Probabilistic and Deterministic Slope Stability Analysis by Random Finite Elements

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ABSTRACT: Program PES (Probabilistic Engineered Slopes) provides a repeatable methodology allowing the user to perform a slope stability analysis on a one-sided and two-sided sloping structure using a deterministic or probabilistic approach. Program PES, in contrast with other deterministic or probabilistic classical slope stability methodologies, is cable of seeking out the critical failure surface without assigning a pre-defined failure surface geometry. The probabilistic approach of program PES applies the Random Finite Element Method (RFEM) by Griffiths and Fenton (1993) taking into account the soil spatial variability and allowing the use of different random fields to characterize the spatial variation of any material type. The methodology is compared against the probabilistic approach proposed by the program SLOPE/W version 7.14 (Geostudio Group, 2007), and demonstrates its potential for predicting probability of failure (p_f) in non-homogeneous soil structures characterized by phreatic conditions and potential post-earthquake liquefiable conditions. The p_f results obtained by program PES have proved that underestimating the influence that the soil material variability has on the computation of p_f will lead to lower results of probability and underestimate of the risk of slope instability. Program PES capabilities could be used by the engineering practice to prioritize intervention activities within a risk context.

INTRODUCTION

Stability analyses are routinely performed in order to assess the equilibrium conditions of natural and manmade slopes. The analysis technique chosen depends on both site conditions and the potential mode of failure, with careful consideration being given to the varying strengths, weaknesses and limitations inherent in each methodology.

The motivation driving this study is closely related to the assessment and mitigation of the hazards caused by the instability processes and the important role that stability analysis of slopes plays in civil engineering applications and design.

For many years the nature of geotechnical slope stability analysis has been predominantly deterministic, whether performed using design charts or computers. It is inherent in this type of approach that the parameters characterizing the soil materials such as friction angle, cohesion, Young's modulus, Poisson's ratio, unit weight and ground water are also treated as deterministic. Intuitively, it can be recognized that, where there are materials more homogeneous than others in nature, there are no perfectly homogeneous natural materials. The deterministic approach, which does not allow any variation in the soil materials properties, clearly introduces a high level of approximation to the analysis and characterization of slope stability. The level of approximation can only be reduced if the natural variation of soil is taken into account, allowing the soil to be characterized by a range of values for each parameter instead of a single value.

Soil properties measurements are usually taken over a finite volume, which represents a local average of the property with respect to the overall size of the site domain. For this reason, the Local Average Subdivision (LAS) method (Fenton and Vanmarcke, 1990) has been used to generate the random fields in all the investigations presented in this work. The random field model provides a useful tool for the generation of spatially variable soil properties. A random field is characterized by sets of soil property values, which are randomly generated around their mean value, and are mapped onto the finite element mesh creating a 2D model of variable soil. Each set of property values (e.g. cohesion and friction) characterizes an element within the domain analyzed. The Monte-Carlo method is lastly applied to this model performing multiple random field realizations. The number of simulation that give a Factor of Safety (FS)<1 divided by the total number of simulations represents the probability of failure.

In the computation of slope stability and probability of failure certainly there are many sources of uncertainty, in addition to those related to soil variability. In current engineering practice, most slope stability analyses following a deterministic approach or characterized by a 1D model, do not account for soil variability. The current work will show that accounting for the influence of soil variability, varying the soil strength parameters and using a 2D model, leads to more conservative probability of failure results compared to those computed using classical approaches to geotechnical problems.

PROGRAM PES CHARACTERISTICS

Deterministic Theory

Program PES (Probabilistic Engineered Slopes) coded in FORTRAN.95 allows the user to perform a slope stability analysis on a one-sided and two-sided sloping structure using a deterministic or probabilistic approach. A brief description of program PES methodology is given below. For more detailed information on the elastic- visco-plastic and the strength reduction algorithms used in this study the reader is referred to Griffiths and Lane (1999) and Smith and Griffiths (2004).
As a first step program PES reads the geometry input parameters from the input data file generating a finite element mesh of the problem. Subsequently the soils

properties recorded in the input data file are assigned to the relative embankment and foundation deterministic mesh regions and random fields.

The program can allow the analysis of a liquefiable layer ether in the foundation or in the embankment as well as the partition of a homogeneous embankment into two materials. Clearly these more complicated components are highly dependent on the problem analyzed and require modifications of the main program code each time a different problem is selected.

After the information from the input data files are read Program PES computes the elastic stress-strain matrix, the shape function at the integrating points, the analytical version of the stiffness matrix for an 8-node quadrilateral element, and the lower triangular global matrix *kv*. Then the program generates the additional loading due to free-standing water outside of the slope, as well as a pore pressure within the slope. The water load is equal to the summation of gravity load and pore pressure load, and is computed before being added to the total load already computed. The program allows for the analysis of submerged slopes as well as slopes characterized by a specific water table which can vary in elevation throughout the mesh.

Subsequently program PES computes the strength reduction factor and then performs a check on whether or not the yield is violated according to the failure criterion. The theory coded in this section of the program is described in more detail in the following paragraphs.

Program PES models a 2D plane strain analysis of elastic-perfectly plastic soils with a Mohr-Coulomb failure criterion using 8-node quadrilateral elements with reduced integration (4 Gaussian-points per element) in the gravity load generation, the stiffness matrix generation and the stress redistribution phases of the algorithm. From the literature, conical failure criteria are the most appropriate to describe the behavior of soils with both frictional and cohesive components, and the Mohr-Coulomb criterion is known as the best of this group of failure criteria. Therefore the program uses the Mohr-Coulomb criterion as failure mechanism in all cases. In terms of principal stresses and assuming a compression-negative sign convention, the Mohr-Coulomb criterion can be written as shown in Eq. 1

$$F_{mc} = \frac{\sigma_1' + \sigma_3'}{2}\sin\phi' - \frac{\sigma_1' - \sigma_3'}{2} - c'\cos\phi' \qquad (1)$$

where σ_1' and σ_3' are the major and minor principal effective stresses.

In cases where the soil is characterized by a frictionless component (undrained clays) the Mohr-Coulomb criteria can be simplified into the Tresca criterion substituting $\phi = 0$ in Eq. 1 and obtaining Eq. 2,

$$F_t = \frac{\bar{\sigma}(\cos\theta)}{\sqrt{3}} - c_u \qquad (2)$$

The failure function F for both criteria can be interpreted as follows:

F<0 stresses inside failure envelope (elastic)
F=0 stresses on failure envelope (yielding)

F>0 stresses outside failure envelope (yielding and must be redistributed)

The soil is initially assumed to be elastic and the model generates normal and shear stresses at all Gauss-points within the mesh. These stresses are then compared with the Mohr-Coulomb failure criterion.

