# Finite element analysis of skirts for gravity base structures

## Analyse des éléments finis pour les structures de base de gravité

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ABSTRACT: Conventional practice in foundation analysis is to reduce the three-dimensional problem to a two-dimensional one. This is commonly done for gravity base structures by considering a "vertical slice". HARMONY - a finite element program for axisymmetric structures under non-axisymmetric load - is described. Unlike conventional analysis methods, the program is truly three-dimensional. The program is used to analyse the behaviour of a single circular skirt embedded in a normally consolidated clay. The skirt was clamped at mudline and subjected to lateral movement. Typical results are shown of the skirt deformations and stresses, plus the non-linear mudline system response. Comparisons are also made with conventional two-dimensional "vertical slice" analyses of the same problem. It is concluded that approximations and uncertanities associated with 2-D analyses have been removed. This leads to more optimum foundation design and an improvement in the prediction of the overall system response.

RESUME: Pour l'analyse du fondement, l'usage courant est de ramener le problème de trois à deux dimensions. C'est ce qu'on fait en général pour les plate-formes poids, en étudiant une "tranche verticale". HARMONY, décrit ici, est un programme des éléments finis pour structures axisymétriques soumises à des pressions asymétriques. Contrairement aux méthodes d'analyses conventionelles pour plate-formes poids, le programmme est réellement à trois dimensions. On utilise le programme pour analyser le comportement d'une jupe circulaire simple encastrée dans de l'argile solidifiée naturellement. Cramponée au fond sous-marin, la jupe est soumise à un mouvement latéral. Les résultats caractéristiques des déformations et des tensions de la jupe sont mis en évidence ainsi que la réponse du système non-linéaire du fond sous-marin. On établit également des comparaisons avec l'analyse classique, à deux dimensions de la "tranche verticale" du même problème. Ainsi disparaissent les approximations et les incertitudes associées aux analyses à deux dimensions. Ceci permet d'obtenir une conception du fondement prèsque optimale et de prévoir avec plus de précision la réponse générale du système.

## 1 INTRODUCTION

The bases of offshore gravity structures founded in deep water are subject to mudline horizontal loads H and overturning moments M. These forces are a large percentage of the vertical load V. If the foundation soils are weak, then bearing capacity failure under inclined (H, M, V) loading is generally critical.

In such cases, large diameter circular foundations (skirts) are used to transfer the loads to more competent soils at depth. This improves the overall factor of safety against general shear failure and stiffens the horizontal sliding response.

The Finite Element Method (FEM) can be used for design. It removes many of the assumptions made in the limit equilibrium method, e.g. material non-homogeneity. However, 3-D analyses are generally not performed, especially if the soil model is non-linear. Hence, FEM analyses are made of 2-D plane strain idealisations. A general problem is the determination of the skirt stresses. The beneficial effects of the lateral rigidity of the circular skirt cannot be readily assessed, except by model tests.

## 2 PROGRAM DESCRIPTION

To avoid using a full 3-D finite element analysis, one can take advantage of material symmetry about the centreline. Fourier series expansions are used to model the displacement, strain and stress variations in the circumferential (θ) direction, and two-dimensional elements in the r-z direction (Figure 1). The problem is then solved of a 2-D axi-symmetric structure under general H, M, V loads (Figure 2).

The Fourier technique enables the solution method, and hence the program, to appear like a 2-D analysis. Thus the computer costs are significantly less than a comparable 3-D analysis, and the program gives an economical exact solution in comparison with approximations derived from simplified 2-D analyses.

The first harmonic analyses for elastic solids were described by Wilson (1965). The present HARMONY development by Griffiths (1985a) for elasto-plastic analyses is based upon the method described by Winnicki and Zienkiewicz (1979). The material is perfectly elastic-perfectly plastic (Mohr-Coulomb) with a no-tension cutoff. This constitutive model has allowed program validation against a number of classical closed-form solutions.

