u/p, where p is the effective overburden stress being essentially constant for a particular deposit. Skempton (1957) proposed the following relationship:

$$\frac{C_u}{p} \simeq 0.11 + 0.0037\ (PI) \qquad (3)$$

which, for a clay of PI = 20%, leads to a rate of increase of cohesion with depth (or Strength Density) of about 2 $kN/m^2/m$.

Upper bound solutions to this problem exist using circular failure surfaces (Raymond, 1967) and Prandtl-like mechanisms (Salencon, 1974). Davis and Booker (1973) quoted "exact" solution by using plasticity theory to find coincident upper and lower bounds to the problem.

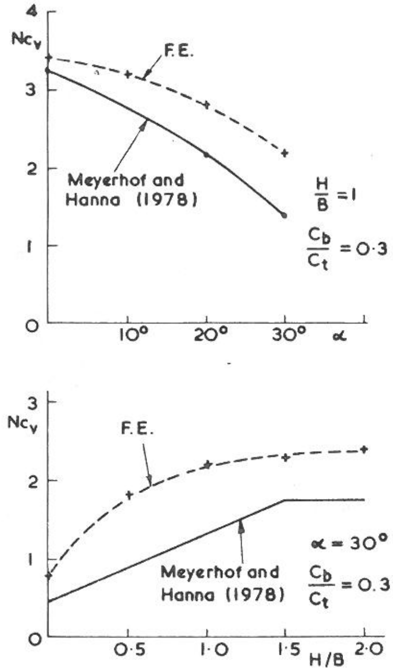

Fig.9 Inclined Loads on Two-Layer Clay

Fig.10 shows the basic problem considered using finite elements. The variables of the problem were C_0, the surface cohesion and λ, the "strength density". As the program used 2-point Gaussian integrations this enabled a cohesion value to be assigned to every row of integrating points leading to two different cohesion values per element.

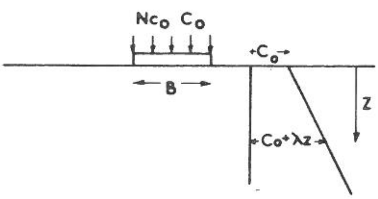

Fig.10 Soil with Increasing Strength with Depth

Results obtained using finite elements have been compared with classical solutions in Fig.11. The ultimate bearing capacity was expressed in terms

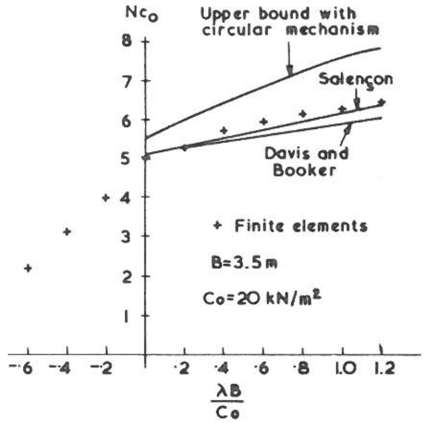

Fig.11 Finite Element Solution for Increasing Strength with Depth

of the surface cohesion according to equation (4):

$$q_{ULT} = N_{c_o} C_o \tag{4}$$

and the "strength density" concealed in the dimensionless grouping $\lambda B/C_0$. From equation (3), for a footing of width $B = 3.5$m and $C_0 = 20$ kN/m^2, a typical value of the dimensionless grouping would be 0.35.

The finite element results were generally good, although a little on the high side. Fig.11 also shows that results can also be obtained using finite elements should the "strength density" become negative.

Davis and Booker (1973) considered the total bearing capacity by superposing the contributions due to a constant surface cohesion and the "strength density" component. This latter component, whose cohesion increases linearly from zero at the ground surface, is analogous to the self-weight component of bearing capacity in frictional materials expressed through the factor N_γ .

The analogy was emphasised in the finite element solutions in that numerical convergence became more difficult as the component due to "strength density" increased relative to the surface cohesion. The worst case numerically was when the surface cohesion was put to zero giving extremely slow convergence.

Another effect of the "strength density" component was its influence on the stress distribution at failure. Two such distributions are plotted in Fig.12 for different values of λ. It may be noted that as λ increases so the stress distribution becomes more triangular, being a maximum at the footing centreline and reducing to 5.14 C_0 at the footing edge.

