The Influence of Random Confining Layer Thickness on Levee Seepage Analysis

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Abstract: Seepage analyses are routinely performed as part of levee analyses. While finite-element analysis is being used with increasing frequency, semi-analytical solutions such as “blanket theory” are still commonly used, especially for simplified reliability analysis in conjunction with first-order methods. Unfortunately, the simplified approaches do not properly account for the spatial variability inherent in geologic materials when considering the blanket thickness. In this paper, random field theory is used to determine the influence of a confining layer of random thickness on levee under seepage and exit gradients. Results indicate that neglecting spatial variation in blanket thickness does not cause significant error in computed exit gradients and flow quantities obtained from simple reliability analysis using blanket theory.

INTRODUCTION

Nearly $3 trillion of flood related damages were sustained by the United States in 2014 (NWS, 2015) despite the more than 100,000 miles of levees that currently span counties accounting for 43 percent of the nation’s population (ASCE, 2013). With increasing urbanization and climate change, the perpetual flood risk associated with existing levee systems is ever increasing. However, the design practices for levees have remained relatively unchanged over the last 60 years (Turnbull and Mansur, 1961a and 1961b; USACE, 2000; Wolff, 1994) and focused primarily on minimizing the potential for heave at the landside levee toe through the analysis of exit gradients.

A simplified levee cross section is illustrated in Figure 1. It is quite common for levee foundations in fluvial environments to have a pervious aquifer overlain by a fine grained confining layer as shown. Due to the common nature of this geometry, simplified semi-analytical solutions (often referred to as “blanket theory”) have been developed to assess the steady state seepage problem for various boundary conditions (Bennett, 1945; Turnbull and Mansur, 1961a).

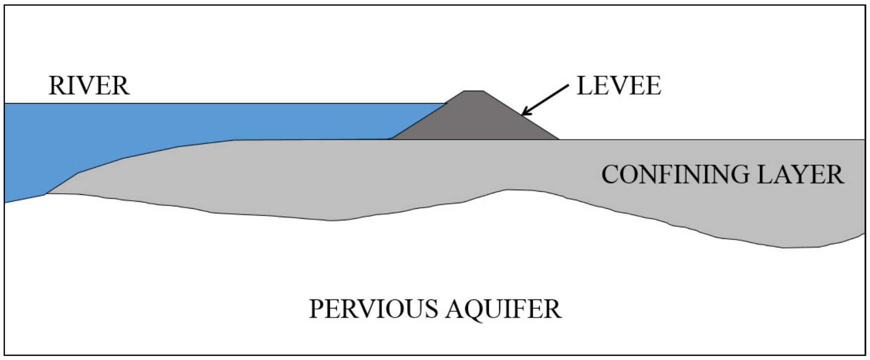

FIG 1. Schematic of Geologic Cross Section Common to Many Levee Systems.

Despite the increasing use of analysis by the finite element method (FEM) in levee design, semi-analytical solutions are still commonly used in practice, especially when simple first order methods of reliability analysis are being used for assessing flood risk. The primary advantage of using semi-analytical solutions for reliability analysis is the ease with which uncertainty in problem geometry (e.g., confining layer thickness) can be considered. To incorporate this uncertainty with FEM analysis would require multiple FEM models be constructed and interpreted, or the use of more sophisticated, random FEM (RFEM) models (e.g. Griffiths and Fenton 1993, Fenton and Griffiths 1993, Fenton and Griffiths, 2008). The disadvantage of semi-analytical solutions is the implicit assumption of uniform geologic conditions (constant layer thickness). This paper explores the influence that this assumption has on reliability calculations by comparing RFEM simulation results incorporating spatially variable soil profiles to the results of simple reliability analysis using blanket theory.

BLANKET THEORY REVIEW

Background

Blanket theory was originally developed by Bennet (1945) and further developed by Turnbull and Mansur (1961a). As noted by Batool et al. (2015), the derivation of blanket theory stems from the Method of Fragments (Harr, 1962). Therefore, numerous cases can be defined using different fragments to represent different boundary conditions. Turnbull and Mansur (1961a) defined seven distinct cases by considering different combinations of semi-pervious, impervious, or no blanket layer on the land and river side of the embankment. The case with a semi-pervious blanket on both the riverside and landside of the levee (case 7c), as shown in Figure 2, is one of the most common scenarios encountered in practice and will be the only scenario considered for this investigation. For a thorough review of all cases, the authors recommend reviewing Batool et al. (2015) and Meehan and Benjasupattananan (2012).