The elastic parameter E' and υ' refer to Young's modulus and Poisson's ratio of the soil, respectively. If a value of Poisson's ratio is assumed (typical drained values lie in the range $0.2 < \upsilon' < 0.3$), the value of Young's modulus can be related to the compressibility of the soil as measured in a 1D oedometer (e.g. Lambe and Whitman 1969) as shown in Eq. 3,

$$E' = \frac{(1+\upsilon')(1-2\upsilon')}{m_\upsilon (1-\upsilon')} \tag{3}$$

where m_v is the coefficient of volume compressibility.

In this study the parameters E' and υ' have the values of ($E'=10^5$ kN/m^2 and $\upsilon'=0.3$) respectively. The total unit weight γ assigned to the soil is proportional to the nodal self-weight loads generated by gravity. The forces generated by the self weight of the soil are computed using a gravity procedure which applies a single gravity increment to the slope. The gravity load vector for a material with unit weight γ is computed at the element level as shown in Eq. 4, and subsequently accumulated from each element at the global level by integration of the shape function [N] as shown in Eq. 5,

$$gravlo^{(e)} = \gamma \int_{V^e} N^T dV^e \tag{4}$$

$$gravlo = \sum_{elemnts}^{all} \gamma \iint [N]^T dxdy \tag{5}$$

where N represents the shape functions of the element and the superscript *e* refers to the element number. This integral evaluates the volume of each element, multiplies by the total unit weight of the soil and distributes the net vertical force consistently to all the nodes.

Others have shown that in nonlinear analyses, the stress paths due to sequential loading versus the path followed by a single increment to an initially stress-free slope can be quite different; however the factor of safety appears unaffected when using elasto-plastic models (e.g. Borja *et al* 1989, Smith and Griffiths 2004). It is also important to remember that classical limit equilibrium methods do not account for loading sequence in their solutions.

In the program the application of gravity loading is followed by a systematic reduction in soil strength until failure occurs. This is achieved using a strength reduction factor *SRF* which is applied to the frictional and cohesive components of strength in the form of Eq. 6

$$\phi'_f = \arctan\left(\frac{\tan\phi'}{SRF}\right) \quad \text{and} \quad c'_f = \frac{c'}{SRF} \tag{6}$$

The factored soil properties ϕ'_f and c'_f are the properties actually used in each trial analysis. When slope failure occurs, as indicated by an inability of the algorithm to find an equilibrium stress field that satisfies the Mohr-Coulomb failure criterion coupled with significantly increasing nodal displacements, the factor of safety is given by Eq. 7

$$FS \approx SRF \tag{7}$$

In the literature this method is referred to as the “shear strength reduction technique” (e.g. Matsui and San 1992).

The reduction of soil strength is followed in the program by the computation of the total body load vectors. A description of generation of the body loads computed in the program can be found in deWolfe (2010) and a detailed description of the algorithm used in the program involving viscoplasticity can be found in Smith and Griffiths (2004).

After the computation of body load vectors is completed the program generates the graphic output files respectively a PostScript image of the nodal displacement vectors and a PostScript image of the deformed mesh. The PostScript plot of the displaced finite element mesh has an optional grey-scale representation of the material property random field.

Probabilistic Theory

With regard to the probabilistic analysis computed by program `PES,` the probability of failure can be calculated using two different approaches. When the program is asked to compute the safety factor (*FS*) for each Monte-Carlo simulation, the probability of failure is described by the proportion of Monte-Carlo simulations with *FS*<1. When the program is asked to compute the probability without determining the exact value of *FS* for each simulation, the probability of failure is described by the proportion of Monte-Carlo slope stability analyses that failed. In this case the SRF is equal to 1(no strength reduction is actually applied). In this case, “failure” was said to have occurred if, for any given realization, the algorithm (Mohr-Coulomb failure criterion) was unable to converge within 500 iterations.

The RFEM code enables a random field of shear strength values to be generated and mapped onto the finite elements mesh, taking full account of element size in the local averaging process. In a random field, the value assigned to each cell (or finite elements in this case) is itself a random variable.

The random variables can be correlated to one another by controlling the spatial correlation length and the cross correlation matrix where the degree of correlation ρ between each property can be expressed in the range of $-1<\rho<1$.

More generally the correlation coefficient between two random variables X and Y can be defined by Eq. 8

$$\rho_{XY} = \frac{COV[X,Y]}{\sigma_x \sigma_y} \tag{8}$$

where COV represents the covariance between the two variables X and Y and their respective standard deviations σ_x and σ_y.

Due to the isotropic approach applied throughout this work the following simplifications can be made with respect to the mean, standard deviation and the spatial correlation length: $\mu_x = \mu_y = \mu_z$, $\sigma_x = \sigma_y = \sigma_z$, and $\theta_x = \theta_y = \theta_z$.

Using an exponentially decaying (Markovian) correlation function, Eq. 8 can be rewritten as in Eq. 9 and Eq. 10

$$\rho = e^{-\frac{2\tau}{\theta_{\ln c}}} \tag{9}$$

$$\rho = \exp\left\{-\frac{2}{\theta_{\ln c}}\sqrt{\tau_x^2 + \tau_y^2}\right\} \tag{10}$$

Where ρ is the familiar correlation coefficient, τ is the distance between two points in the random field and θ_{lnc} represent the spatial correlation length.

The spatial correlation length (θ), also referred to in literature as the “scale of fluctuation”, describes the distance over which the spatially random values will tend to be significantly correlated in the underlying Gaussian field. Mathematically θ is defined as the area under the following correlation function (e.g. Fenton and Griffiths, 2008 from Vanmarcke, 1983);

$$\theta = \int_{-\infty}^{\infty} \rho(\tau)\,d\tau = 2\int_{0}^{\infty} \rho(\tau)\,d\tau \tag{11}$$

where τ represents the distance between two positions in the random field. A large value of θ will imply a smoothly varying field, while a small value will imply a ragged field.

Another important dimensionless statistical parameter involved in this probabilistic approach is the coefficient of variation v, which for any soil property can be defined as

$$v = \frac{\sigma}{\mu} \tag{12}$$

where σ is the standard deviation and μ the mean value of the property.

