Probabilistic Design of Slopes in Normally Consolidated Clays

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Abstract

Natural soils always exhibit spatial variability with the properties varying from point to point. The majority of probabilistic slope stability analyses model strength parameters as stationary random fields, i.e. the mean and standard deviation of strength are constant everywhere. Due to the influence of the effective overburden pressure however, normally consolidated soils regularly display an increasing undrained strength trend with depth. Using recently published deterministic solutions as a benchmark, this paper uses random finite element method (RFEM) to investigate the influence of spatial variability on the undrained stability of non-stationary random slopes, where the mean strength increases linearly with depth while the coefficient of variation remains constant. Results are presented in the form of charts that give the mean and standard deviation of the dimensionless stability number. By reading the charts presented in this paper, engineers can obtain a preliminarily assessment of the probability of slope failure for normally consolidated clay slopes.

INTRODUCTION

In the world of probabilistic geotechnical analysis, slope stability analysis seems to have received more attention in the literature than any other geotechnical application. Important early contributions appeared in the 1970s (e.g., Matsuo and Kuroda 1974; Alonso1976; Tang et al. 1976; Vanmarcke 1977). Recognition that geotechnical engineering is highly amenable to probabilistic treatment goes back much further. In his foreword to the inaugural issue of *Géotechnique* in 1948, Karl Terzaghi talked about the properties of the soil material varying

"*...from point to point.*" Various probabilistic tools have subsequently been developed for tackling probabilistic geotechnical analysis, such as event trees, first order second moment (FOSM) method and first order reliability method (FORM) (e.g., Whitman 1984; Wolff 1996; Lacasse 1994; Christian et al. 1994; Hassan and Wolff 1999; Duncan 2000).

It is only recently however, that Terzaghi's observation of spatially varying soil properties has been tackled explicitly by an advanced numerical method called the Random Finite Element Method (RFEM), with initial application to seepage problems (Griffiths and Fenton 1993; Fenton and Griffiths 1993), and later to slope stability analysis (e.g., Griffiths and Fenton 2000, 2004; Griffiths et al. 2009). In these studies, slope stability was investigated systematically using the finite element method combined with random field theory in a Monte-Carlo framework. The random fields were generated by the Local Averaging Subdivision (LAS) method (Fenton and Vanmarcke 1990) which can account for spatial variability and local averaging over each finite element. All the RFEM analyses mentioned previously considered slopes with stationary random properties, i.e. the mean and standard deviation of strength are constant everywhere. Due to the influence of the effective overburden pressure however, normally consolidated clays regularly display an increasing undrained strength trend with depth. Hicks and Samy (2002) considered some non-stationary random slopes using RFEM for the special case of zero strength at the ground surface (e.g., Gibson and Morgenstern 1962), while this paper will present some results for slopes in which the mean strength increases linearly with depth from a non-zero value at the ground surface (e.g., Hunter and Schuster 1968).

The geometry and parameters for this problem are shown in Fig. 1, where the slope angle is β, the slope height is H and the foundation depth ratio is D. The mean undrained strength is a linear function of depth given by

$$\mu_{c_{uz}} = \mu_{c_{u0}} + \rho z \tag{1}$$

where $\mu_{c_{uz}}$ is the mean strength at depth z; $\mu_{c_{u0}}$ is the mean strength at the ground surface and ρ is the strength gradient. In this paper a constant coefficient of variation v_{c_u} is assumed. For the random field modeling, a dimensionless and isotropic spatial correlation length $\Theta = \theta / H$ is