Problem Description

Blanket theory separates the under seepage problem into three components, i.e., the flow through the riverside blanket, the flow through the pervious foundation, and the flow through the landside blanket. It is assumed that the flow in the foundation is

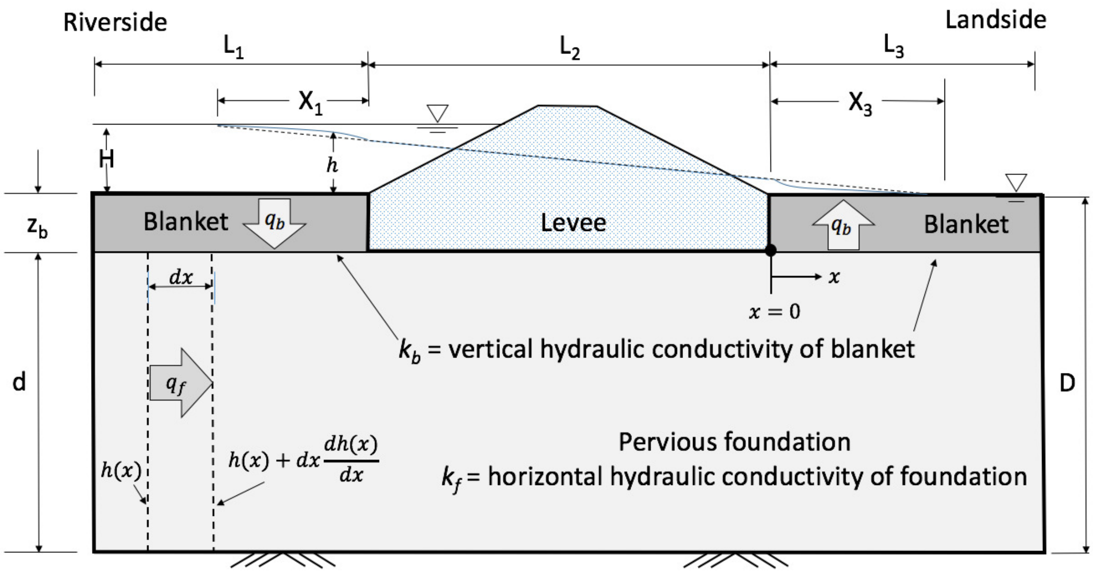

FIG. 2. Representation of Levee Geometry for Blanket Theory Derivation (modified from Batool et al., 2015).

horizontal, while the flow through the blanket is vertical. As noted by Bennet, this assumption is reasonable provided the ratio of the foundation permeability, k_f, to the blanket permeability, k_b, is greater than ten (i.e., $k_f/k_b > 10$). The foundation has a finite depth, *d*. The length of the riverside blanket, levee base, and landside blanket are denoted as L_1, L_2, and L_3, respectively. The top stratum has a thickness of z_b on both sides of the levee. The total head, referenced to the ground surface, varies from a value of *H* on the riverside boundary to a value of zero on the landside boundary. The total head at any point in the foundation is denoted as $h(x)$.

If the confining layers were impervious, the headloss would vary linearly from the seepage entrance to the seepage exit. However, the flow through the semi-pervious confining layers allows the head to reach equilibrium with the ground surface boundary over distances denoted as the effective seepage entrance length (X_1) and the effective seepage exit length (X_3). If the distance to the seepage entrance, L_1, is larger than X_1, then all head loss will occur over the effective seepage entrance length and X_1 will control the problem. Increasing L_1 beyond distance X_1 has negligible influence on the problem. Likewise, increasing L_3 beyond distance X_3 has negligible influence on the problem. If either L_1 or L_3 is smaller than X_1 or X_3, respectively, then the distance over which the headloss occurs is reduced, thereby increasing foundation gradients (and discharge quantities) and altering the excess head along the foundation profile. As such, X_1 and X_3 must be known to compute the exit gradient and flow quantities using blanket theory. The following section derives relationships for X_1 and X_3, $h(x)$ landward of the levee, and the total quantity of flow through the levee foundation.

Solution Derivation

Considering any x location in the foundation, the total quantity of flow passing beneath the blanket layer is given by

$$q_f = k\,A\,i = k_f d\frac{dh(x)}{dx} \quad \text{(Eq. 1)}$$

where *A* is the area ($d \cdot 1'$ cfs per unit width of levee), *i* is the hydraulic gradient, and *h(x)* denotes the total head with respect to some datum (taken as the ground surface). The vertical flow through a differential width *dx* of the landside blanket is

$$dq_b = k\,i\,dx = \frac{k_b \cdot h(x) \cdot dx}{z_b} \quad \text{(Eq. 2)}$$

and, from the continuity equation

$$\frac{dq_f}{dx} + \frac{dq_b}{dx} = 0 \quad \text{(Eq. 3)}$$

Taking the derivative of Equation 1 with respect to *x* yields

$$\frac{dq_f}{dx} = k_f d\frac{d^2h(x)}{dx^2} \quad \text{(Eq. 4)}$$

Substituting Equations. 2 and 4 into Equation 3 yields

$$\frac{d^2h(x)}{dx^2} = \frac{k_b \cdot h(x)}{z_b k_f d} \quad \text{(Eq. 5)}$$

Recognizing that the blanket layer thickness, foundation layer thickness, blanket permeability, and foundation permeability are all constants, Equation 5 becomes

