Bearing Capacity Assessment of a Shallow Foundation on a Two-Layered Soil Using the Random Finite Element Method

Ł. Zaskórski[1]; W. Puła, Ph.D.[2]; and D.V. Griffiths, Ph.D., M.ASCE[3]

[1,2]Faculty of Civil Engineering, Wrocław Univ. of Science and Technology, Wybrzeże St. Wyspiańskiego, 27 50-370, Wrocław, Poland. E-mail: lukasz.zaskorski@pwr.edu.pl; wojciech.pula@pwr.edu.pl
[3]Dept. of Civil and Environmental Engineering, Colorado School of Mines, 1500 Illinois St., Golden, CO 80401. E-mail: d.v.griffiths@mines.edu

Abstract

In this paper results of stochastic analysis of bearing capacity of a shallow foundation on a two-layered soil are presented. Computations use the random finite element method (RFEM) which allows soil strength parameters to be modeled as random fields. In this study, both the friction angle and the cohesion were characterized by two dimensional random fields using bounded and lognormal distributions respectively. Other soil properties such as Young's modulus, Poisson's ratio and soil unit weight were assumed to be deterministic, as their influence on bearing capacity can be neglected. Results are presented for the cases of "weak" over "strong"; and "strong" over "weak" while simultaneously considering various thicknesses of the layers, anisotropic correlation lengths and the coefficients of variation of the friction angle and cohesion. Conclusions indicate the cases in which stochastic characterization of soil parameters can significantly affect the stochastic bearing capacity of shallow foundations.

INTRODUCTION

The spatial variability structure of soils strongly influences the shallow foundation designing when it is taken into account. Therefore the theory of random fields seems to be an appropriate tool in a description of soil properties if bearing capacity is under consideration. Several papers have been published that have utilized random finite element method (RFEM) to evaluate probabilistic characteristic of bearing capacity (Griffiths and Fenton, 2001; Fenton and Griffiths, 2003; Pieczyńska et al., 2011; Pieczyńska-Kozłowska et al., 2015; Zaskórski and Puła, 2016). In the paper by Pieczyńska et al. (2015) bearing capacity of an embedded shallow foundation has been evaluated under an assumption that soil strength parameters have been described by anisotropic random fields. However, only one soil layer was considered. In the present study the authors made an attempt to generalize former papers for the case layered subsoil.

SOIL PROPERTIES

Random fields in RFEM are generated by local average subdivision – LAS method (Fenton and Vanmarcke, 1990). In this study both soil layers are cohesive and shear strength parameters (the cohesion and the friction angle) are described by random fields. The cohesion is characterized by a lognormal distribution obtained by the transformation $X = \exp\{Z\}$. Z is a normally distributed random field. The probability density function of *X* is given by the following equation

$$f(x) = \frac{1}{x\sigma_{\ln X}\sqrt{2\pi}} \cdot \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu_{\ln X}}{\sigma_{\ln X}}\right)^2\right\}, \tag{1}$$

where $\mu_{\ln X}$ is a mean value and $\sigma_{\ln X}$ denotes a standard deviation of an underlying Gaussian distribution of *Z*. The friction angle is described by a bounded distribution which the probability density function takes form

$$f_x(x) = \frac{\sqrt{\pi}(b-a)}{\sqrt{2}s(x-a)(b-x)} \cdot \exp\left\{-\frac{1}{2s^2}\left[\pi\ln\left(\frac{x-a}{b-x}\right) - m\right]^2\right\}, \tag{2}$$

where *a* and *b* are min. and max. values of a parameter, *s* is a scale factor correlated with a standard deviation of the property, *m* is a location parameter and $x \in (a, b)$. The above distribution can be generated from a standard normal random field Z_0 by the following transformation

$$X = a + \frac{1}{2}(b-a)\left\{1 + \tanh\left(\frac{sZ_0}{2\pi}\right)\right\}. \tag{3}$$

More details can be found in Fenton and Griffiths (2008). Moreover each random field is characterized by its correlation structure. Within this study the ellipsoidal correlation function for anisotropic case was considered

$$\rho(\tau) = \exp\left(-\sqrt{\left(\frac{2|\tau_2|}{\theta_x}\right)^2 + \left(\frac{2|\tau_1|}{\theta_y}\right)^2}\right), \tag{4}$$

where θ_x and θ_y denote fluctuation scales along directions x and y. Furthermore τ_1 and τ_2 are the vertical and horizontal distances respectively, between two points in two-dimensional space. The random fields of cohesion and friction angle are assumed to be stochastically independent. Table 1 presents applied soil properties of each layer of soil.