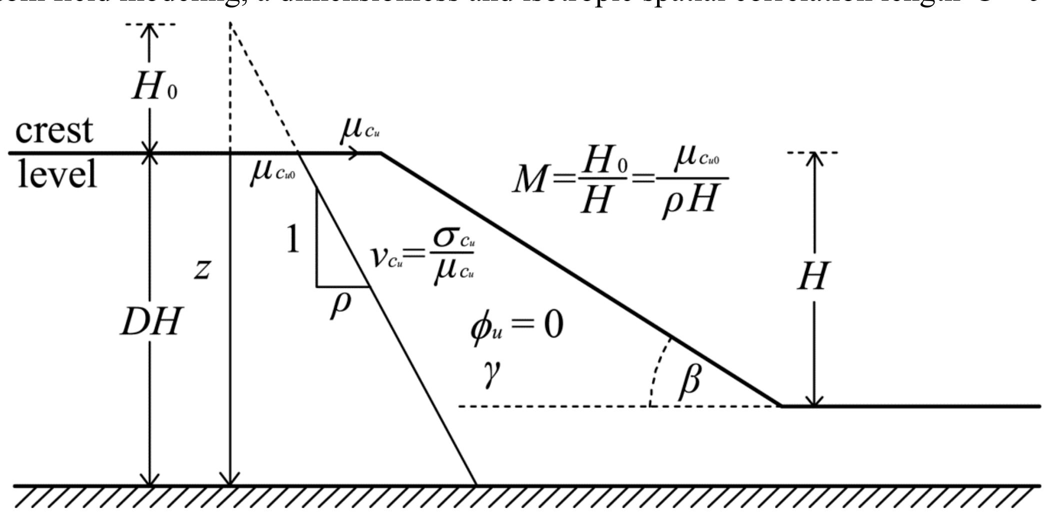


Figure 1. Undrained slope with linearly increasing mean strength.

used for parametric studies. The spatial correlation length may exhibit anisotropy, especially in the horizontal direction. For simplicity, the assumption of isotropy is made throughout this paper and the option relating to anisotropy may be a topic for future studies. Other deterministic parameters are the undrained friction angle $\phi_u = 0$ and the saturated unit weight γ.

NON-STATIONARY RANDOM FIELD GENERATION

With reference to Fig. 1, in order to generate a random field with the properties given by Eq. (1)

Step 1

Initially generate a homogeneous stationary lognormal random field across the mesh based on the parameters at $z = 0$, i.e. mean $\mu_{c_{u0}}$, coefficient of variation v_{c_u} and dimensionless spatial correlation length $\Theta = \theta / H$. Let the initial values assigned to all elements at this stage be c_{0i}, $i = 1,\ 2,\cdots,\ n$ where n is the number of elements in the mesh.

Step 2

The element values are then adjusted to account for other depths using the scaling factor

$$c_{zi} = c_{0i} \frac{\mu_{c_{u0}} + \rho z}{\mu_{c_{u0}}},\ i = 1,\ 2,\ldots,\ n \tag{2}$$

where z is sampled at the centroid of each element.

Figure 2 shows a typical realization of a random field with the properties indicated in the figure caption. Dark and light regions depict high and low values of soil strength, respectively. It can be seen from Fig. 2 that high values of strength are more likely to occur at greater depths (higher values of z).

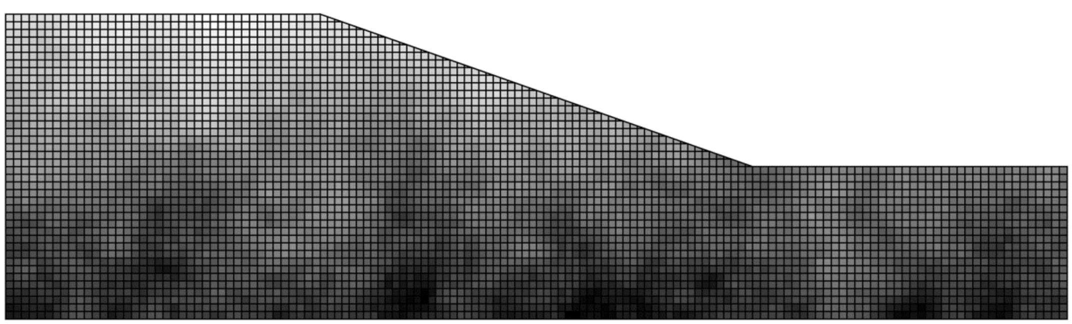


Figure 2. A typical realization of a random field for a slope with $\beta = 20°$, $M = 1.0$, $D = 2.0$, $v_{c_u} = 0.1$ and $\Theta = 1.0$.