In brief, the analyses involve the application of gravity loading, and the monitoring of stresses at all the Gauss points. The program uses the Mohr-Coulomb failure criterion, which if violated, attempts to redistribute excess stresses to neighboring elements that still have reserves of strength. This is an iterative process which continues until the Mohr-Coulomb criterion and global equilibrium are satisfied at all points within the mesh under quite strict tolerances. Plastic stress redistribution is accomplished using a visco-plastic algorithm with 8-node quadrilateral elements and reduced integration in both the stiffness and stress redistribution parts of the algorithm. For a given set of input shear strength parameters (mean, standard deviation and spatial correlation length), Monte-Carlo simulations are performed until the statistics of the output quantities of interest become stable.

A more comprehensive explanation of the random finite elements method, including local averaging approach and discussion on spatial correlation length can be found in Fenton and Griffiths (2008).

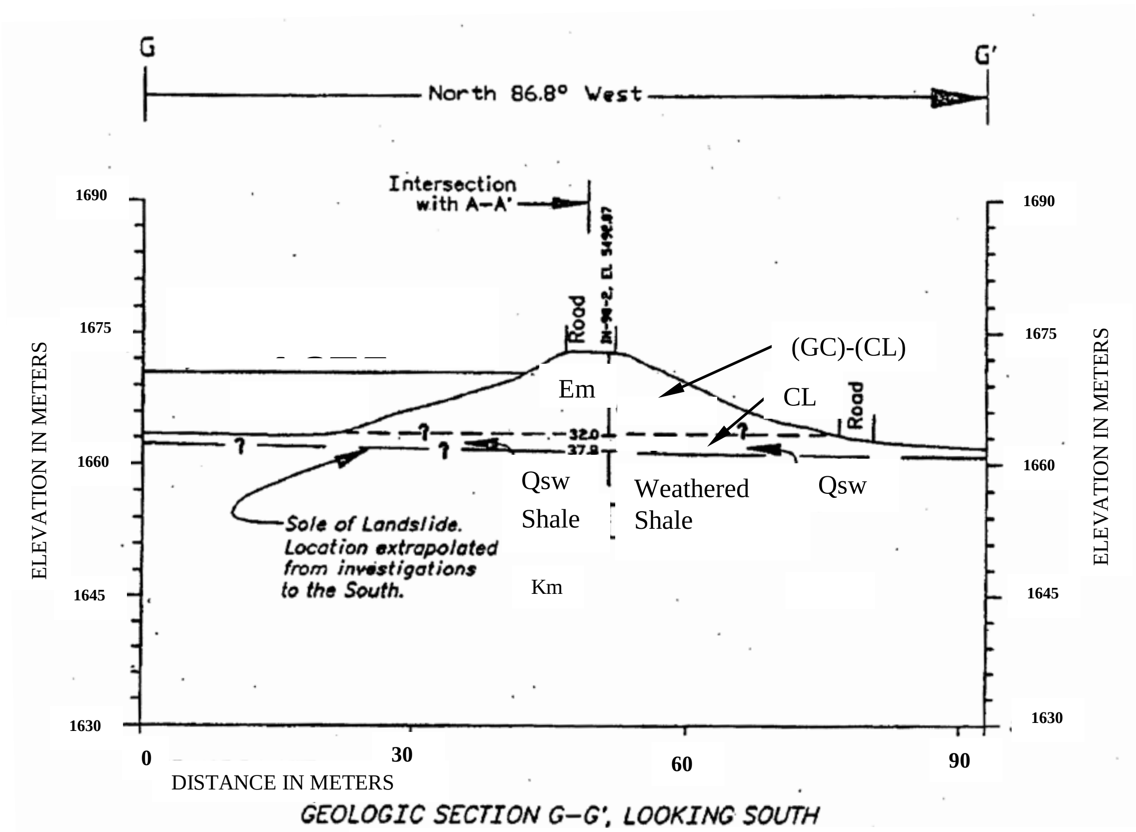
PROGRAM PES APPLICATIONS

Fruitgrowers Dam Deterministic and Probabilistic Slope Stability Analyses

In this section program PES is tested in the analysis of a dam case history. Fruitgrowers Dam is located in Delta County, Colorado, 6.4 kilometers upstream from Austin, Colorado on Alfalfa Run, a tributary of the Gunnison River. The dam was constructed by the Bureau of Reclamation from 1938 to 1939 for the primary purpose of irrigation. The crest of the dam is at elevation 1674.0 meters (5493 feet). The dam has a structural height of 16.8 meters (55 feet), hydraulic height of 12.2 meters (40 feet), crest width of 7.6 meters (25 feet), and crest length of 463.3 meters (1520 feet). An aerial view of Fruitgrowers dam is shown in Figure 1.

The dam is a compacted zoned earthfill structure consisting of a wide central core protected by a riprap layer on the upstream slope and by a thin gravel shell on the downstream slope. The embankment core is composed of clay, sand and gravel, grading to gravel at the outer slopes as shown in Figure 2. A cut-off trench was excavated to impermeable material. The trench has a bottom width of 2.4 meters (8 feet) and is located 10.7 meter (35 feet) upstream of dam centerline. The surficial material beneath the dam shell upstream and downstream of the cut-off trench was stripped to remove top soil and organic material.

FIG. 1. Aerial view of Fruitgrowers dam (Photo courtesy of the BOR)



Geologic label description:

EM: Embankment material
GC-CL: gravel with clay and sand to lean clay
CL: Lean Clay
Qsw: Quaternary slope wash alluvium
Km: Mancos Shale Formation

FIG. 2. Cross section G-G' showing post construction actual dimensions of Fruitgrowers Dam

The case history of Fruitgrowers Dam was selected because past studies of the site conducted by the Bureau of Reclamation presented possible post-earthquake liquefiable conditions in the foundation.

A seismic hazard assessment, conducted by the Bureau of Reclamation (2003), concluded that the background earthquake sources present in the area will not likely result in a large liquefaction potential. In August 2004, to address new concerns created by the presence of silty sand material on the dam abutment, a study was conducted using data collected from five field explorations performed between 1980 and 1999.

The results of this latest study showed a low likelihood of foundation liquefaction at the dam site. According to this study, to produce the failure of the embankment a liquefied continuous lens, longer than 19.5 meters (64 feet), should be present in the foundation under the right abutment, and from the drill log data collected on each side of the embankment during the field explorations the presence of such a long continuous layer is unlikely. As shown in Figure 3, a deterministic post liquefaction FS of 1.05 was computed for the structure assuming the presence of a 18.3-meter (60-foot) long liquefiable layer.