Table 1. Summary of soil properties

Parameter	Symbol	Unit	Weak layer			Strong layer		
Unit weight	γ	**[kN/m^3]**	20			21		
Friction angle	**Distribution**	[-]	bounded			bounded		
	μ_φ	[°]	10			21		
	σ_φ	[°]	1,5			3,20		
	φ_{min}	[°]	5			11		
	φ_{max}	[°]	15			31		
	s	[-]	1,94			2,29		
Cohesion	**Distribution**	[-]	lognormal			lognormal		
	μ_c	[kPa]	20	20	20	38	38	38
	σ_c	[kPa]	5	10	15	9,5	19	28,5
	COV	[%]	25	50	75	25	50	75
Dilation angle	ψ	[°]	0			0		
Young modulus	E	[MPa]	15			35		
Poisson ratio	ν	[-]	0,3			0,3		
Scale of fluctuation	θ_x	[m]	10			10		
	θ_y	[m]	0,4; 0,8; 1,6; 2,4; 3,2			0,4; 0,8; 1,6; 2,4; 3,2		

NUMERICAL MODELLING OF THE SOIL PROFILES

Stochastic analysis were performed in program created in FORTRAN which was based on RBEAR2D code (this code can be found on the website http://www.engmath.dal.ca/rfem/). The size of soil model was calibrated so that the boundary conditions had no influence on the bearing capacity results. A 12,0×4,0m mesh was considered the suitable mesh size, which is equal to 120 elements in the X direction and 40 elements in the Y direction. Each element is 0,1×0,1m. The nodes on the bottom boundary of the mesh model are fixed. The left and right boundaries are constrained against horizontal displacement and are free to slide vertically.

Footing width was set on 0,8m and embedment on 1,0m. The depth of the foundation in the model is represented by the load applied on the level of the footing equal to the unit weight of 1st layer multiplied by H=1,0m. The soil model is shown in Figure 1.

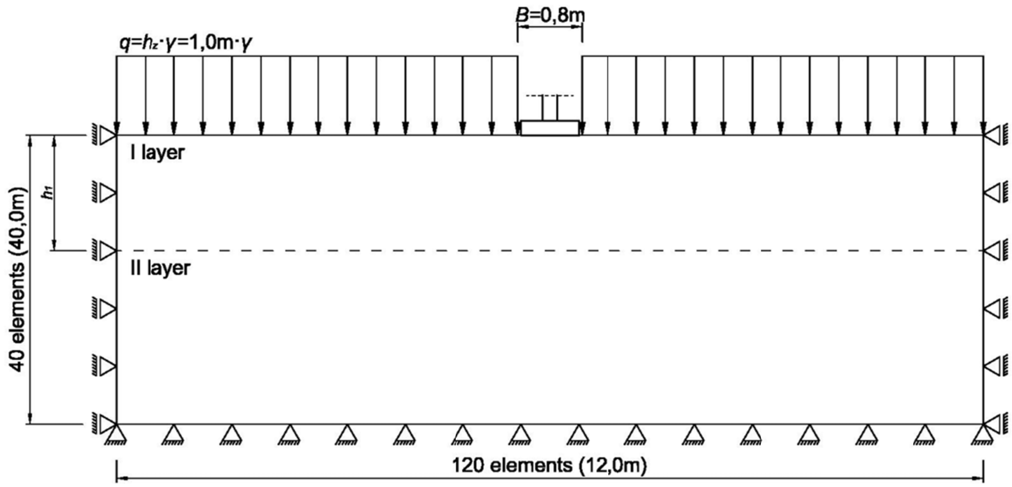


Figure 1. Soil model

This paper is focused on testing the influence of variability of four parameters on bearing capacity of a shallow foundation on layered soil. These parameters are:

