Finite Elements in Geotechnical Engineering

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The finite element (FE) method *per se* has nothing much to do with geotechnical analysis. It is a numerical method for obtaining approximate solutions to differential equations with appropriate boundary conditions. Where differential equations arise in geotechnical analysis, the FE method may be used to solve them. Admittedly, when we flick through the pages of a typical soil mechanics textbook, we do not usually see many differential equations, but on closer inspection, we realize that nearly all important areas of geotechnical analysis are governed by underlying differential equations (e.g., Terzaghi's 1-D consolidation equation, Laplace's equation for steady seepage, and elastic equilibrium equations for settlement analysis). The FE method is ideally suited to solving all problems of this type.

Limit analysis problems in geotechnical analysis involving failure of soil masses are traditionally tackled using classical techniques such as Rankine's earth pressure theory, Terzaghi's bearing capacity equation, or methods of slices. These approaches are “rigid-plastic” with no concept of deformations prior to failure. If we allow elastic deformations prior to failure, however, we can use the FE method to solve the elastic differential equations, and deal with failure by placing a limit on the stresses using Mohr-Coulomb’s failure criterion.

Nearly all areas of geotechnical analysis are amenable to solution by the FE method. The most attractive features of the method are 1) soil property variability due to stratification and/or stress-level dependence is readily modeled, and 2) FE meshes can easily be constructed to model awkward shapes (e.g., embankments, tunnels, cofferdams). Finite element analysis can be performed in 1-, 2- or 3-dimensions as necessary.

Soil and rock are the most variable of all engineering materials, and geotechnical engineers, (unlike their structural colleagues), must often make do with the local, in-situ conditions. This makes the FE method an even more valuable tool for geotechnical analysis. Finite element approaches enable us to solve problems relating to deformations, failure, seepage, transient effects and dynamics that have no convenient “textbook” or chart solution.

The relatively slow growth of FE usage in geotechnical engineering practice is partially due to a perception that the method is complicated and uses “advanced” models that are hard to understand. It is certainly true that the FE method opens up many opportunities for advanced constitutive modeling; however, the most useful applications of finite elements require exactly the same soil properties that would be taught in an undergraduate curriculum, namely the friction angle, cohesion, compressibility, permeability and coefficient of consolidation.

This article demonstrates two applications of the FE method to classical geotechnical problems using free FE software (Smith and Griffiths,2004) that can be downloaded from the web: 1-D consolidation and 2-D slope stability analysis. These applications have been chosen because they are very familiar to geotechnical engineers, yet are usually solved using quite different approaches.

One-Dimensional Consolidation

Terzaghi's 1-D consolidation equation given by is probably the most "famous" differential equation in soil mechanics. The most commonly used solution method for this equation involves replacing it by a finite difference approximation. This is simple enough in the middle of a consolidating homogeneous layer, but becomes less convenient where there are changes of properties or impermeable (undrained) boundaries. The FE approach, on the other hand, is easier to generalize for variable properties and boundary conditions. A string of 1-D finite elements are stuck together, end to end, across the consolidating layer. Each element can have different c_v-values and lengths, so analysis of layered soils such as shown in Figure 1 is easily implemented. Before modeling a layered soil, however, we will validate the program by solving the classical "oedometer problem." In this case, all elements have been assigned the same c_v (=1), the drainage path is set to unity, and the initial excess pore pressure distribution is set to 100 at all depths. Figure 2 shows a dimensionless plot of the average degree of consolidation, U, versus the time factor, T, for this case, indicating almost perfect agreement between the numerical solution (using Program 8.1 in Smith and Griffiths 2004) and the exact series solution to be found in any soil mechanics textbook.