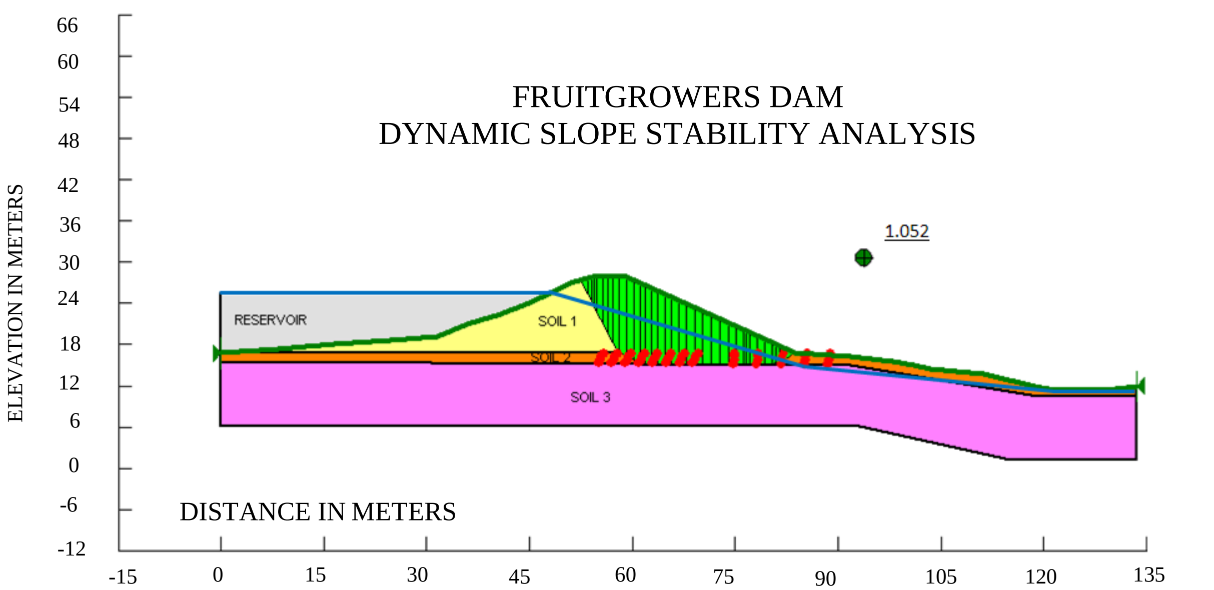


Figure 3: Deterministic post-liquefaction steady state analysis computed in 2004 using the software `SLOPE/W` version 7.4

From the computer program `SLOPE/W` version 7.4, the method of analysis used to compute this result was the Spencer method, coupled with a rigid block theory technique for the evaluation of the failure surface.

In the "Evaluation of Liquefaction and Post Earthquake Stability" conducted by the Bureau of Reclamation in August 2004, as well as in previous studies, the dam is essentially modeled as a homogeneous embankment. Similar to the study conducted in 2004 the geometry of the current model is based on cross section G-G, Figure 2 (post construction actual dimensions) and also represents a homogeneous embankment.

The phreatic condition characterizing the analysis is also adopted from the model constructed in 2004 which shows the reservoir elevation at 1672 meters (5485) (top of active conservation) with 2.44 meters (8 feet) of freeboard, and a downstream toe water elevation of 1662 meters (5453 feet), 1.22 meter (4 feet) below ground surface.

This piezometric line was developed during a study also conducted in 2004 investigating the effect of the artesian pressure on the site foundation and embankment structure (Technical Memorandum No. FW-8312-2, 2004). Figure 4 shows the piezometric line, the geometry and the major units characterizing the deterministic model created in 2004.

The model representing Fruitgrowers Dam is characterized by the following 3 soil materials.

- The embankment core is composed of clay, sand and gravel, grading to gravel at the outer slope.
- The foundation material consists of the Mancos Shale Formation (Km) and is modeled with a thickness of 11 meters (36 feet).
- The Quaternary alluvium (Qal) is characterized by recent alluvial deposits of the Alfalfa Run and is modeled with a thickness of about 1.83 meters (6 feet).

Before diving into the probabilistic analysis, initial deterministic static analyses modeling pre- and post-liquefaction conditions were conducted using program PES.

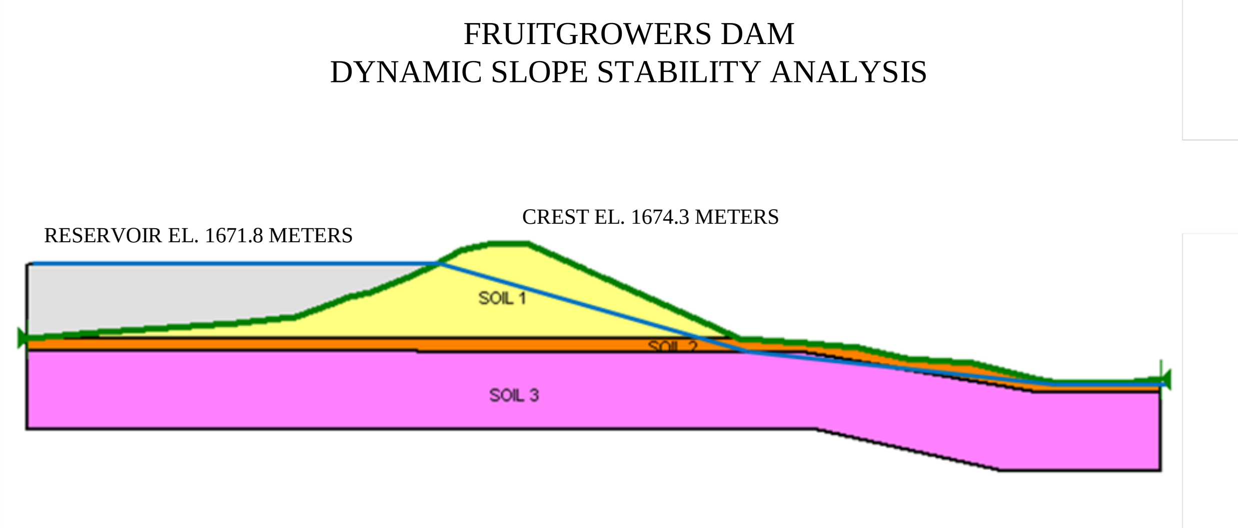


FIG. 4. Representation of the 2004 model used in the deterministic post liquefaction analysis.