- thickness of first layer
  - values: h_1=0,4m; 0,8m; 1,6m; 2,4m; 3,2m,
- position of layers
  - cases: 1st layer – weak, 2nd layer – strong; 1st layer – strong, 2nd layer – weak,
- vertical scale of fluctuation of the cohesion in both layers
  - values: θ_y=0,4m; 0,8m; 1,6m; 2,4m; 3,2m,
  - cases: θ_{y1} – variable, θ_{y2} – const (θ_{y2} =0,8m); θ_{y1} – const (θ_{y1} =0,8m), θ_{y2} – variable; θ_{y1} – variable, θ_{y2} – variable,
- mean value and standard deviation of the cohesion
  - values of COV of the cohesion: 25%; 50%; 75%.

Values of thickness of 1st layer and vertical scales of fluctuation of the cohesion in 1st and 2nd layer are related with the width of the footing (B=0,8m). In Table 2 can be found characterization of considered cases. Cases are divided on three groups A, B and C in which COV of the cohesion is different. In each case 30 examples were conducted that gives 540 examples overall. Stochastic bearing capacity (a mean value and a standard deviation) was calculated based on 500 realizations. Number of realizations was selected in preliminary tests and is the smallest number which ensures the stabilization of values of random bearing capacity.

Table 2. Summary of considered cases

Group	Case	Thickness of 1st layer h_1 [m]	COV of soil parameters φ [%]	c [%]	Vertical scale of fluctuation θ_y	1st layer	2nd layer
A	A1	0,4-3,2	15	25	$\theta_{y1} = \theta_{y2}$ – variable	strong	weak
	A2				$\theta_{y1} = \theta_{y2}$ – variable	weak	strong
	A3				θ_{y1} – variable	strong	weak
	A4				θ_{y1} – variable	weak	strong
	A5				θ_{y2} – variable	strong	weak
	A6				θ_{y2} – variable	weak	strong
B	B1	0,4-3,2	15	50	$\theta_{y1} = \theta_{y2}$ – variable	strong	weak
	B2				$\theta_{y1} = \theta_{y2}$ – variable	weak	strong
	B3				θ_{y1} – variable	strong	weak
	B4				θ_{y1} – variable	weak	strong
	B5				θ_{y2} – variable	strong	weak
	B6				θ_{y2} – variable	weak	strong
C	C1	0,4-3,2	15	75	$\theta_{y1} = \theta_{y2}$ – variable	strong	weak
	C2				$\theta_{y1} = \theta_{y2}$ – variable	weak	strong
	C3				θ_{y1} – variable	strong	weak
	C4				θ_{y1} – variable	weak	strong
	C5				θ_{y2} – variable	strong	weak
	C6				θ_{y2} – variable	weak	strong

RESULTS

On Figures 2, 3 and 4 are presented results for minimum and maximum values of the considered vertical scale of fluctuation (θ_y=0,4m; 3,2m) to make them more transparent and clear.

