# Probabilistic Finite Element Analysis of a Raft Foundation Supported by Drilled Shafts in Karst

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## ABSTRACT

The paper describes probabilistic analyses performed as part of a large expansion to an existing cement manufacturing plant. A raft supported by drilled shafts was proposed for the project, but during installation, significant slurry and concrete loss began to occur indicating numerous voids existed in what was previously considered competent limestone bedrock. Since the possibility of voids, especially at the shaft tip, could serious reduce the shaft capacity, a probabilistic Monte Carlo 3D finite element simulation was proposed for the most heavily loaded raft foundation. The purpose of the simulation was to determine the probability of adverse performance, giving guidance as to whether any remedial measures (e.g., additional structural elements or thickened raft) might be required.

## INTRODUCTION

This paper details analysis and results obtained from 3D finite element deterministic and probabilistic studies of a heavily loaded area of a factory expansion project.

The foundation for a large expansion to a manufacturing facility was under construction. The proposed foundation described in this paper was to consist of a reinforced concrete raft foundation, 9.5 feet thick, supported by 5 ft diameter shafts founded in competent rock as shown in plan-view in Figure 1. During construction of the shafts, karst was discovered when large quantities of drilling slurry were lost during drilling of some of the rock sockets. A subsequent, detailed subsurface investigation of the rock beneath the remainder of the shafts to be constructed, revealed that 35% of the exploratory borings contained randomly distributed voids that were expected to significantly undermine the tip resistance of some of the shafts. Little was known about the locations of the karst voids underlying shafts that had been successfully constructed, so a probabilistic approach was adopted to determine the effect of the voids on the performance of the raft.

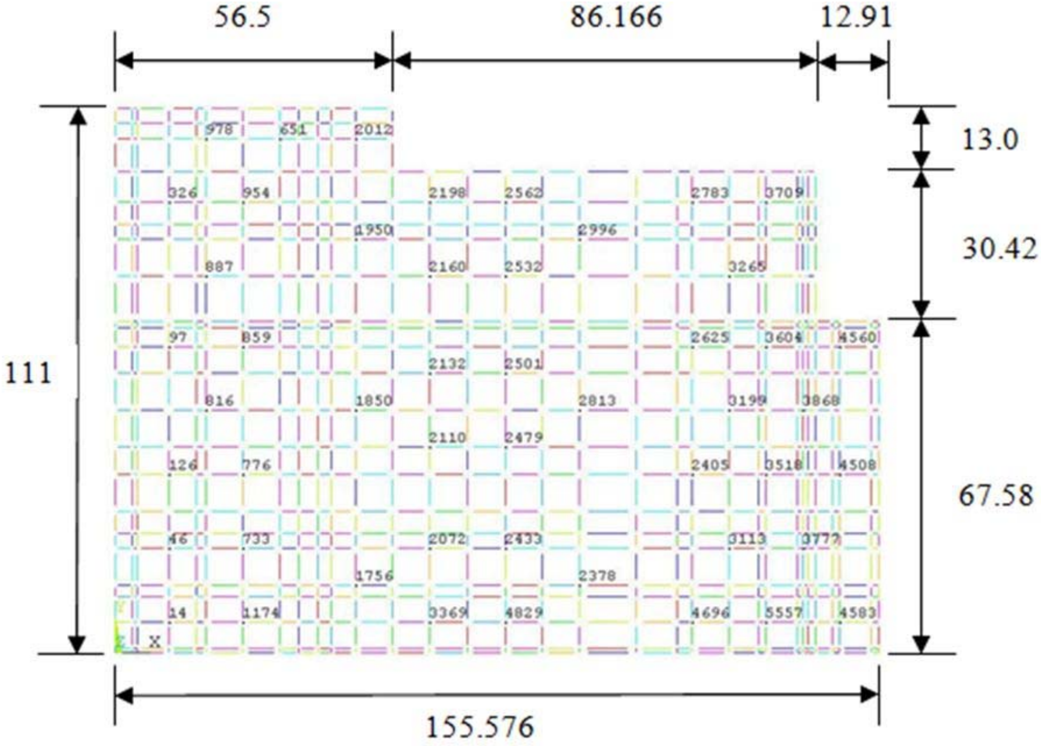

**Figure 1. Plan view of finite element mesh for raft showing the numbered locations of shafts (dimensions in ft).**

The analyses were performed using the software suite described by Smith and Griffiths (2008) which can be downloaded from the authors web site at `www.mines.edu/~vgriffit/4th_ed` In particular, Program 5.4 from that system, which models 3D elastic solids using 20-node hexahedral elements, was modified to include vertical springs to represent the supporting shafts. The springs were assigned one of two possible stiffness values. A shaft founded on competent rock was assigned twice the stiffness of a shaft founded in a void. The rationale for this 2:1 ratio was that results from an O-test performed on a shaft in competent rock, which indicated a similar stiffness contribution from the tip and the sides as shown in Figure 2. A conservative value of about 89,000 kip/ft was selected as the spring stiffness for a shaft in competent rock.