The soil properties used in the 2004 slope stability analysis to characterize the embankment, foundation, and liquefiable layer are considered generally appropriate for these two deterministic analyses and are summarized in Table 1.

Table 1. Deterministic soil properties used in the Fruitgrowers Dam pre and post-liquefaction analyses

	Material	Unit weight (kg/m^3)	$\phi^{'}$ (º)	$c^{'}$ (kPa)
Post liquefaction conditions	Embankment	2050	32	20.68
	Foundation	2082	30	0.05
	Quaternary alluvium	2082	0	14.36
Pre liquefaction conditions	Embankment	2050	32	20.68
	Foundation	2082	30	0.05
	Quaternary alluvium	2082	30	0.05

Subsequently the post liquefaction deterministic model was run using the probabilistic capability offered in program PES.

The soil properties as probabilistic variables and their statistical parameters used during the probabilistic analysis are summarized in Table 2.

Table 2. Probabilistic soil properties used in the Fruitgrowers post-liquefaction analyses

Material	μ	σ characterize by lower *v*	σ characterize by higher *v*	Distribution Type
Embankment $\phi^{'}$ (º)	32	3.2	6.4	Lognormal
Embankment $c^{'}$ (kPa)	20.68	2.07	4.14	Lognormal
Foundation $\phi^{'}$ (º)	30	6	15	Lognormal
Foundation $c^{'}$ (kPa)	0.05	0.009	0.02	Lognormal
Quaternary alluvium $\phi^{'}$ (º)	0	0.2	0.5	Lognormal
Quaternary alluvium $c^{'}$ (kPa)	14.36	2.87	7.18	Lognormal

The probabilistic analysis associates one random field with the embankment, one with the foundation and the liquefiable layer is described by the foundation random field which is modified to address the new values describing the liquefiable material. In this probabilistic model only the strength parameters of friction and cohesion are analyzed in a probabilistic approach; the other parameters, dilation angle, unit weight, Young's modulus and Poisson's ratio are analyzed following a deterministic approach.

To address the level of uncertainty incorporated with the mean values describing the properties the same probabilistic model is run one time with a higher Coefficient of Variation (*v*) and one time with a lower *v*. The *v* values chosen represent suggested values available in the literature for similar soil material. (e.g Phoon and Kulhawy, 1999). The *v* values used in this analysis for all material types are summarized in Table 3

Table 3. *v* values characterizing Fruitgrowers probabilistic runs.

Material	lower v	higher v
Embankment $\phi^{'}$ (º) and $c^{'}$ (kPa)	0.1	0.2
Foundation $\phi^{'}$ (º) and $c^{'}$ (kPa)	0.2	0.5
Quaternary alluvium $\phi^{'}$ (º) and $c^{'}$ (kPa)	0.2	0.5

The *v* values characterizing the probabilistic analyses were chosen evaluating suggested values

Another critical value in the analysis is the spatial correlation length used to determine the soil spatial variability. The set of isotropic values chosen to investigate the spatial correlation length θ for all probabilistic runs is reported in Table 4.

Table 4. Isotropic θ values characterizing Fruitgrowers spatial variation of soil.

θ=	1.22 m
θ=	7.62 m
θ=	18.288 m
θ=	30.48 m
θ=	60.96 m
θ=	91.44 m
θ=	152.4 m

All the probabilistic analyses are run using 1000 Monte-Carlo simulations. It has been observed during this investigation that the probabilistic model representing Fruitgrowers dam associated with 1000 Monte-Carlo simulations returns a probability that can vary up to 2.7% as showed in Figure 5, which represent a repeatable computation. During all probabilistic and deterministic analyses all soil properties are considered uncorrelated between each other.
The results of the probabilistic analyses as well as the comparison with the results generated by the program Slope\W version 7.14 are described in the following section

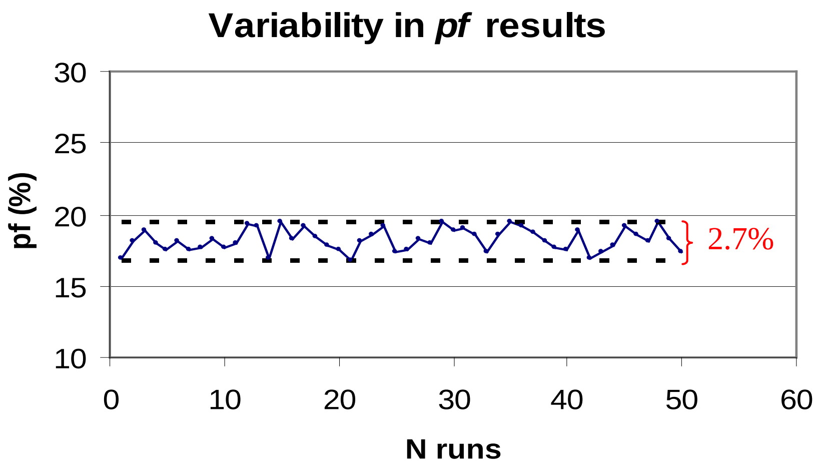


FIG. 5. Variability in p_f results using 1000 Monte-Carlo simulations. To recognize how much the p_f computed by the Fruitgrowers model could vary in a probabilistic setting the same data file was run 50 times.

Programs `PES` and `SLOPE/W`: Deterministic and Probabilistic Slope Stability Results Comparison.

The result from the deterministic pre-liquefaction model run using program `PES` shows a FS=1.66 (Figure 6) while the `SLOPE/W` result according to Spencer's Method returns a FS=1.746 (Figure 7). The deterministic post-liquefaction model computed by `PES` returned a value of FS=1.09 (Figure 8) when the `SLOPE/W` result on the same model according to Spencer's Method returned a FS=1.06 (Figure 9).