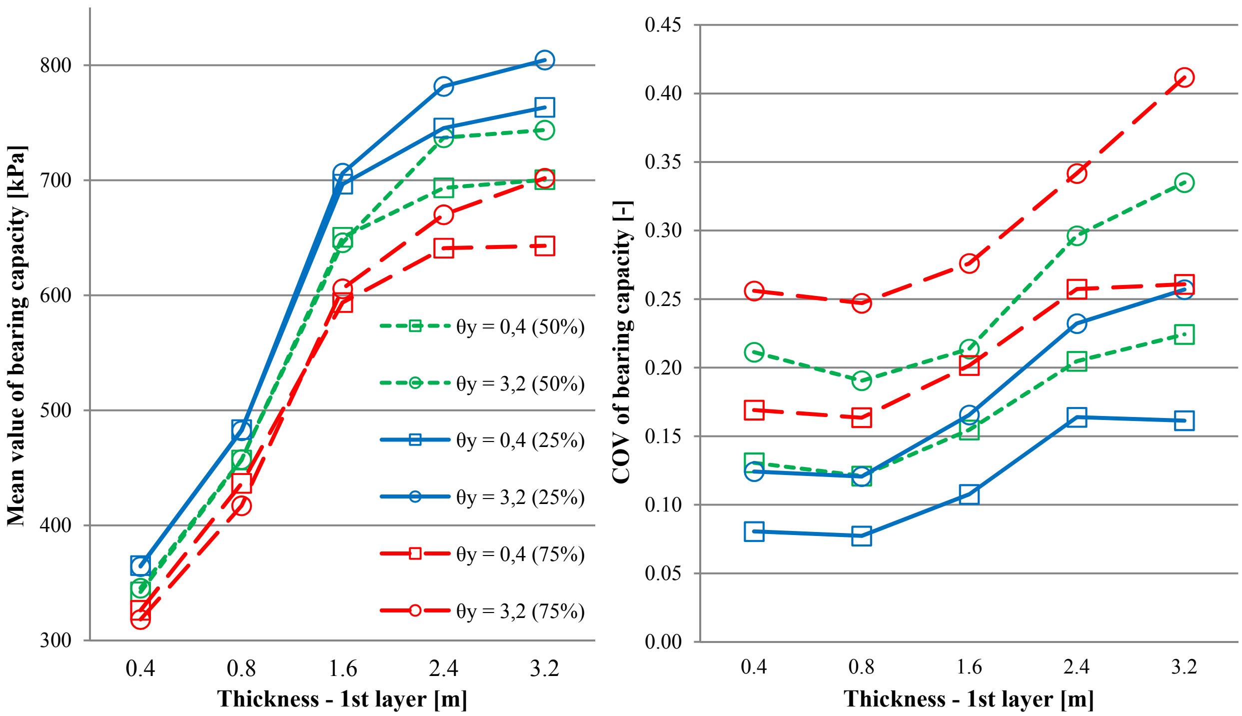
It can be noticed that mean values of bearing capacity increase with thickness of 1st layer in case strong-weak (Figures 2a, 3a, 4a). This phenomenon is cost by increase of the strong layer thickness. Furthermore mean values of bearing capacity stabilize substantially on level of 3*B* (h_1=2,4m) regardless parameters of cohesion. This is the consequence of the vanishing influence of the weak 2nd layer.

Opposite situation is in case weak-strong – the thicker 1st layer the smaller mean value of bearing capacity. In this situation mean values of bearing capacity stabilize on level of 1*B* (h_1=0,8m). In Figure 2b the case C2 (θ_y=3,2m, COV of cohesion equals 75%) is an exception. It can be observed that for h_1=3,2m, the mean value of bearing capacity rises. Most probably it is associated with the number of realization which in this case shall be greater than 500.