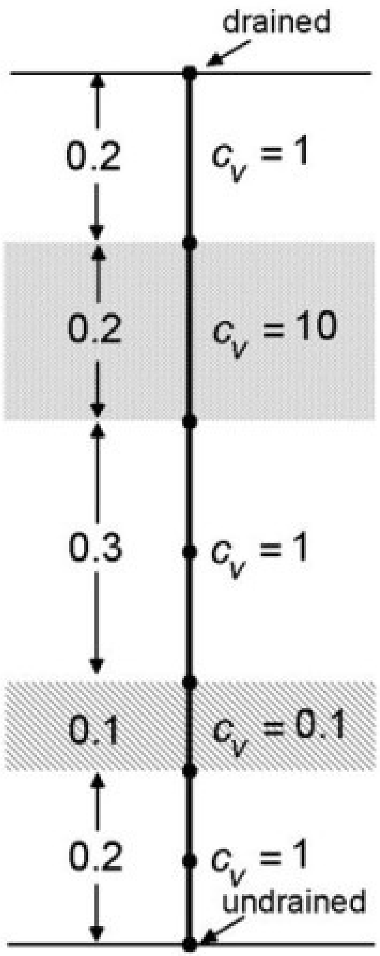

Figure 1: 1-D FE Mesh for Consolidation Analysis

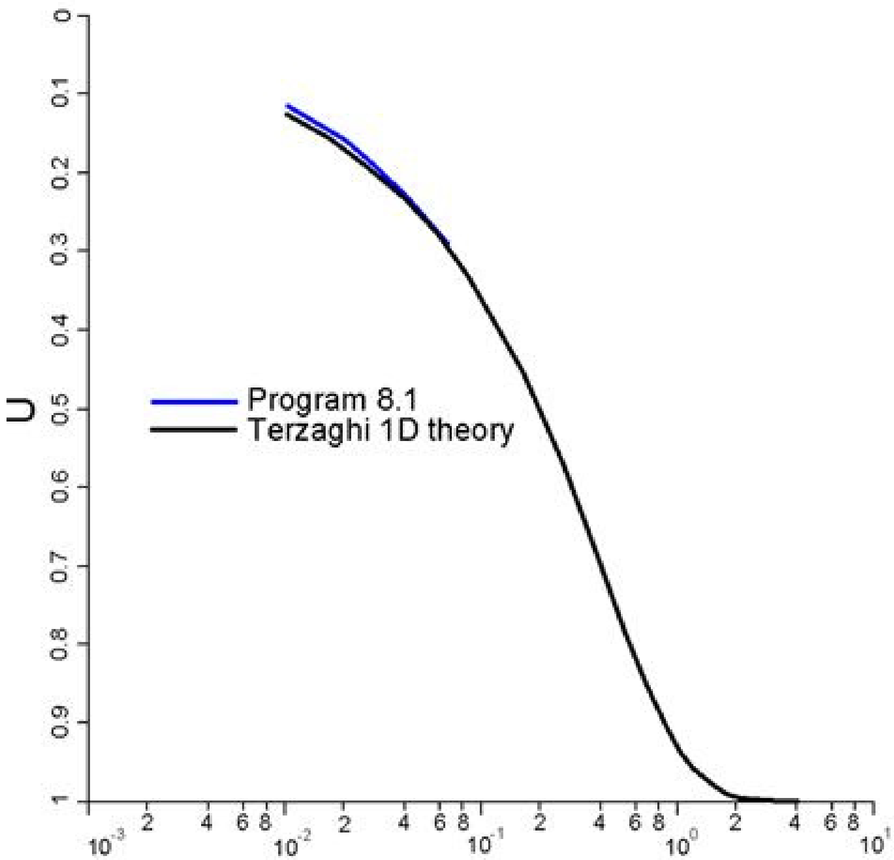

Figure 2: Computed and Exact Plots U vs. T For Homogeneous Case

Figure 3 shows results for the distribution of excess pore pressure, u, at three different time factors, T, for the layered soil example with the same boundary and initial conditions. The influence of the different layer properties is clearly visible.

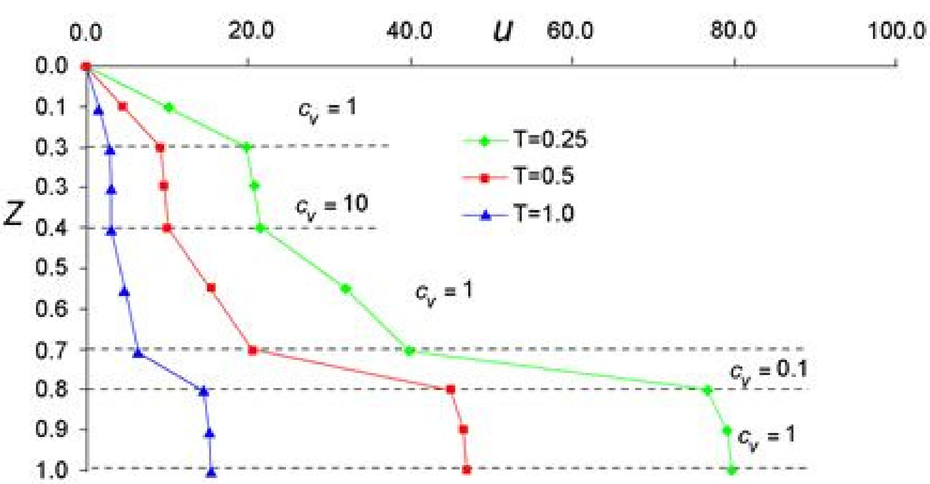

Figure 3: Excess Pore Pressure Distribution at Different Time Factors for Layered 1-D Consolidation