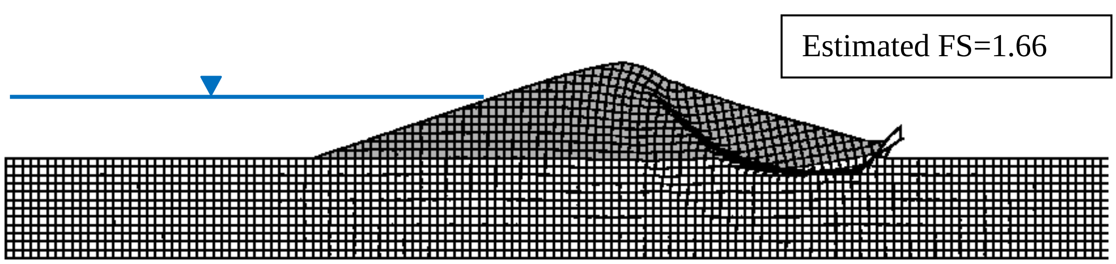


FIG. 6. Displacement file showing displacement associated with the deterministic pre-liquefaction conditions at Fruitgrowers Dam.

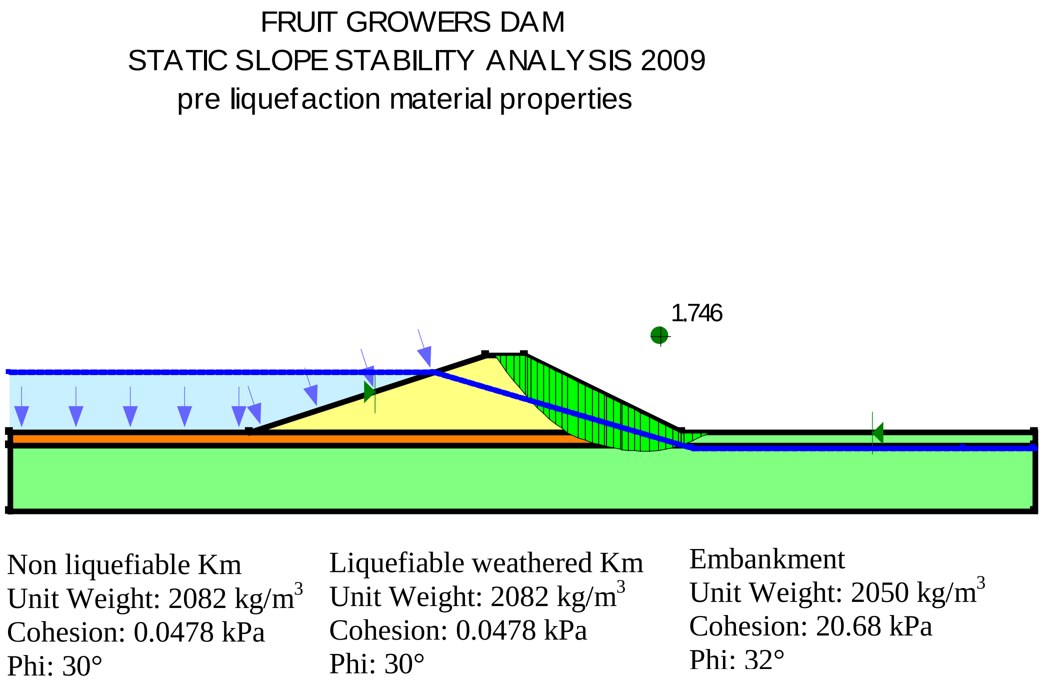


FIG. 7. Graphic representation according to Spencer's Method of the `SLOPE/W` results describing the deterministic pre-liquefaction conditions at Fruitgrowers Dam.

The loading applied in the post liquefaction analysis are vertical gravity load only. The post liquefaction analysis results from both programs assumes the presence of a liquefiable layer, 84.12 meter (276 feet) long (16.45 meter or 54 feet downstream from the centerline of the dam), while the post liquefaction deterministic analysis computed in the 2004 obtained a FS=1.05 assuming the presence of a continuous liquefiable layer 18.29 meter (60 feet) downstream of the centerline of the dam.

In the probabilistic analysis computed by `PES` the deterministic variables are characterized by the same values used in the post-liquefaction analysis and the probabilistic values are described by the statistical parameters summarized in the previous section. In the probabilistic analysis computed using `SLOPE/W`, the failure surface associated with the FS of 1.06 (Figure 9) was chosen as the critical one to test with the probabilistic approach offered by `SLOPE/W.`

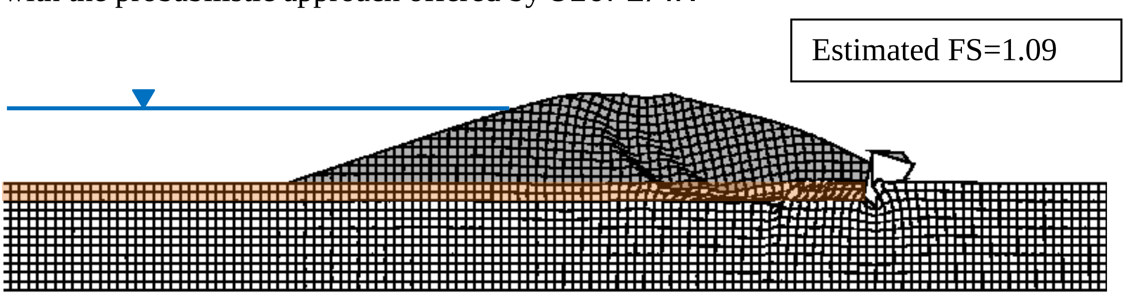
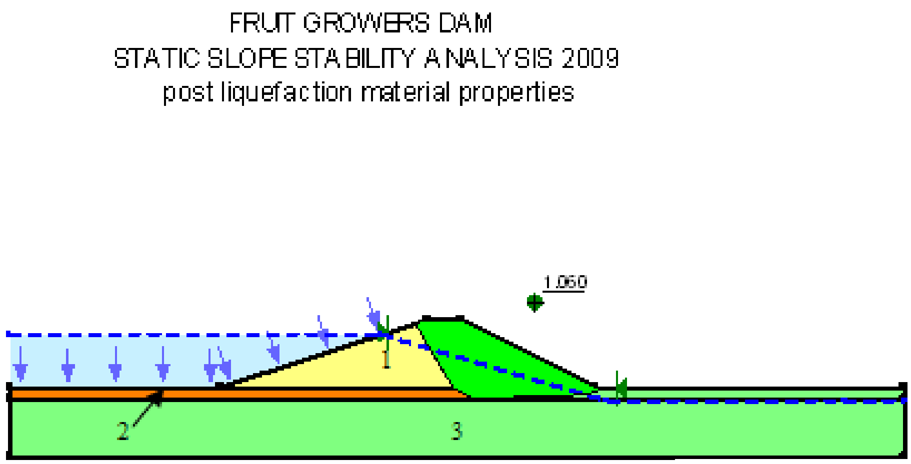


FIG. 8. Representation of the displacement associated with the deterministic post-liquefaction conditions at Fruitgrowers Dam (program `PES).`

The soil properties statistical parameters and soil spatial variation parameters used in this analysis are the same as those used in the analysis run with program `PES`, and are summarized in Tables 2, 3, and 4.