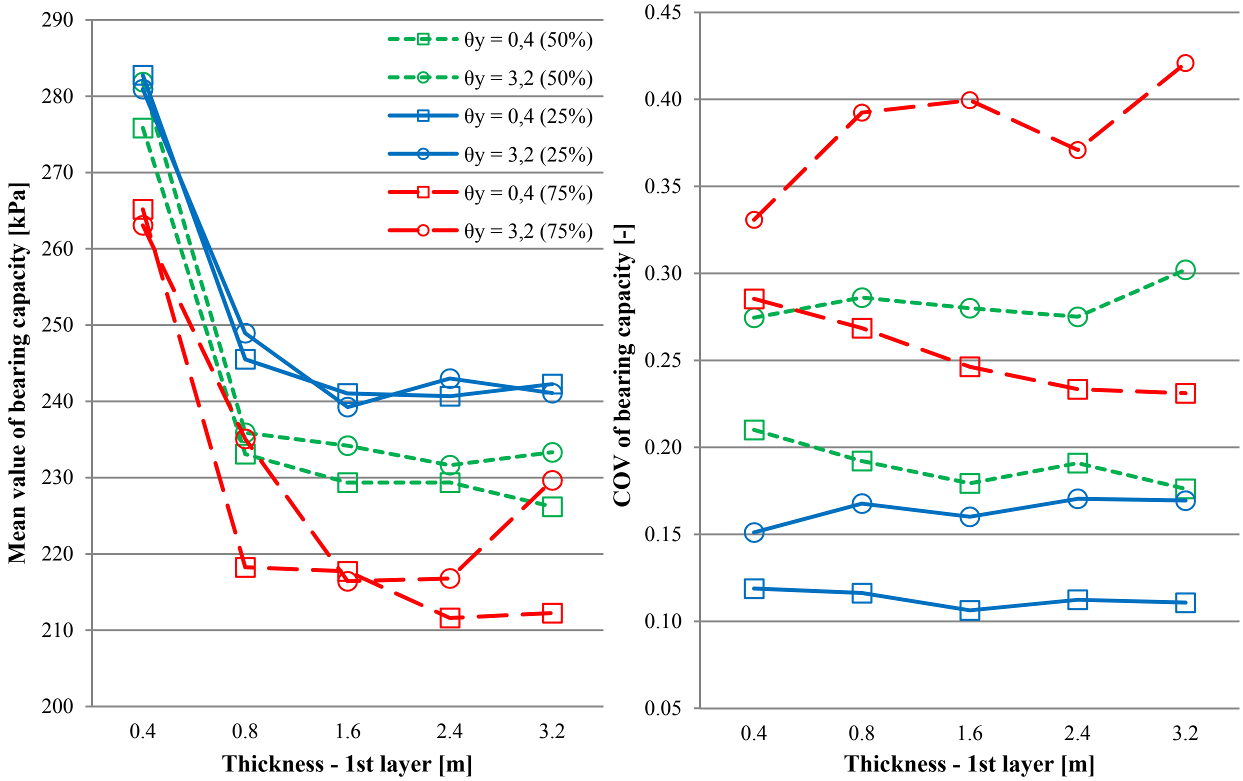
The conclusions formulated above are not identical with engineering intuition. In many deterministic approaches it is believed that the presence of the second layer is negligible if the thickness of 1st layer is greater than 2*B*.

COV of bearing capacity increases with the thickness in case strong-weak as it can be observed on Figures 2a, 3a and 4a.

Greater mean values of bearing capacity are achieved for greater values of vertical scales of fluctuation for case strong-weak (see Figures 2a and 3a), however the differences are not large.
As regards COV of bearing capacity it can be noticed that their values are practically independent on the thickness of 1st layer.
In cases weak-strong (A6, B6, C6) coefficients of variation of bearing capacity are almost independent on the vertical fluctuation scale in 2nd layer.