RESULTS

In the framework of deterministic analysis of the problem shown in Fig. 1, Hunter and Schuster (1968) presented charts to facilitate calculation of the factor of safety, which have been recently refined by Griffiths and Yu (2015). The fundamental solution is a stability number N expressed as

$$N = f\left(\beta, D, M\right) \tag{3}$$

where M is a dimensionless strength gradient parameter defined as

$$M = \frac{\mu_{c_{u0}}}{\rho H} = \frac{H_0}{H} \tag{4}$$

Koppula (1984) collected published field test result for ρ, with typical values in the range $0 < \rho < 3.5\ \mathrm{kN/m^3}$. This led to a dimensionless parameter $c_\mathrm{R} = 1/M$ in the range $0.1 < c_\mathrm{R} < 5$, which cover the selected M in the current study.

The factor of safety FS is equal to the ratio of the restoring moment to the overturing moment. By using an optimisation approach, the minimum factor of safety can be obtained. Then the factor of safety FS can be expressed by

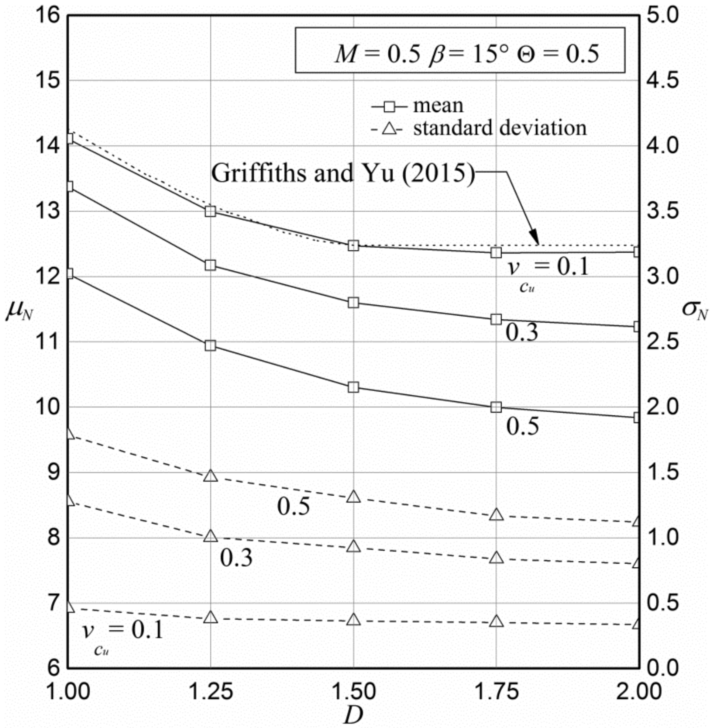
$$FS = N\frac{\rho}{\gamma} \tag{5}$$

from which the stability number can be computed as

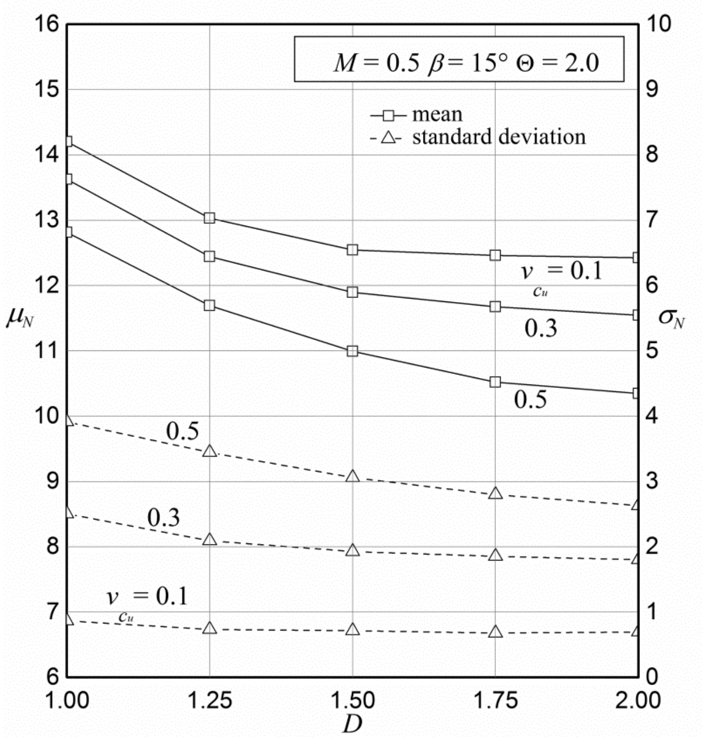
$$N = FS\frac{\gamma}{\rho} \tag{6}$$

In the context of a Monte-Carlo analysis, each realization of an RFEM analysis of the problem shown in Fig. 1 involves generation of a non-stationary random field as described in the previous section, followed by a conventional deterministic slope stability analysis using strength reduction to calculate the factor of safety FS. Finally the stability number N is derived from Eq. (6). The process is then repeated; a new non-stationary random field is generated leading to a different stability number N and so on. Following each suite of 1000 Monte-Carlo simulations, the mean and standard deviation of stability number (μ_N and σ_N) can be obtained. Use of a stability number is a convenient and more fundamental way of normalizing the slope dimensions and the soil unit weight to render the problem dimensionless. For consistency, this paper uses the same stability number used by Hunter and Schuster (1968) and Griffiths and Yu (2015). The authors have performed a comprehensive parametric study of this problem, but for the purposes of this paper, a subset of results are shown in Fig. 3, for the case of $M = 0.5$, $\beta = 15°$ with dimensionless spatial correlation lengths $\Theta = 0.5$ and 2.0.