1-Embankment
Unit Weight: 2050 kg/m^3
Cohesion: 20.68 kPa
Phi: 32°

2-Liquefiable weathered Km
Unit Weight: 2082 kg/m^3
Cohesion: 13.36 kPa
Phi: 0°

3-Non liquefiable Km
Unit Weight: 2082 kg/m^3
Cohesion: 0.0478kPa
Phi: 30°

FIG. 9: Graphic representation according to Spencer's Method of the `SLOPE/W` results describing the deterministic post-liquefaction conditions at Fruitgrowers Dam.

Tables 5 and 6 respectively summarize the results from the `SLOPE/W` analyses and the analyses run with `PES`. Figure 10 shows a direct comparison of the results from the two programs for both lower and higher *v*.

The results showed in Figure 10 outlines fundamental differences between the two programs. A detailed effort has been made during this study to comprehend the differences among the two programs, but while for the program `PES` a full version of the program's code is available, for the program `SLOPE/W` the author of this research has to solely rely upon the program manual, published by Geostudio, which does not provide detailed information on the program code.

Table 5. Results from the Fruitgrowers probabilistic analyses run with the program `SLOPE/W`.

(θ) m	Low v p_f %	High v p_f %
1.22	3.8	20.12
3.05	12.53	34.23
4.57	19.37	39.02
6.09	23.37	43.48
7.62	26.41	45.48
9.14	28.47	45.95
10.67	28.21	46.39
12.19	28.69	46.4
15.24	29.14	46.35
152.4	29.28	46.72

Table 6. Results from the Fruitgrowers probabilistic analyses run with the program `PES`.

(θ) m	Low v p_f %	High v p_f %
1.22	94.7	98.6
7.62	72.9	95.8
18.29	70.3	89
30.48	66.7	82.7
60.96	66.4	78.6
91.44	65.9	77.9
152.40	67.3	73.5

The p_f trend shown in Figure 10, corroborated by the trend results between program `PES` and the program `SLOPE/W` compared in the probabilistic validation presented in deWolfe (2010). The results presented in Figure 10 confirm that the probability of failure computed by `SLOPE/W` is unconservative with respect to the probability of failure estimated by program `PES`.

Figure 10 shows that for high values of spatial correlation the p_f results from both programs will show very little variation which is expected because high values of spatial correlation correspond to a virtually homogeneous soil material at each simulation. Lower values of spatial correlation instead emphasize a very different trend between the two programs.

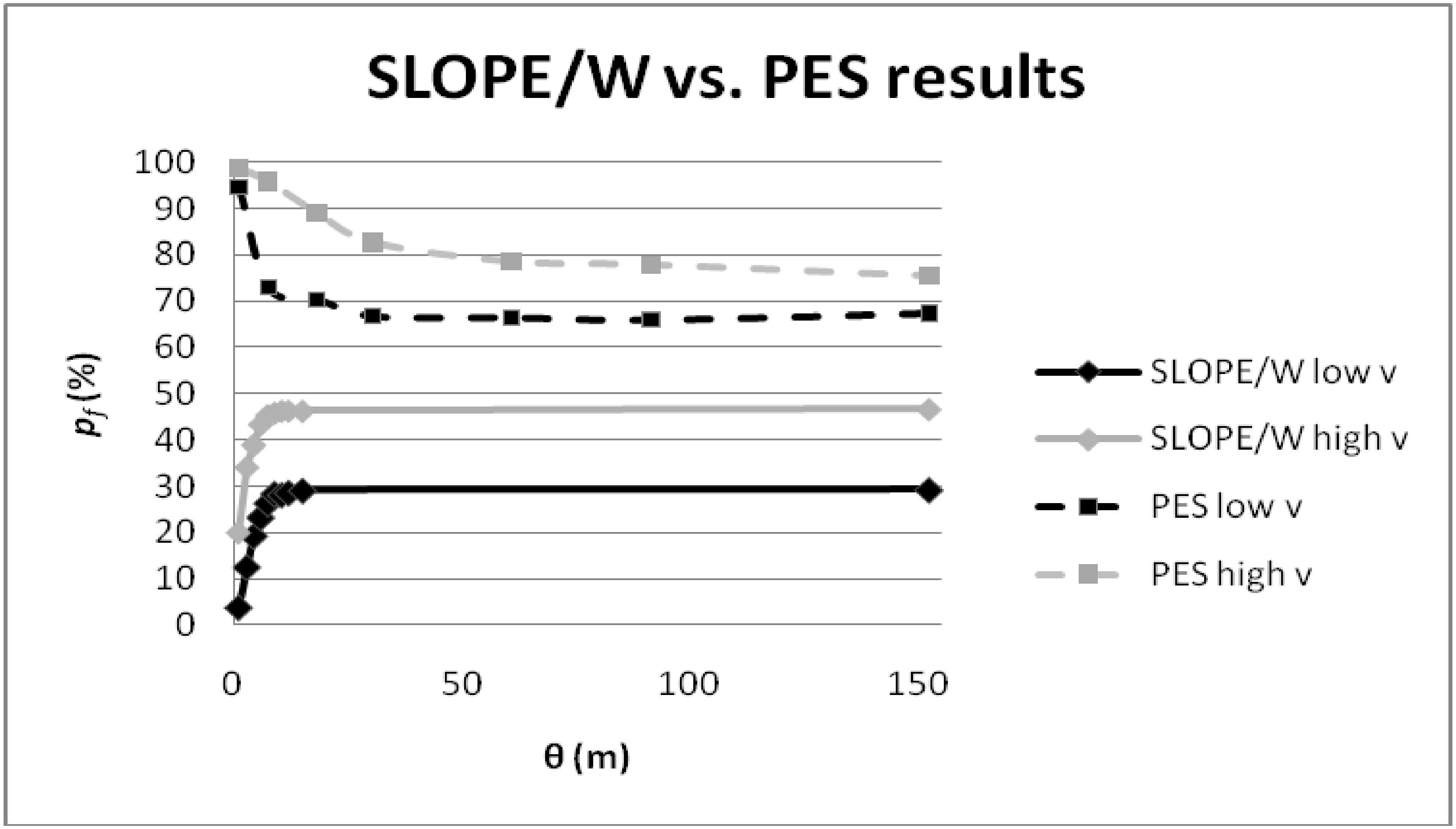


FIG. 10: Comparison of the results from programs PES and SLOPE/W for both lower and higher *v*.
PES results are based on the deterministic F.S of 1.09 and the SLOPE/W results on the deterministic F.S of 1.06
To gain a better prospective on the comparison between the element size and the spatial correlation length in this model, it is important to remember that a single square element size is equal to 0.91 meter (3 feet), and the total dimensions of the problem are approximately, 21.95 meters (72 feet) in height and 146.3 meters (480 feet) in length.