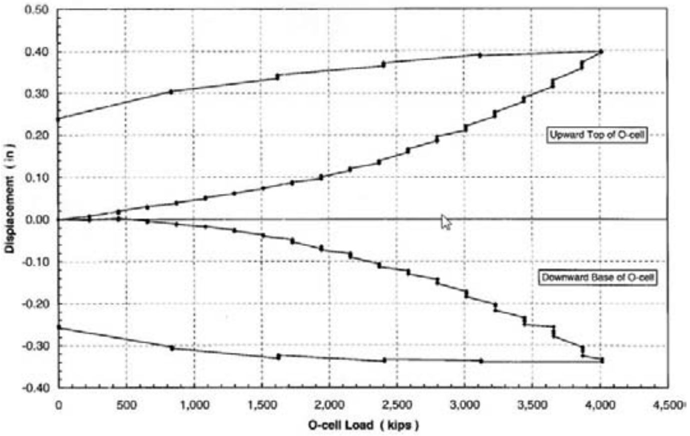

**Figure 2. Results of O-test performed on a shaft founded in competent rock.**

## FOUNDATION ANALYSES

Referring to Figure 1, the nodes signified by a small dot and a number indicate the plan locations of the 49 shafts situated below the raft. The thickness of the raft was 9.5 ft, and two rows of elements were included in the depth direction. The full model consisted of 1210 20-node hexahedral elements and 7084 nodes. Reduced Gaussian integration (8 sampling points per element) was used to generate the stiffness matrix of the raft after which the shaft spring stiffnesses were added to the appropriate diagonal terms of the matrix. The properties used for the analysis are given in Table 1.

**Table 1. Data for raft/shaft foundations.**

| | |
|---|---|
| Young’s modulus of slab | 453,600.0 kip/ft$^2$ |
| Poisson’s ratio of the slab | 0.17 |
| Likelihood of tip hitting a void | 35% |
| Shaft stiffness (tip in competent rock) | 89,000 kip/ft |
| Shaft stiffness (tip in void) | 44,500 kip/ft |
| Total vertical load on the raft | 47,870 kip |

## LOADING

Vertical forces were applied at numerous locations on the slab as shown in Figure 3, in addition to two moments to model wind loading on the superstructure. The figure also shows the location of two fixed boundary nodes to eliminate horizontal movement and rotation of the raft about a vertical axis.

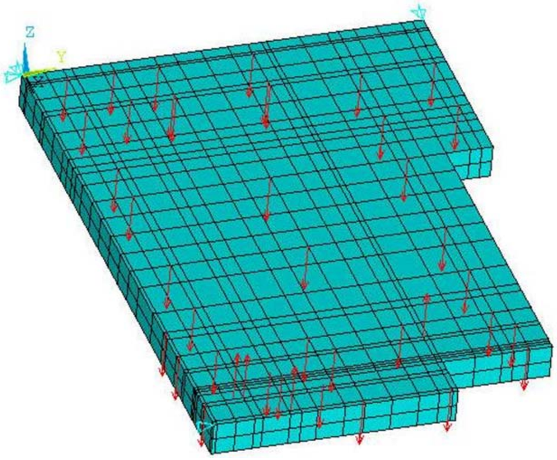

**Figure 3. Finite element mesh for raft showing applied loads and fixed nodes.**

## DETERMINISTIC ANALYSES

Two deterministic analyses were initially performed; one in which shafts were assigned stiffness corresponding to intact rock (89,000 kip/ft) and one in which all shafts were assigned stiffness corresponding voids (44,500 kip/ft). These analyses established lower and upper bounds on raft displacements.

**Results.** With all springs set to 89,000 kip/ft a maximum vertical displacement of 0.0275 ft (0.33 in) was obtained in close agreement with an independent analysis using the package STAAD that gave 0.32 in. The maximum in-plane moments back-figured from the stresses across the slab sections were $M_x = 575$ kip ft/ft and $M_y = 1301$ kip ft/ft. The corresponding displacement with all springs set to 44,500 kip/ft was 0.0455 ft (0.55 in) with essentially the same maximum moments.

## STOCHASTIC ANALYSES

Probabilistic analysis of foundation settlement is a rapidly growing area of interest to engineers and academics (e.g. Baecher and Ingra 1981, Beacher and Christian 2003, Brzakała and Puła 1996).