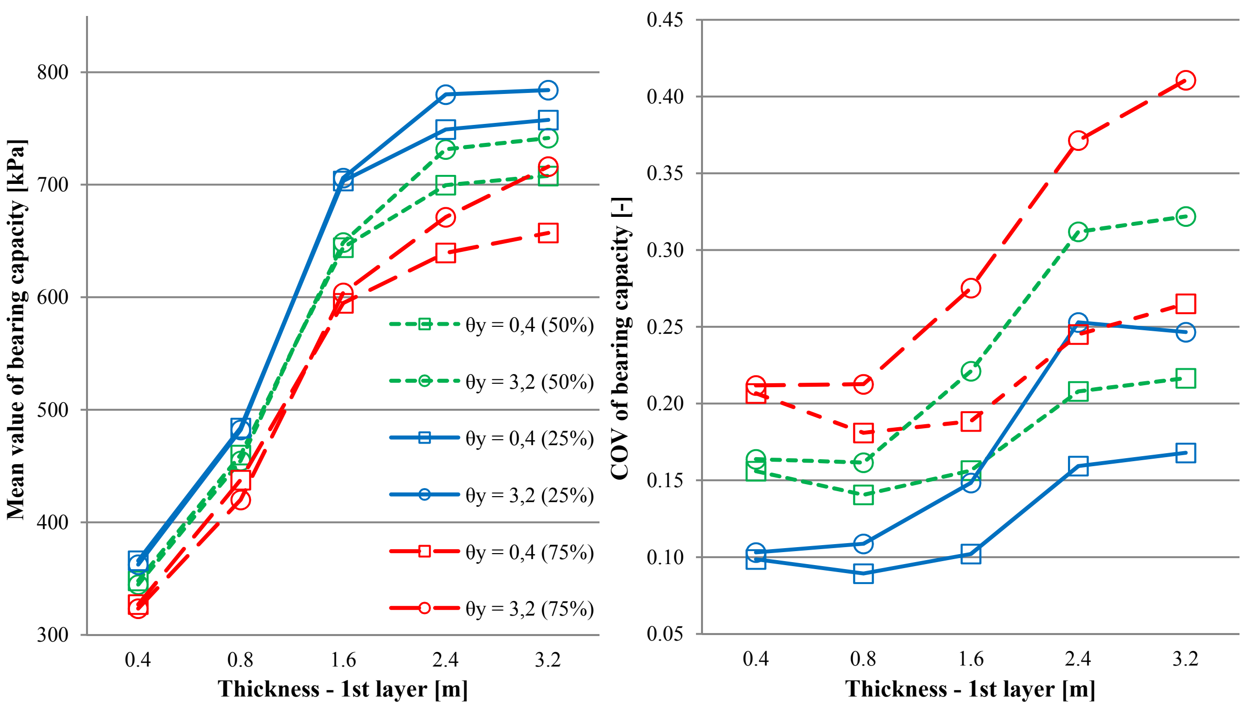


a) Cases A1, B1, C1 (1st layer – strong, 2nd layer – weak)