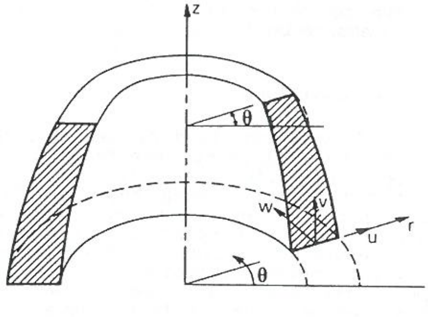


Fig. 1 Axisymmetric solid - coordinates and displacements

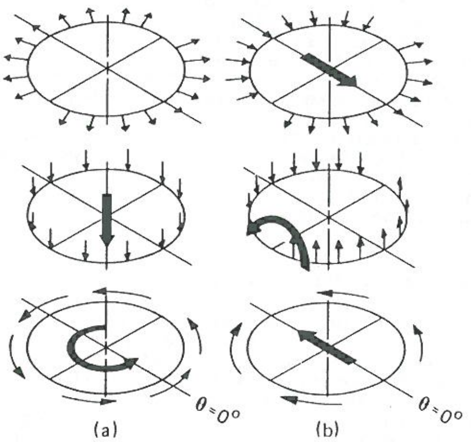


Fig. 2 Nett load components
(a) Zero harmonic
(b) First harmonic

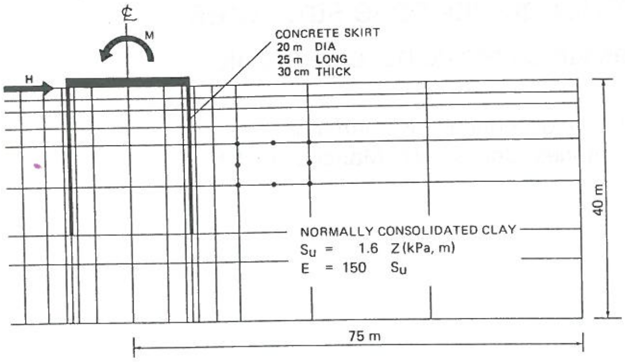


Fig. 3 Concrete Skirt - Analysis

To date, **HARMONY** has been used for non-linear analysis of geotechnical problems including laterally loaded piles in clay (Kay et al., 1986) and sand (Lane and Griffiths, 1988), and spudcans for offshore jack-up rigs under general (H, M, V) loading (Fugro, 1987).

## 3 LATERALLY LOADED SKIRT ANALYSIS

### 3.1 *Problem*

The concrete skirt is 20 m diameter, 25 m high and 30 cm wall thickness (Figure 3). The dimensions are comparable with skirts used in cellular foundations of deep water structures in the Norwegian Trench in the North Sea.

The skirt was assumed to be elastic (i.e. tensile stresses were permitted), with a Young's Modulus E of 30,000 MPa and a Poisson's Ratio $\nu$ of 0.3.

The soil is a normally consolidated clay. The in-situ parameters chosen for a total stress undrained analysis are shown on Figure 3. No tensile stresses were permitted in the clay. No account was taken of softening due to cyclic wave loading and associated pore water pressure build-up.

### 3.2 *Boundary Conditions*

Response bounds were obtained by assuming the whole perimeter of the soil/skirt interface to be either perfectly rough or perfectly smooth.

This was achieved by using the "tied freedoms" technique (Griffiths, 1985b). It fixes the freedoms of nodes such that the displacement components are identical. The use of special interface elements and unnecessary iterations are avoided.

The aim of the analyses presented herein was to investigate second order effects numerically for comparison with small scale model tests on clamped skirts. Hence the skirt was clamped at mudline (i.e. rotation was prevented) and displaced laterally. The final lateral displacement dx = 1.20 m was achieved in 24 equal increments.

An anti-clockwise moment (Figure 3) is induced by prevention of rotation at mudline. This moment acts in the reverse sense to that commonly encountered in design of offshore structures.

Three harmonics were used (0,1 and 2). Only the latter harmonic gives second order effects. The soil stresses were checked for yield at 30 degree intervals in the θ-direction.

Since harmonics other than the first were used, special centreline boundary conditions were necessary to prevent mesh "splitting". This phenomenon is caused should unequal displacement occur at angles θ and θ+π radians on the r = 0 axis.