The stochastic analyses described in this paper used a standard normal distribution to assign a reduced stiffness of 44,500 kip/ft to 35% of the shafts and the full stiffness of 89,000 kip/ft to 65% of the shafts. There are 49 shafts and the loads were the same as in the deterministic analyses. The analysis with stochastic spring stiffnesses was repeated 1600 times using Monte-Carlo simulations. With the assumption of independence between shafts (i.e. one shaft encountering a void would not change the likelihood of one of its neighbors also encountering a void), the analysis is a Bernoulli process with a mean of $49 \times 0.35 = 17$ and a standard deviation of $\sqrt{49 \times 0.35 \times (1 - 0.35)} = 3.34$ as shown in the histogram of Figure 4.

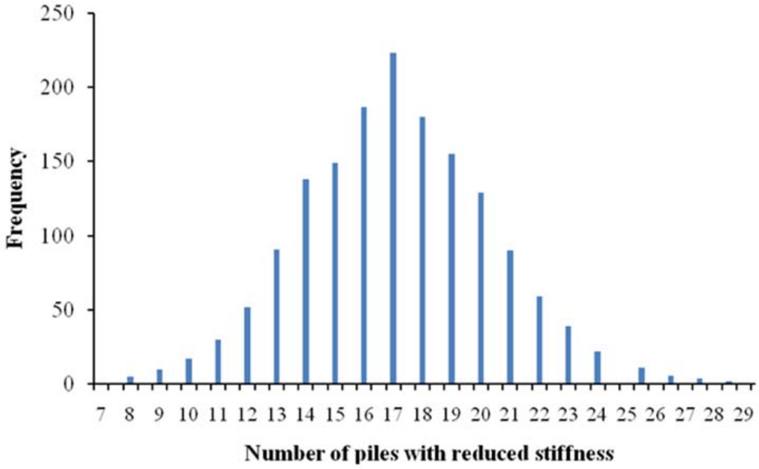

**Figure 4. Histogram of number of shafts with reduced stiffness under raft Following 1600 Monte-Carlo simulations.**

From each simulation, the maximum vertical displacement and the maximum in-plane moments in the slab in two directions were recorded. Following the suite of Monte-Carlo simulations, histograms of these quantities were plotted and their means and standard deviations computed.

**Maximum Vertical Displacement.** Figure 5 shows a histogram of the maximum vertical displacement. In the vast majority of simulations it occurred at the same place as in the deterministic analysis. A summary of statistical results for the maximum displacement following 1600 Monte-Carlo simulations is given in Table 2.

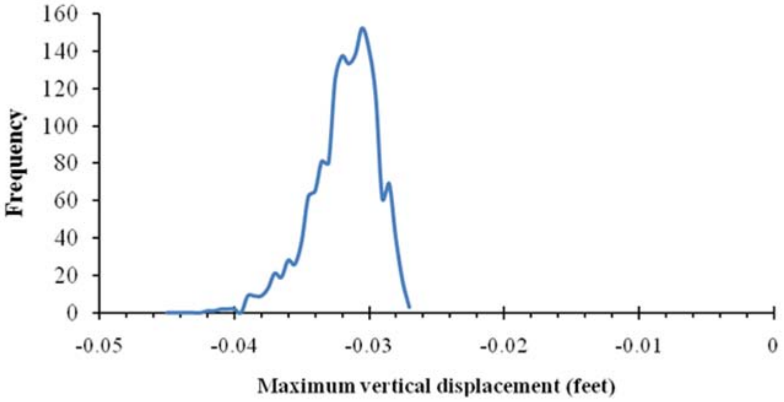

**Figure 5. Histogram of the maximum vertical displacement in the raft following 1600 Monte-Carlo simulations.**

**Table 2. Statistical output for the maximum vertical displacement in the raft.**

| | |
|---|---|
| Mean | $\mu_{\delta(\max)} = 0.0321$ ft (0.385 in) |
| Standard deviation | $\sigma_{\delta(\max)} = 0.0024$ ft (0.029 in) |
| Minimum | $\delta_{(\min)} = 0.0273$ ft (0.327 in) |
| Maximum | $\delta_{(\max)} = 0.0423$ ft (0.507 in) |

It can be noted that the coefficient of variation of the maximum displacement is $v_{\delta(\max)} = \sigma_{\delta(\max)} / \mu_{\delta(\max)} = 0.07$, or 7% which is quite low, presumably due in part to the stiff 9.5 ft thick reinforced concrete raft.