The trend showed in Figure 10, by SLOPE/W results, associates lower p_f to a highly spatially variable soil (low spatial correlation) and a higher p_f with a more homogeneous soil (high spatial correlation). In the other hand, program PES show results which associate higher p_f with more variable soils and lower p_f to a more homogeneous soil. As mentioned in program SLOPE/W manual, the program does not apply any reduction to the standard deviation or the mean values of a random property unless the length between two sections, ΔZ, is equal to or greater than the scale of fluctuation or spatial variation length.

In the specific case of the model representing Fruitgrowers Dam the average distance between two slices is approximately 1.22 meters and therefore no reduction was ever applied to the standard deviation or the mean values of a random property through all analyses. In general in the case of a deterministic FS>1 a random field characterized by a reduced mean and variance values will lead to higher probability of failure, and that could explain why the SLOPE/W results are consistently unconservative with respect to the results computed by program PES. Instability in the results produced by program PES can be observed when the spatial correlation length value is equal to or smaller than the element size. In general, cases where the element size is greater than the spatial correlation length do not represents a very meaningful model, when instead, if many elements are able to define the variability inside the spatial correlation length, this can be considerate a representative model.

Even for the cases when this may apply, one unstable result certainly cannot in anyway change the overall interpretation of the analysis results trend.
Without a doubt it is quite difficult to determine the correct value of a soil variability and this parameter represents a key component of this probabilistic analysis. Only expert engineering judgment supported by exploration can truly lead to the understanding of what that meaningful range of soil variability is for a specific material. The results computed by the program PES and shown in Figure 10 clearly emphasizes that not accounting properly for soil variability will lead to unconservative results of p_f or non-convergence and underestimate the probability of slope instability. It needs to be remembered that the high probability of failure computed by program PES associated with Fruitgrowers dam is strictly dependent on the liquefaction of a continuous layer approximately 1.5 to 2 times the height of the embankment. Even though the presence of potentially liquefiable material has been corroborated by field testing in the area, the absolute continuity of the potentially liquefiable layer still remains uncertain. Furthermore, based on the blow counts values describing the strength of the weathered shale characterizing the potentially liquefiable layer, liquefaction can occur only for an event associated with a high seismic return period, such as the 50,000-year return period characterized by an acceleration value of 0.27g. The probability of such event occurring in this area is highly unlikely. For further information on the seismicity associated with Fruitgrowers Dam the reader is referred to the Bureau of Reclamation seismic study conducted in 2004 (Bureau of reclamation 2004).

CONCLUDING REMARKS

Program PES provides a repeatable methodology able to improve the confidence associated with the computation of probability of slope instability, which is a key component of risk assessment for an engineering structure.

The probabilistic approach used in program PES applies a combination of the random field technique and the finite element method.

At the core of the RFEM approach is the capability of accounting for spatially random shear strength parameters and spatial correlation. This methodology combines a non-linear elasto-plastic finite element analysis with random field theory generated using the Local Average Subdivision Method (Griffiths and Fenton, 2004). More specifically the spatially variable soil properties are correlated through the parameter spatial correlation length or scale of fluctuation (θ), which indicates the distance within which the values of a property show a relatively strong correlation, and the parameter correlation coefficient (ρ). The main advantage of the RFEM over traditional probabilistic slope stability techniques is that RFEM enables slope failure to develop naturally by "seeking out" the most critical mechanism.

The methodology utilized in program PES is compared against the probabilistic approach proposed by the program SLOPE/W version 7.14, and demonstrates its potential for predicting probability of failure in a non-homogeneous soil structure characterized by phreatic conditions and a possible liquefiable layer. While the results computed from the deterministic analyses using programs PES and SLOPE/W show a very close agreement, the results from the probabilistic analyses from the two programs are generally in disagreement, and the SLOPE/W results consistently show lower values of p_f than obtained using program PES.

In the author's opinion the difference in p_f computed by the two programs can be explained by the following three observations:

1. Both programs PES and SLOPE/W produce results of deterministic FS, pf, mean and standard deviation of FS, but it is important to remember that, for both probabilistic and deterministic analyses, program SLOPE/W represents a 1D model of the soil property correlations along the potential failure surface, while PES characterizes the soil property correlations using a 2D model. In the probabilistic approach, the program PES investigates the soil variability through the spatial correlation length over the entire foundation and embankment zones while SLOPE/W investigates the soil variability only along the line characterizing the critical slip surface.
2. Another major difference between the two programs is that SLOPE/W will perform the probabilistic analysis on a failure surface found using traditional slope stability methods (Jambu, Spencer, Bishop etc.) that require a subdivision of the slope into columns, while the program PES based on a strength reduction allows the modeled slope to fail naturally by "seeking out" the path of least resistance of each Monte-Carlo simulations. In the author's opinion, the number of columns initially selected by the user in program SLOPE/W not only

influences the precision of the deterministic FS, but also influences the computation of the probability of failure.

3. Another component that may lead to the low values of probability by `SLOPE/W`, especially at lower values of the spatial correlation length (θ), is the difference in the way local averaging is implemented in the two programs.

The establishment of a robust methodology provided by this research will not only allow testing of the stability conditions of dams during modification phases, but will also help estimate the probability of failure in cases involving post-earthquake liquefaction. Although in the current study interest was concentrated on a classical two-sided embankment geometry, the methodology can be applied to a wide range of geotechnical engineering problems, taking into account the soil spatial variability and its capability of "seeking out" the critical failure surface without assigning a pre-defined failure surface geometry.

The current work has proven that not accounting for spatial variability can lead to unconservative results with respect to more classical approaches computing probability of failure in geotechnical problems.

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