### 3.3 *Response of Skirt and Soil*

The skirt mudline behaviour is shown on Figure 4. Smoothness reduces the collapse lateral load by 25% and decreased the initial stiffness by a similar magnitude. The moment reductions were smaller - 15%.

At design load levels, the response is essentially linear. Hence the determination of the appropriate soil E value is more important than the soil/skirt interface condition assigned.

Figure 5 summarises the different displacement mechanisms. In both cases the skirt (and the enclosed soil) moves essentially as a rigid body.

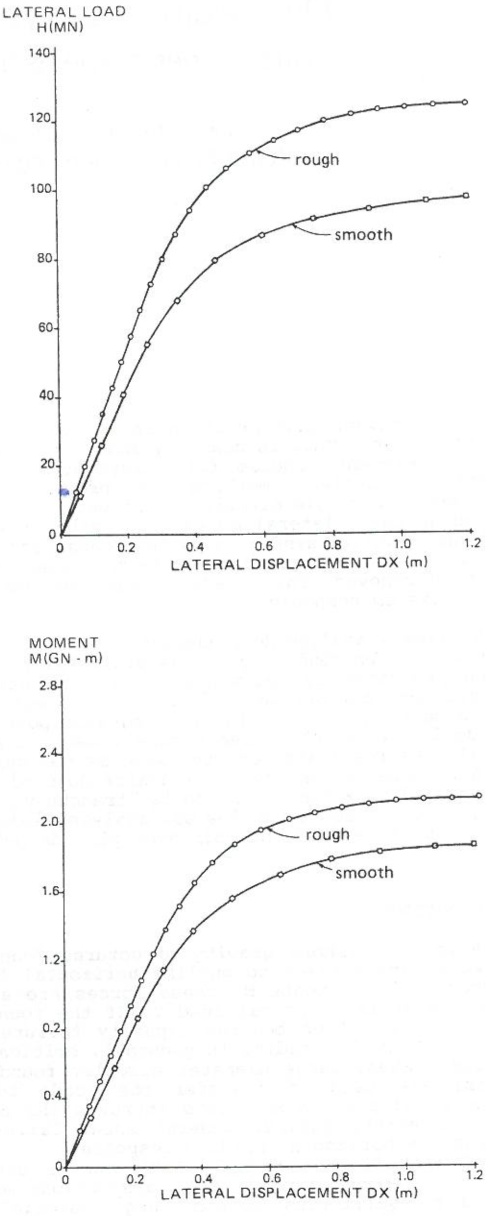
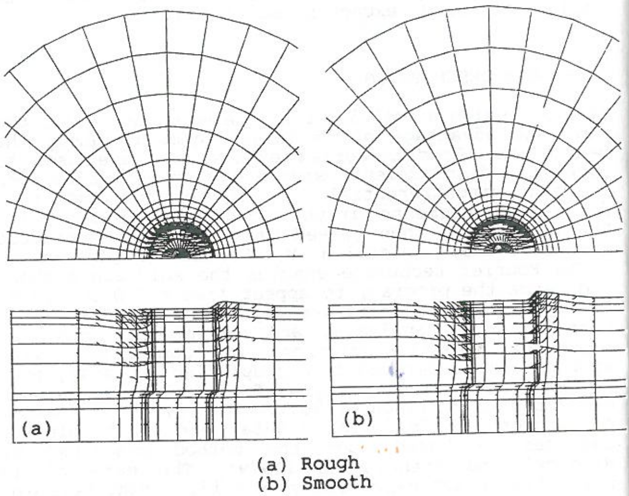