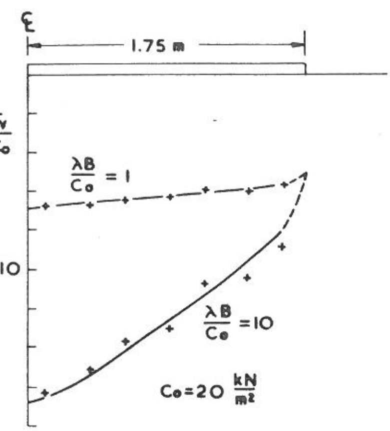

Fig.12 Failure Stress Distribution for Different Strength Densities

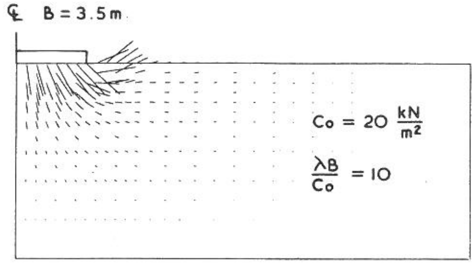

Fig.13a Displacement Vectors for High Strength Density

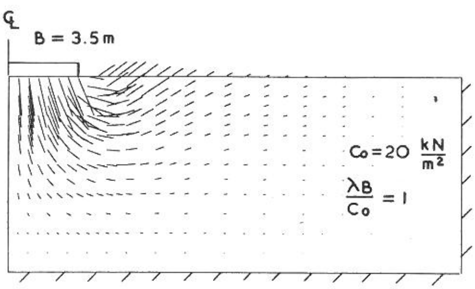

Fig.13b Displacement Vectors for Low Strength Density

This stress distribution was predicted by Davis and Booker (1973) and is similar to the observed behaviour of footings on $c' - \phi'$ soils as the frictional contribution is increased.

Finally, the influence of "strength density" on the kinematics of the problem is also indicated by the finite element analysis. The effect on the failure mechanisms as the "strength density" was increased is shown in Figs.13a and 13b. The larger "strength density" encouraged a more shallow mechanism as the soil tried to avoid shearing through the rapidly strengthening material below.

CONCLUSIONS

A finite element program in conjunction with plasticity theory has been shown to give accurate bearing capacity predictions for a wide range of homogeneous soils. Cohesionless soils, with high friction angles, remain the most difficult area numerically.

It is relatively easy using finite elements to assign different properties to different regions of the mesh, so the same program was used to examine bearing problems in non-homogeneous soils.

Two-layer problems were examined, and generally good agreement achieved with existing upper bound and exact solutions. As well as collapse stresses, the finite elements emphasised the dramatic difference between the displacement fields at collapse for the cases of strong clay overlying weak clay and vice versa. Due to the nature of those mechanisms it was found that footing roughness had considerably more influence in the latter case. Numerical convergence was also considerably slower when the upper soil was weakest.

The case of a thin dense sand layer on a weak clay was also considered and found to agree well with available theories. Provided the sand layer was thin relative to the footing width, reasonable results were achieved and the solution dominated by the clay. For thicker layers, however, convergence was slowed considerably as usually observed in the analysis of homogeneous cohesionless soils with high friction angles.

A brief study was made of the effects of inclined loading on two-layer soils. The problem is an awkward one in that no symmetry exists, and tensile effects may need accounting for. Although reasonable results were obtained on homogeneous clays, the finite elements predicted considerably higher strengths for layered clay than obtained by Meyerhof and Hanna (1978). This discrepancy is not fully understood and requires further consideration.

Finally, the problem of an undrained clay with linearly increasing strength with depth was considered. Numerically, this was found to be similar to a cohesionless soil in that convergence was slowed down by any increase in the "strength density". Provided the surface cohesion was finite, good agreement was obtained with "exact" and upper bound solutions. For a surface cohesion of zero however, convergence was extremely slow and the problem seemed numerically similar to a cohesionless material.

The finite element analysis again gave a good indication of the mechanisms of failure with various strength densities. This of course was within the limitations imposed by any analysis in which the material is assumed to remain a continuum.

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