$$\frac{d^2h(x)}{dx^2} - \alpha^2 h(x) = 0 \quad \text{(Eq. 6)}$$

where $\alpha = \sqrt{k_b/(z_b k_f d)}$. Equation 6 is a second order, linear, ordinary differential equation with constant coefficients for which the general solution is

$$h(x) = c_1 e^{\alpha x} + c_2 e^{-\alpha x} \quad \text{(Eq. 7)}$$

Applying the boundary conditions $h = h_{toe}$ at $x = 0$ and $h = 0$ at $x = L_3$, the coefficients c_1 and c_2 are

$$c_1 = h_{toe} - c_2 \quad \text{(Eq. 8)}$$

$$c_2 = \frac{h_{toe}e^{\alpha L_3}}{e^{\alpha L_3} - e^{-\alpha L_3}} \quad \text{(Eq. 9)}$$

Substituting Equations 8 and 9 into Equation 7 and recognizing that $\sinh(x) = \frac{e^x - e^{-x}}{2}$, $h(x)$ can be defined as

$$h(x) = h_{toe} \frac{\sinh(\alpha(L_3 - x))}{\sinh(\alpha L_3)} \quad \text{(Eq. 10)}$$

for positive values of *x,* with *x* increasing landward from *x*=0 at the landside levee toe (Figure 2). To determine the effective seepage exit length, X_3, the slope of $h(x)$ at *x*=0 is projected landward until $h(x) = 0$. The slope of $h(x)$ is

$$\frac{dh(x)}{dx} = h_{toe} \frac{\alpha \cosh(\alpha(L_3 - x))}{\sinh(\alpha L_3)} \quad \text{(Eq. 11)}$$

Evaluating the slope at $x = 0$, and extrapolating landward until $h(x = X_3) = 0$ yields

$$\frac{dh(0)}{dx} = h_{toe} \frac{\alpha \cosh(\alpha L_3)}{\sinh(\alpha L_3)} = \frac{h_{toe}\alpha}{\tanh(\alpha L_3)} = \frac{h_{toe}}{X_3} \quad \text{(Eq. 12)}$$

Solving for X_3 yields

$$X_3 = \frac{\tanh(\alpha L_3)}{\alpha} \quad \text{(Eq. 13)}$$

The derivation for X_1 is nearly identical to that of X_3, yielding an equation identical to Equation 13 (see Meehan and Benjasupattananan 2012 for full details). For this paper, $L_1 = L_3$, such that $X_1 = X_3$. Given the values of X_1 and X_3 calculated from Equation 13, the excess head at the levee toe can be computed through similar triangles as

$$h_{toe} = \frac{H\, X_3}{X_1 + L_2 + X_3} \quad \text{(Eq. 14)}$$

Using Equations 10, 13 and 14, the head can be computed for positive values of x. For complete details on calculating the excess head for all values of *x*, refer to Meehan and Benjasupattananan (2012). The total flow per unit width through the foundation (q_f) can be computed as

$$q_f = \frac{d\, k_f\, H}{X_1 + L_2 + X_3} \quad \text{(Eq. 15)}$$

Equation 14 can then be used to compute the factor of safety with respect to initiation of piping (heave).

$$FS = \frac{i_c}{i_e} = \frac{(\gamma'/\gamma_w)}{(h_{toe}/z_b)} \quad \text{(Eq. 16)}$$

If the blanket thickness z_b is treated as the only random input variable in this analysis, the variance in the factor of safety can be computed using the first order second moment (FOSM) method, as outlined by Wolff (1994) and Duncan (2000), where

$$\sigma_{FS}^2 = \left[\frac{FS(\mu_{z_b}+\sigma_{z_b})-FS(\mu_{z_b}-\sigma_{z_b})}{2}\right]^2 \quad \text{(Eq. 17)}$$

Where μ_{z_b} and σ_{z_b} represent the mean and standard deviation of z_b respectively. The variance computed with Equation 17 considers only the variance of z_b and not the spatial variability. The following section uses Monte-Carlo FEM analyses to examine the influence of spatial variability on the variance of the head at the toe and corresponding exit gradient for comparison.

RANDOM FINITE ELEMENT METHOD

Random fields have been combined with FEM (RFEM) to assess the influence of property variation on many types of geotechnical problems (Fenton and Griffiths, 2008). In many of these cases, the random field is used to represent physical soil properties; but in this particular investigation, only the blanket thickness is treated as a random variable.

Problem Description

The blanket thickness z_b, is assumed to be log-normally distributed with a mean value of μ_{z_b} and a standard deviation of σ_{z_b}. Except for the addition of this random variable and the consideration of a continuous blanket beneath the levee, all other aspects of the problem (geometry and material properties) are identical to the simplified, blanket theory analysis problem.