Fig. 4 Skirt - Mudline Response



(a) Rough
(b) Smooth

Fig. 5 Skirt - Deformed Mesh

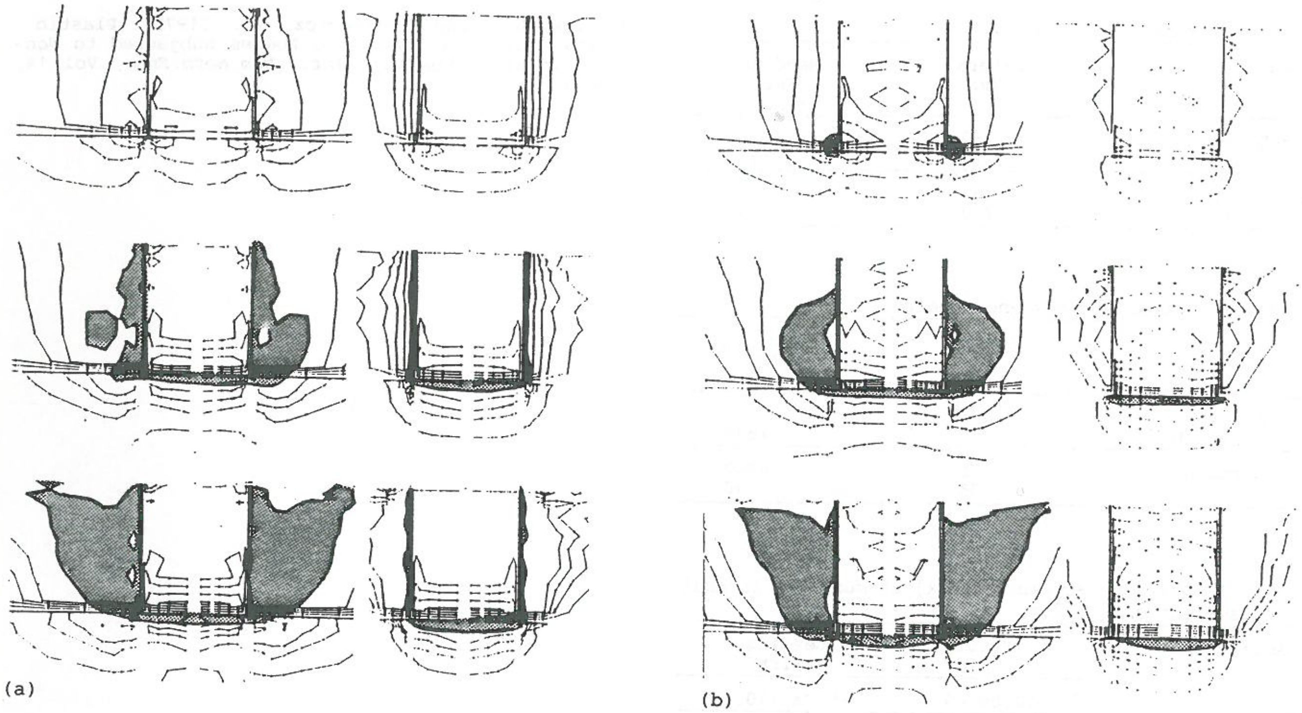


Fig. 6 Yielded Soil Zones at dx = 0.25, 0.50 and 1.0 m
(a) Rough
(b) Smooth

Figure 6 compares the development of yielded soil zones. The results are given in a 3 by 2 matrix. The first column is a section along the direction of movement; the second in the transverse direction. The failure mechanism is wedge-shaped in front of, and trailing, the skirt. Horizontal shear occurs under the skirt base. The soil inside the skirt does not fail.

The zones are asymmetric due to tensile failure occurring at/near mudline behind the skirt.

### 3.4 *Skirt Stresses*

Figure 7 shows typical skirt principal stresses. The maxima occur at mudline in the z-direction (tensile at leading edge, compressive at trailing edge).

At skirt tip level, significant stresses also occur. The pattern is reversed. The maxima are hoop stresses in the θ-direction (compression at leading edge, tension at trailing edge). This is caused by second order effects - "dimpling" of the skirt.

## 4 EQUIVALENT PLANE STRAIN ANALYSES

Conventional 2-D plane strain analyses were made for comparison with the above HARMONY results. The mesh was widened from 75m to 150m. The skirt E value was increased by a factor 10e3 to 300e6 MPa to provide the same equivalent bending stiffness (EI = 27e6 MNm2).

The "tied freedoms" technique was again used (in addition to the rough/smooth idealisations) to ensure pairs of skirt nodes had identical lateral displacements. Hence second order effects were precluded.