The abscissa is the depth ratio D, the ordinate to the left is μ_N and the ordinate to the right is σ_N. Consider for example, the case of a slope with $M = 0.5$, $\beta = 15°$ and $\Theta = 0.5$, as shown in Fig. 3(a). As might be expected, for slopes with low values of the input coefficient of variation, the mean stability numbers agree well with those from a deterministic analysis. As the input coefficient of variation increases however, the mean stability number decreases, implying a decreasing value of the mean factor of safety. On the other hand, the standard deviation of the stability number increases as the input coefficient of variation is increased.



(a)



(b)

Figure 3. Mean and standard deviation of stability number (μ_N and σ_N) against depth ratio (D) for $M = 0.5$ and $\beta = 15°$.

An interesting observation from Fig. 3(a) is that the mean stability number for higher values of v_{c_u} continues to fall with increasing foundation depth ratio (in the range $1.5 < D < 2.0$) where the deterministic results would remain constant. This phenomenon further emphasises the "seeking out" effect of the critical failure mechanism in a finite element approach to slope stability analysis, where the analysis allows the soil mass to "fail where it wants to fail". Figure 4 shows a failure mechanism from a suite of Monte-Carlo simulations for the same case with $v_{c_u} = 0.5$ and $D = 2.0$. In the deterministic case with $M = 0.5$ and $\beta = 15°$, the mechanism can never go deeper than $D = 1.5$ (see Fig. 5(b) in Griffiths and Yu 2015), however in the probabilistic simulations, the failure mechanism can and does go deeper in some simulations as shown in Fig. 4. This can happen probabilistically, because some random field simulations generate sufficiently low strengths in this deeper range to attract the critical mechanism.

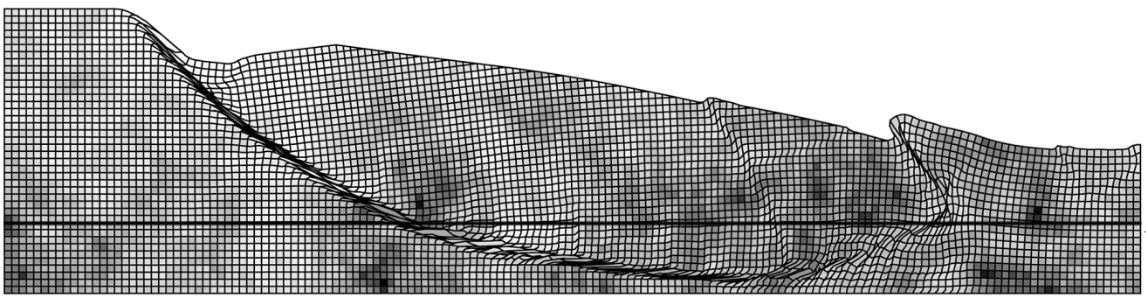


Figure 4. Failure mechanism for the case slope with $v_{c_u} = 0.5$ and $D = 2.0$.

Example problem

Since each non-stationary random field simulation computes a different stability number N, the sample probability density function (pdf) of N values can be plotted for analysis. Figure 5 shows a histogram of N values for an example slope problem with geometry and properties shown in Table 1, together with a lognormal fit. As shown in Fig. 5, the fitted curve agrees well with random field results. The histogram is obviously positive-skewed, and a goodness of fit test indicates that a p-value is about 0.45.