In a two-dimensional seepage analyses, the blanket thickness can be represented as a one-dimensional random field. The blanket thickness at each value of x was computed using a one-dimensional random field generator based on the local average subdivision method (Fenton and Vanmarcke, 1990). This method readily allows correlated sampling to be implemented such that differing frequencies of blanket variation can be generated as shown in Figure 3 through the definition of a spatial correlation length, θ. Large values of θ correspond to correlated sampling over large distances, such that segments of rather uniform thickness are obtained (Figure. 3b). Smaller values of θ cause z_b to vary at a higher frequency (Figure 3a), with $\theta = 0$ corresponding to a "white noise" process. Given the random blanket thickness, the foundation thickness at each location x was

$$D(x) = d + \mu_{z_b} - z_b(x) \quad \text{(Eq. 18)}$$

such that the FEM ground surface elevation was fixed to the quantity $(d + \mu_{z_b})$.

The problem geometry obtained from the random field generator (for each simulation) was then overlaid onto a FEM mesh with 2 ft square elements. Elements were classified as blanket material if the element centroid was located above the random field line and foundation material if the element was below the random field line. The finite element problem was then solved yielding head quantities and flows. The FEM solution for the geometry shown in Figure 3b is illustrated in Figure 4.

To quantify the variance in the computed head and flow quantities, a RFEM analysis with 900 simulations was conducted. Each simulation led to a different value of the flow quantity and the exit head from which the mean and standard deviation of these output quantities could be computed. All possible combinations of the independent variable values listed in Table 1 were evaluated resulting in 108 different RFEM analyses being conducted for a total of 97,200 finite element problems being solved.

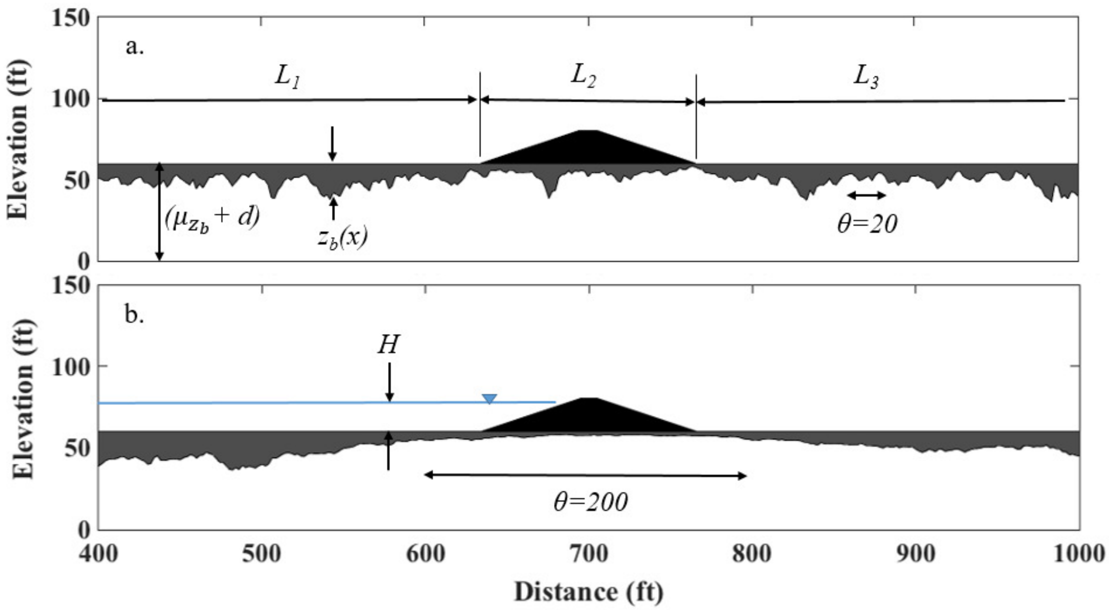

FIG. 3. Sample z_b Realizations with L_1=L_3=634 ft, L_2=132 ft, μ_{z_b}=10 ft, σ_{z_b}=5 ft, d=50 ft, for Spatial Correlation Lengths (θ) of (a) 20 ft and (b) 200 ft.

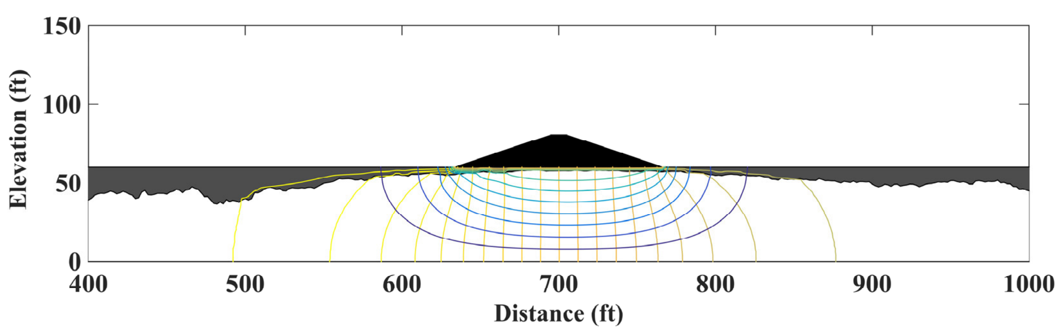

FIG. 4. FEM Solution for the Geometry Shown in FIG. 3b.