**Moments.** Figures 6 and 7 show histograms of the maximum moments in the raft following 1600 Monte-Carlo simulations.

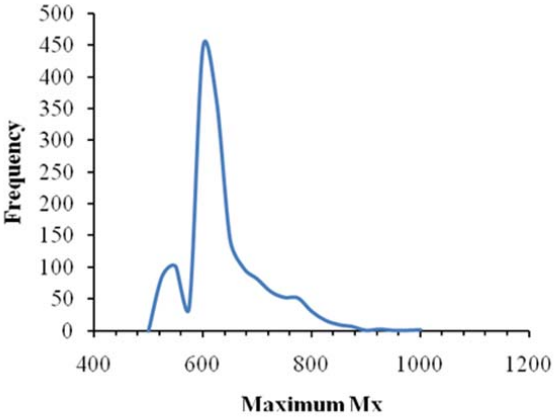

**Figure 6. Histogram of the maximum $M_x$ moment in the raft following 1600 Monte-Carlo simulations (moments in kip ft/ft)**

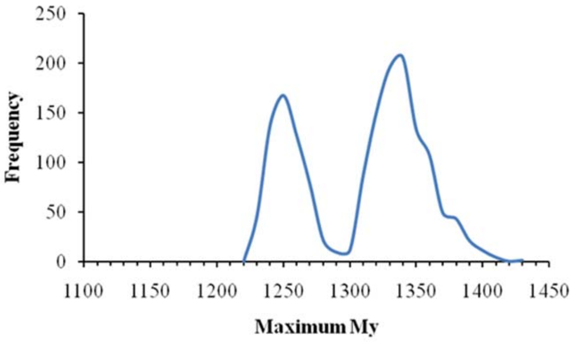

**Figure 7. Histogram of the maximum $M_y$ moment in the raft following 1600 Monte-Carlo simulations (moments in kip ft/ft).**

A summary of statistical results for the maximum $M_x$ and $M_y$ moments in the raft following 1600 Monte-Carlo simulations is given in Table 3.

**Table 3. Statistical output for the maximum $M_x$ and $M_y$ moments in the raft.**

| | | |
|---|---|---|
| Mean | $\mu_{M_{x(\max)}} = 626$ kip ft/ft | $\mu_{M_{y(\max)}} = 1304$ kip ft/ft |
| Standard deviation | $\sigma_{M_{x(\max)}} = 72$ kip ft/ft | $\sigma_{M_{y(\max)}} = 46$ kip ft/ft |
| Minimum | $M_{x(\max)\max} = 978$ kip ft/ft | $M_{y(\max)\max} = 1422$ kip ft/ft |
| Maximum | $M_{x(\max)\min} = 509$ kip ft/ft | $M_{y(\max)\min} = 1224$ kip ft/ft |

It can be noted that the coefficients of variation of the maximum moments are given as $v_{M_{x(\max)}} = \sigma_{M_{x(\max)}} \big/ \mu_{M_{x(\max)}} = 0.12$ and $v_{M_{y(\max)}} = \sigma_{M_{y(\max)}} \big/ \mu_{M_{y(\max)}} = 0.04$

While the variability of $M_{x(max)}$ is relatively higher than $M_{y(max)}$, it can still be concluded that the level of variability of the maximum moment in both directions is low, with a worst-case coefficient of variation of about 12%.

The histogram of $M_{y(max)}$ shown in Figure 7 displays an unusual bi-modal distribution which might warrant further investigation. At this stage the bi-modal behavior is thought to be due to a nearby critical underlying shaft that causes the moment to switch between the two values depending on whether its stiffness is high or low.

## CONCLUDING REMARKS

Deterministic and stochastic analyses have been performed on the raft foundation of a factory expansion project. The motivation for the analyses was that randomly located voids in the underlying karst could lead to significantly reduced shaft resistance. Monte-Carlo simulations were performed with the assumption that 35% of shafts had no tip resistance and would have side resistance only.

An initial deterministic analysis gave very close agreement for the maximum displacement with previously obtained results using the STAAD package. Stochastic analyses on the raft with randomly reduced shaft stiffness gave a quite low coefficient of variation of the maximum displacement of about

7%. The coefficients of variation of the maximum in-plane moments were somewhat higher with a maximum of 12% for the moment about the x-direction.

Subsequent analyses could be performed using these values to estimate the probability of threshold displacements or moments being exceeded. The low variations observed for these quantities however suggest that this might be of marginal value, since relatively small safety margins would lead to extremely low probabilities of unacceptable performance. Ultimately, based on the probabilistic analyses described in this paper, it was concluded that poor performance of the raft foundation was unlikely and that the project could continue without extensive mitigation measures such as grouting of the underlying karst or installation of additional drilled shafts.

## ACKNOWLEDGEMENTS

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