Slope Stability Analysis

Slope stability analysis remains one of the most important areas of geotechnical analysis, yet the methods of slices (e.g., Bishop, Spencer) used in the vast majority of cases have remained essentially unchanged for decades. The elasto-plastic FE approach to slope stability analysis has received increased attention in recent years and offers the following advantages over traditional methods:

- No assumptions need to be made in advance about the shape or location of the failure surface. Failure occurs "naturally" through the zones within the soil mass in which the shear strength is unable to support the gravity-induced shear stresses.
- Because there is no concept of slices in the FE approach, there is no need for assumptions about slice side forces. The FE method preserves global and local equilibrium until "failure" is reached.
- If realistic soil compressibility data is available, the FE solutions will provide information about deformations at working stress levels.
- The FE method can monitor progressive failure up to and including overall shear failure.

The elasto-plastic FE slope stability method follows these main steps:

- Gravity loads are applied to the slope.
- An elastic analysis is performed to compute stresses.
- Stresses in each element are compared with the Mohr-Coulomb failure criterion.
- If stresses violate the Mohr-Coulomb criterion, they are redistributed to neighboring elements that still have reserve strength.
- Slope failure occurs if the algorithm is unable to find a stress redistribution that satisfies Mohr-Coulomb and global equilibrium. Failure is signaled by significantly increased nodal displacements.

The Factor of Safety, FS, in the FE approach is obtained by gradually weakening the soil until the slope fails. This is achieved using a strength reduction factor (SRF) which is applied simultaneously to tanand so that the analysis actually uses factored soil properties and . The SRF is gradually increased until slope failure occurs, at which point the FS = SRF. This approach is entirely consistent with the classical definition of FS used in limit equilibrium methods. Griffiths and Lane (1999) give several examples of the method applied to simple slopes with validation against more traditional solutions using slope stability charts and methods of slices.

Of course the benefits of the finite element slope stability method become more apparent when applied to problems with more complicated geometries and soil property variations, such as the James Bay Dike slope configuration shown in Figure 4. The geometry has a terraced cross-section with four different soil types consisting of cohesionless soil in the embankment and undrained clays in the foundation. This slope has attracted considerable interest (see e.g., Duncan, et al., 2003) because published limit equilibrium solutions that assumed circular failure mechanisms (e.g., Bishop's method), led to unconservative estimates of the FS. Although limit equilibrium procedures are available for estimating the FS associated with non-circular surfaces (e.g., Spencer's method), it is still hard to be certain that the critical surface corresponding to the *minimum* factor of safety has been found.

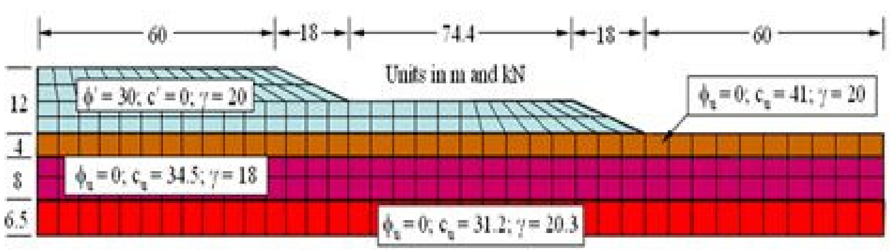

Figure 4: FE Geometry and Soil Properties Assigned to James Bay Dike

By strength reduction to failure as described previously, the FS can be accurately estimated, *and* the corresponding failure mechanism observed. The sudden displacement increase shown in Figure 5 indicates that FS » 1.27 and the deformed mesh at failure given in Figure 6 clearly demonstrates the non-circular critical failure mechanism.

Insert Figure 5

Insert Figure 6

Conclusions

The finite element (FE) method is a mature methodology that has been around for decades; however, the benefits it offers have been slow to gain acceptance in routine geotechnical practice. The method can be applied to geotechnical analyses across the full spectrum of applications, ranging from deformations and steady seepage, to consolidation, limit analysis and dynamics. The effectiveness of the method has been demonstrated by solving two classical geotechnical problems for which no "textbook" solutions are readily available. In addition to proprietary FE software packages, there is now a growing body of free geotechnical FE software available online. This software, combined with the remarkable increase in processing power and decreasing costs of personal computers, means that the FE method should now be an important component of every geotechnical consultant's "tool-box."

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