Table 1. Values of Independent Variables Considered in Analyses.

Variable	Values
L_1 (ft)	634
L_2 (ft)	132
L_3 (ft)	634
H (ft)	20
μ_{z_b} (ft)	10, 20
σ_{z_b} (ft)	2, 5
d (ft)	10, 50, 100
k_f (ft/s)	4.92×10^{-3}
k_b (ft/s)	4.92×10^{-4}, 4.92×10^{-6}, 4.92×10^{-8}
θ (ft)	20, 50, 200

FEM Analyses Results

The pressure head beneath the blanket, blanket thickness, and hence the exit gradient were calculated for each FEM analyses using the mesh node at the interface of the blanket and foundation materials along a vertical line passing through the landside levee toe as illustrated in Figure 5. The blanket thickness was the difference between the elevation of the interface node and the ground surface. The exit gradient was then calculated using the nodal pressure head and computed blanket thickness. Because 900 realizations were run for each of the RFEM analyses, 900 values of the exit gradient were computed from each analysis. The resulting distributions of computed quantities for the analysis shown in Figure 5 are provided in Figure 6.

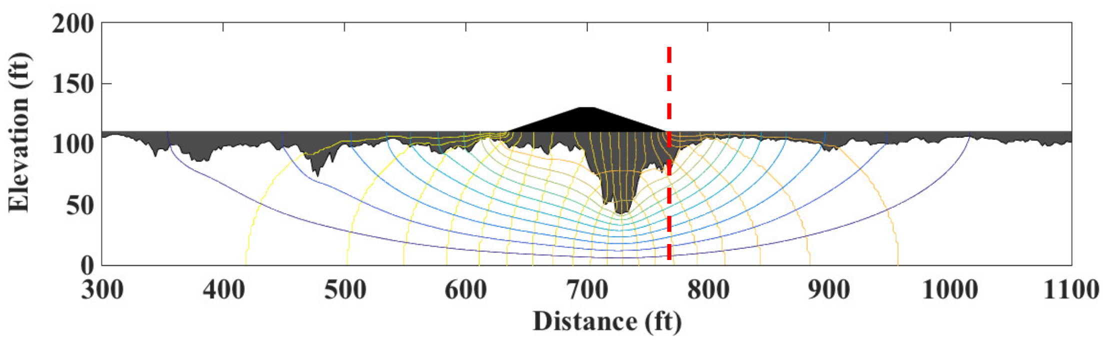

FIG. 5. FEM Realization with L_1=L_3=634 ft, L_2=132 ft, μ_{z_b}=10 ft, σ_{z_b}=5 ft, d=100 ft, and θ=50 ft.

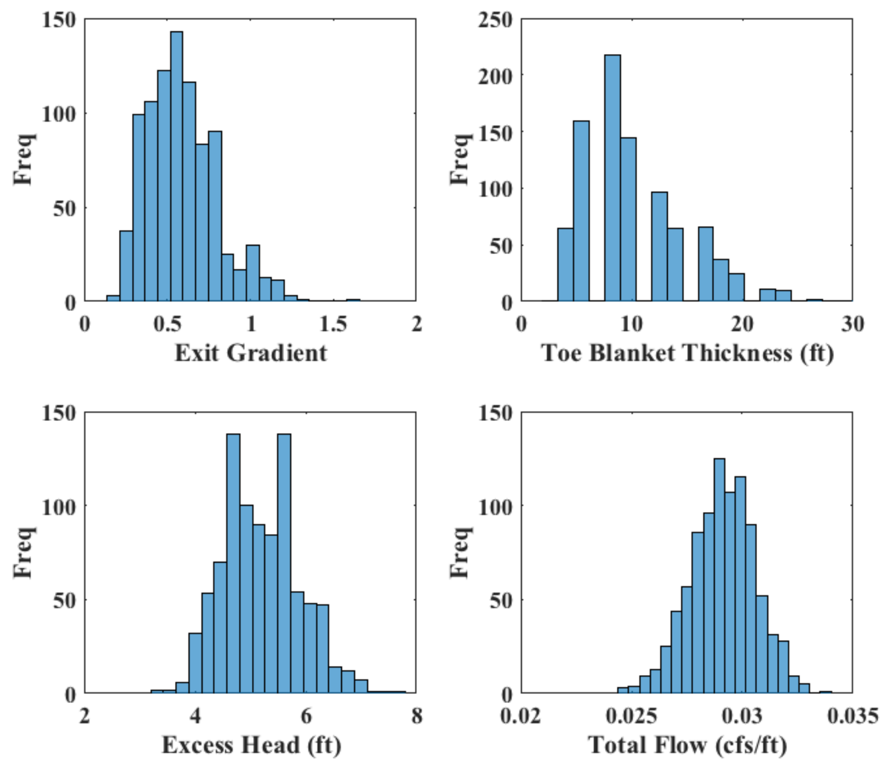

FIG. 6. Distributions of Exit Gradient, Blanket Thickness, Excess Head, and Total Flow Through Section from RFEM Analysis Shown in FIG. 5.