Table 1. Geometry and properties for the example problem

β	H	D	$\mu_{c_{u0}}$	v_{c_u}	Θ	ρ	γ	M
15°	10 m	1	15 kPa	0.5	2.0	3 kN/m^3	20 kN/m^3	0.5

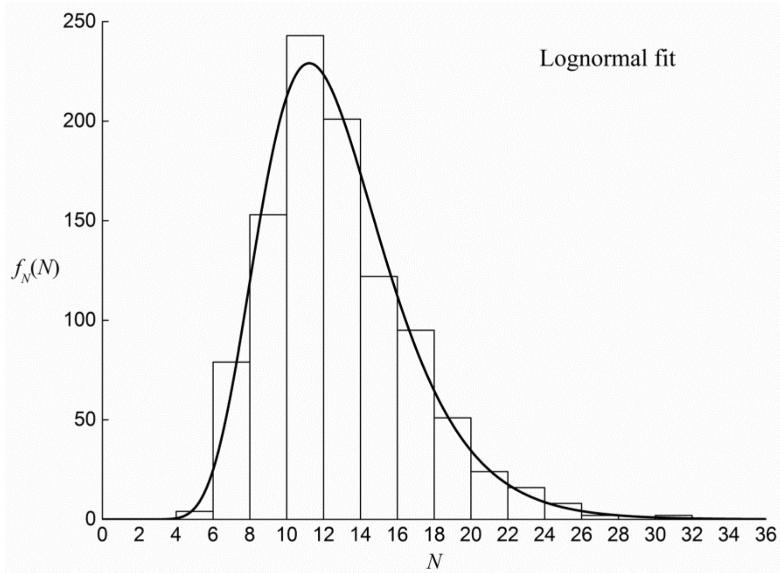


Figure 5. Histogram and lognormal fit for the example problem.

Use of Fig. 3 is now demonstrated to estimate the probability of failure. Figure 3(b) with $v_{c_u} = 0.5$ (a suggested upper bound for the coefficient of variation of undrained strength, e.g., Lee et al. 1983; Phoon and Kulhawy 1999) and $D = 1$, gives $\mu_N \approx 12.81$ and $\sigma_N \approx 3.91$. Assuming that N is lognormal, as suggested above, the standard deviation and mean of the underlying normal distribution of $\ln N$ are given by

$$\sigma_{\ln N} = \sqrt{\ln\left(1 + v_N^2\right)} = \sqrt{\ln\left(1 + 0.31^2\right)} = 0.303 \tag{7}$$

$$\mu_{\ln N} = \ln \mu_N - \frac{1}{2}\sigma_{\ln N}^2 = \ln\left(12.81\right) - \frac{1}{2}0.303^2 = 2.504 \tag{8}$$

Finally the probability of failure (p_f) is given by

$$p_f = p\left[FS < 1\right] = p\left[N\frac{\rho}{\gamma} < 1\right] = p\left[N\frac{3}{20} < 1\right] = p\left[N < 6.67\right]$$

$$= \Phi\left(\frac{\ln 6.67 - \mu_{\ln N}}{\sigma_{\ln N}}\right) = \Phi\left(\frac{\ln 6.67 - 2.504}{0.303}\right) = \Phi\left(-2.00\right) = 1 - \Phi\left(2.00\right) = 0.023 \tag{9}$$

where $\Phi(.)$ is the standard normal cumulative distribution function.

CONCLUSION

The paper has described RFEM analyses of undrained slopes with non-zero mean strength at the ground surface and linearly increasing mean strength with depth. A constant coefficient of variation was assumed in the current work. An algorithm to generate the non-stationary random field for this normally consolidated clay slopes was proposed. Although the values of μ_N for slopes with low values of the coefficient of variation were in good agreement with the recently published deterministic results of Griffiths and Yu (2015), for higher values of the coefficient of variation, μ_N fell below the deterministic lower bound as the foundation depth ratio D was increased. Results presented in this paper are a subset of a comprehensive probabilistic study on normally consolidated clay slopes with linearly increasing mean strength with depth, which will be reported elsewhere.

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