Results similar to those shown in Figures 5 and 6 were obtained, save for the smooth case, where the skirt (and soil contents) deformed in shear.

The results are summarised in Tables 1 through 3 together with the corresponding HARMONY analyses.

Table 3 shows that, due to specifying the same displacement profiles, the 2-D stress distributions are similar for both skirts. The non-linear stress variation across each skirt provides compressive amd tensile axial loads respectively for the rough skirts and zero axial thrust for the smooth case.

This thrust variation causes the variations in bending values (Tables 1 and 2).

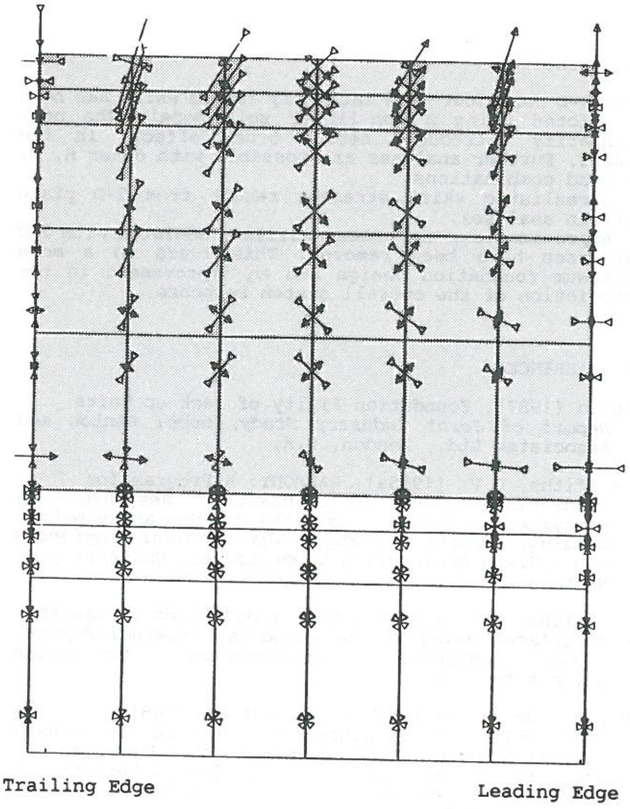


Fig. 7 Rough Skirt - Stresses at dx = 0.25 m

The lateral values (Tables 1 and 2) are underpredicted by 2-D analyses, since they ignore the shear contribution on the sides of the failure plane.

Table 1. Elastic stiffnesses.

| Analysis | Lateral Kxx [MN/m] | Bending Kxz [MNm/m] |
|---|---|---|
| 3-D rough | 280 | 4920 |
| smooth | 215 | 4150 |
| 2-D rough | 265 | 250000 |
| smooth | 250 | 190 |

Table 2. Ultimate Loads and Moments.

| Analysis | Lateral Fx [MN] | Bending Mxz [MNm] |
|---|---|---|
| 3-D rough | 125 | 2150 |
| smooth | 100 | 1890 |
| 2-D rough | 75 | 74300 |
| smooth | 55 | 40 |

Table 3. Skirt $\sigma z$ Stresses* [MPa] at mudline, lateral displacement dx = 0.25m.

| Analysis | Trailing Skirt | | Leading Skirt | |
|---|---|---|---|---|
| | -10.0m | -9.7m | +9.7m | +10.0m |
| 3-D rough | -13 | -13 | +13 | +13 |
| smooth | -11 | -11 | -11 | -11 |
| 2-D rough | -860 | +60 | -60 | +860 |
| smooth | -1212 | +1212 | -1212 | +1212 |

* Compressive stresses are negative.

## 5 CONCLUSIONS

The 3-D behaviour of a laterally loaded skirt has been predicted using a non-linear soil model. The non-linearity introduces second order effects in the skirt. Further analyses are possible with other H, M, V load combinations.

Unrealistic skirt stresses result from 2-D plane strain analyses.

Approximations and uncertanities associated with 2-D analyses have been removed. This leads to a more optimum foundation design and an improvement in the prediction of the overall system response.

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