Values of interest from the FEM analyses include the mean and standard deviation of the following quantities:

$\mu_{h_{toe}}, \sigma_{h_{toe}}$ mean and standard deviation of excess head at the bottom of the blanket directly beneath the levee toe

μ_{i_e}, σ_{i_e} mean and standard deviation of exit gradient

μ_{Q_T}, σ_{Q_T} mean and standard deviation of total flow through the section

μ_{Q_b}, σ_{Q_b} mean and standard deviation of total flow exiting the landside blanket

The above quantities were calculated for all 108 combinations of independent variables. To quantify the influence of θ on the simulation results, the linear correlation coefficient was computed between θ and the mean and standard deviation of the above quantities yielding the results presented in Table 2. The results indicate that, for the cases analyzed, θ only has an influence on the standard deviation of the total flow and the flow exiting through the blanket. To further evaluate the influence of θ on the flow quantities, the distributions of computed flows from two analyses in which only θ differed were compared (Figure 7). As shown, increasing θ leads to increasing variance in computed total flow all else equal.

Table 2. Linear Correlation Coefficients Between θ and the Monte Carlo FEM Result Statistics.

	$\mu_{h_{toe}}$	$\sigma_{h_{toe}}$	μ_{i_e}	σ_{i_e}	μ_{Q_T}	σ_{Q_T}	μ_{Q_b}	σ_{Q_b}
θ	0.05	0.04	0.06	0.06	0.00	0.25	0.00	0.21

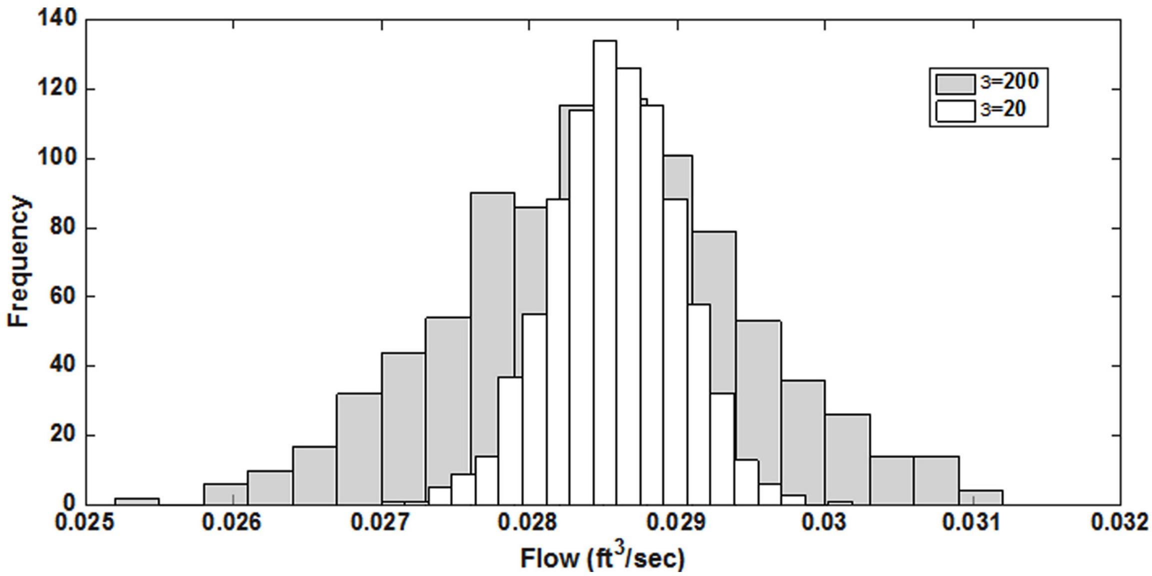

FIG. 7. Distribution of Total Flow Through Section for θ=20 ft and θ=200 ft with d=100 ft, μ_{z_b}=10 ft, σ_{z_b}=2 ft, and k_b= 4.92×10^{-4} ft/s.

DISCUSSION

To determine the error associated with neglecting spatial variability in the simplified FOSM reliability analysis using blanket theory, the mean and standard deviation of the exit gradient were calculated using Equations 8 through 12 for all 108 cases assessed. The computed exit gradients were selected for further comparison as the exit gradient has typically been the performance criteria for reliability analyses. The values computed using blanket theory were compared to the mean and standard deviations computed from the FEM analyses (Figure 8). The largest deviation between FEM and blanket theory results is seen for analyses in which *d*=10 ft and σ_{z_b}=5 ft. The reason for the significant deviation is due to the blanket thickness exceeding the aquifer depth at locations (Figure 9) creating a seepage cut-off. This condition invalidates all of the assumptions of blanket theory, thus explaining the error in the blanket theory results. With the exception of the cases with *d*=10 ft and σ_{z_b}=5 ft, all other FEM results yield nearly the same values as blanket theory using FOSM reliability analysis. The following paragraphs compare the cumulative distribution functions for the cases resulting in the maximum deviation (without cut-offs) in order to visually observe the significance of the differences obtained.

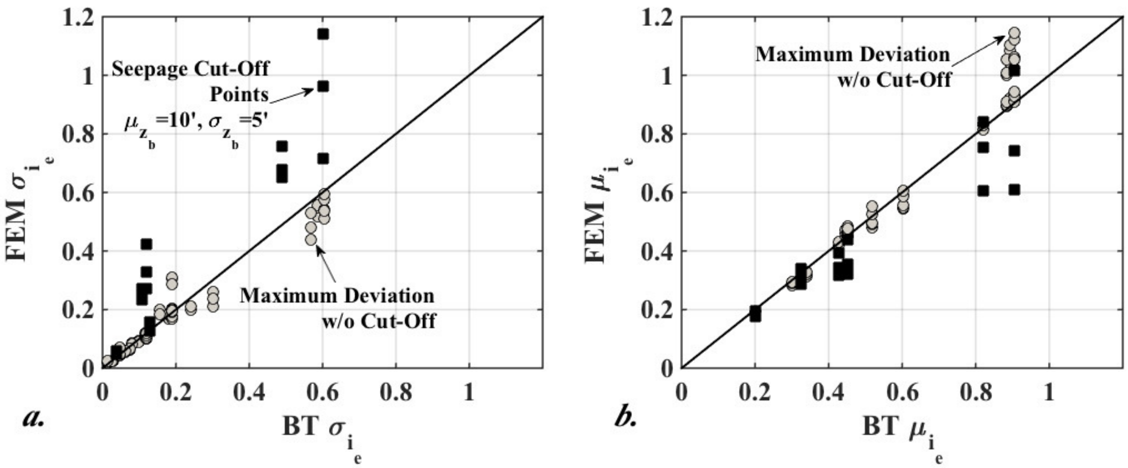

FIG. 8. Comparison of (a) Standard Deviation of Exit Gradient and (b) Mean Value of Exit Gradient Obtained from Blanket Theory (BT) and RFEM (FEM).

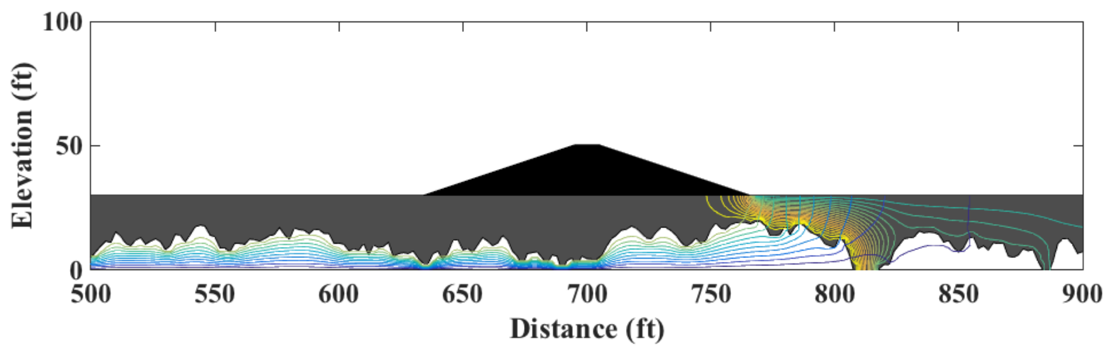

FIG. 9. Example Realization for Cases with d=10 ft and σ_{z_b}=5 ft. The blanket is the full thickness of the aquifer in places creating seepage cut-offs.

The analyses that yielded the largest difference in σ_{i_e} between blanket theory and the FEM analysis was the case with d=50 ft, μ_{z_b}=10 ft, σ_{z_b}=5 ft, θ=20 ft, and k_b=4.92× 10^{-6} ft/s (Case A). The analyses that yielded the largest difference in μ_{i_e} between blanket theory and the FEM analysis was the case with d=50 ft, μ_{z_b}=10 ft, σ_{z_b}=5 ft, θ=200 ft, and k_b=4.92× 10^{-8} ft/s (Case B). For both FEM analyses, the distribution of factor of safety is calculated as

$$FS_j = i_c/i_{e_j} \quad, j = 1, 2, \ldots, n \qquad \text{(Eq. 19)}$$

where n = 900. The value of i_c was taken as 0.92, which corresponds to a soil with a saturated unit weight of 120 pcf.

The blanket theory mean and standard deviation of the factor of safety was computed using Equations 13, 14, 16 and 17 as follows. As an example, the mean factor of safety for Case A was computed as

$$\alpha = \sqrt{k_b/(\mu_{z_b} k_f d)} = \sqrt{(4.92 \times 10^{-6}/(10\ (4.92 \times 10^{-3})50)} = 0.0014 \text{ ft}^{-1}$$

$$X_3 = X_1 = \frac{\tanh(\alpha L_3)}{\alpha} = \frac{\tanh(0.0014(634))}{0.0014} = 505 \text{ ft}$$

$$\mu_{h_{toe}} = \frac{H\,X_3}{X_1+L_2+X_3} = \frac{20*505}{505+132+505} = 8.84 \text{ ft}$$

$$\mu_{FS} = \frac{i_c}{\mu_{i_e}} = \frac{i_c}{(\mu_{h_{toe}}/\mu_{z_b})} = \frac{0.92}{8.84/10} = 1.04$$

Performing the same calculations two more times with z_b increased and decreased by one standard deviation leads to factors of safety of 1.55 and 0.53. Substituting these values into Equation 13, the variance of the factor of safety can be calculated as

$$\sigma_{FS}^2 = \left[\frac{1.55-0.53}{2}\right]^2 = 0.26$$

Assuming the blanket theory factor of safety is normally distributed, the resulting cumulative distribution function for the mean and standard deviation computed above is compared to the empirical cumulative distribution function for all 900 FEM results in Figure 10a. The same procedure was followed for Case B, with the results shown in Figure 10b. From these examples, it is readily observed that FOSM blanket theory computations adequately describe the variability in exit gradients even when the confining layer thickness varies spatially.

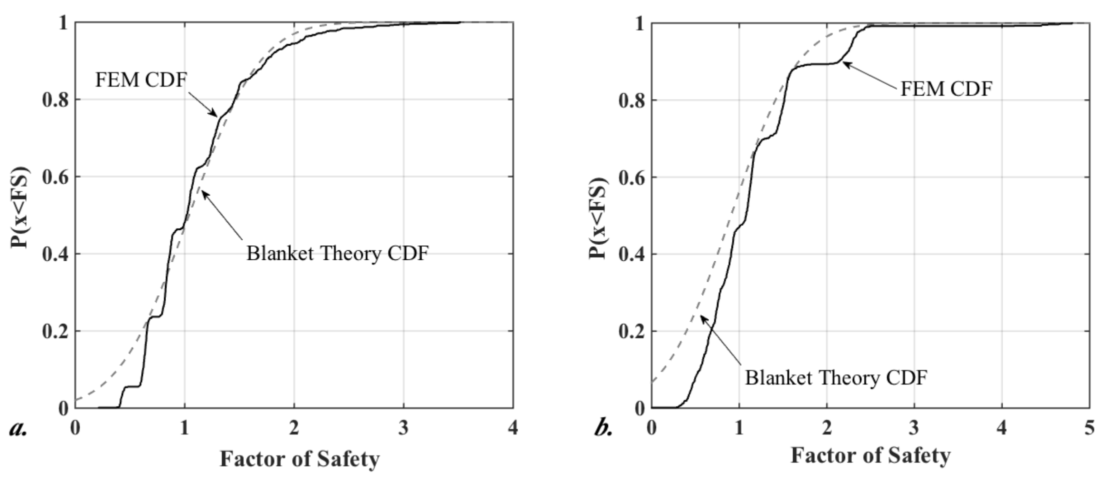

FIG. 10. Comparison of the Cumulative Distribution Functions of Uplift Factor of Safety at the Levee Toe Computed from the FEM Analyses and Blanket Theory for Cases of Maximum Deviation as Shown in FIG. 8.

CONCLUSIONS

RFEM analyses were conducted to evaluate the influence of spatial variation in confining layer (blanket) thickness on levee under seepage calculations. The spatial variation in blanket thickness was simulated using a one dimensional random field generator with spatially correlated sampling. The results of the random finite element analysis were compared to the results of FOSM reliability analysis using blanket theory as the performance function. Results indicate that the FOSM results adequately approximate the cumulative distribution of exit gradients and uplift factors of safety provided two distinct layers exist across the entire length of the model.

Permission to publish was granted by Director, Geotechnical and Structures Laboratory.

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SYMBOLS

A = area of flow per unit width (ft^2/ft)
d = thickness of pervious layer beneath blanket (ft)
h = excess head above the ground surface (ft)
h_{toe} = excess head at the landside levee toe (ft)
H = head loss across the model (ft)
i = hydraulic gradient (-)
i_c = critical vertical hydraulic gradient (-)
k_b = vertical hydraulic conductivity in confining layer (ft/sec)
k_f = horizontal hydraulic conductivity in foundation (ft/sec)
L_1 = length from riverside boundary to riverside levee toe (ft)
L_2 = width of levee base (ft)
L_3 = length from landside levee toe to landside boundary (ft)
q_b = flow through the blanket (ft^3/sec-ft)
q_f = flow through the foundation (ft^3/sec-ft)
q_T = total flow through the section (ft^3/sec-ft)
z_b = blanket thickness (ft)
γ' = buoyant unit weight of soil (pcf)
γ_w = unit weight of water (pcf)
μ_X = mean value of random variable X
θ = spatial correlation length (ft)
σ_X = standard deviation of random variable X