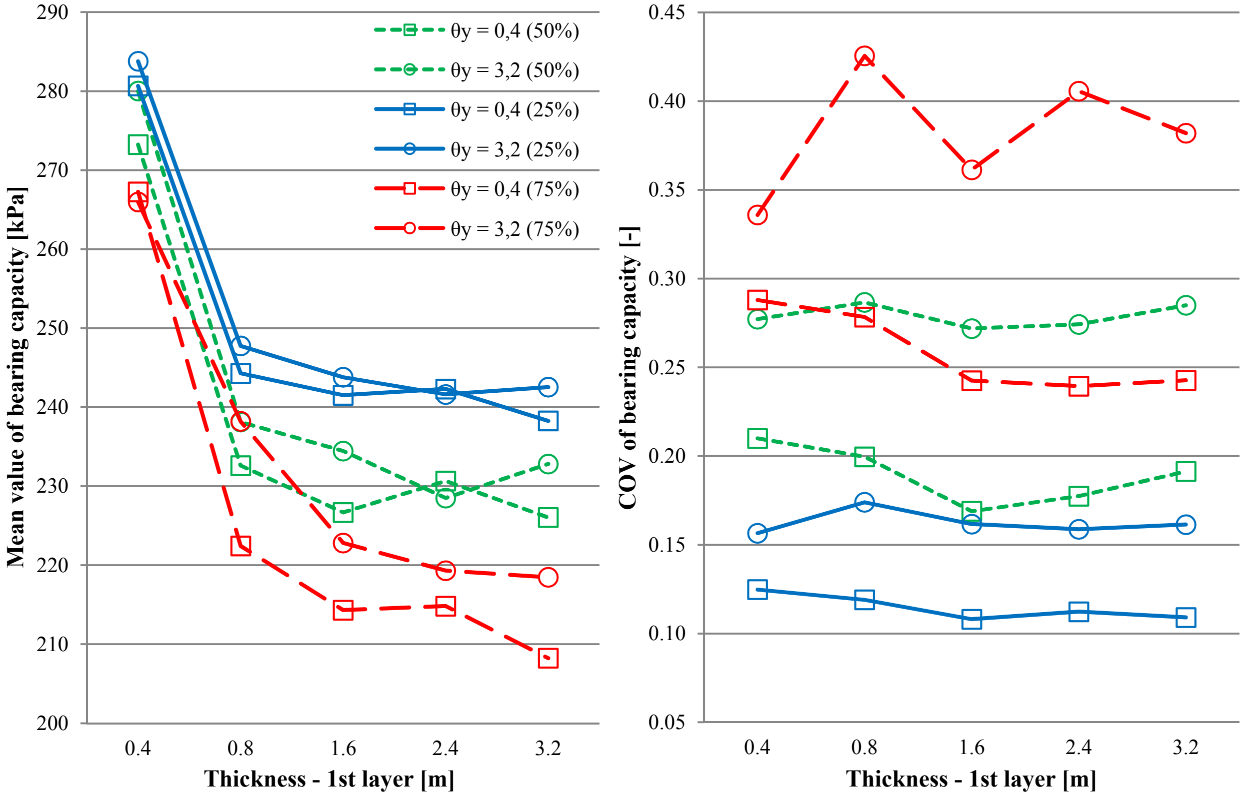


b) Cases A2, B2, C2 (1st layer – weak, 2nd layer – strong)

Figure 2. Mean values and the COV of bearing capacity in dependence of thickness of 1st layer (in legend in brackets are presented COV of cohesion) – $\theta_{y1}=\theta_{y2}$ - variable

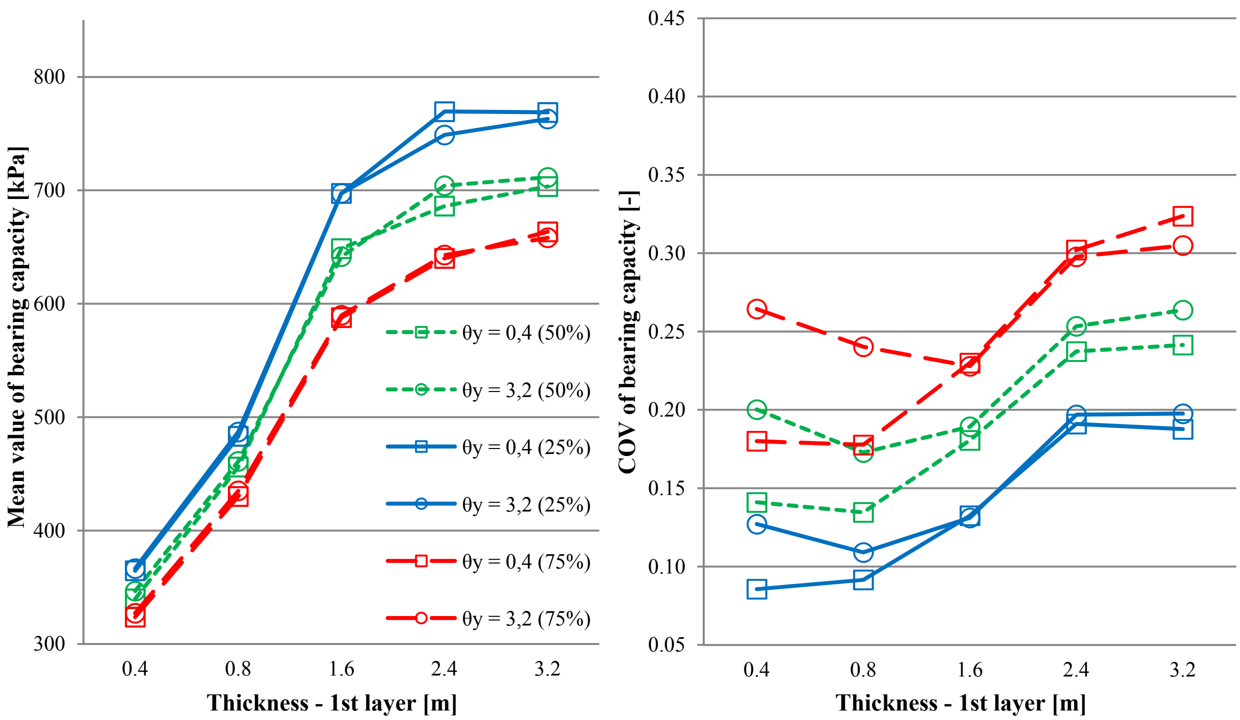


a) Cases A3, B3, C3 (1st layer – strong, 2nd layer – weak)

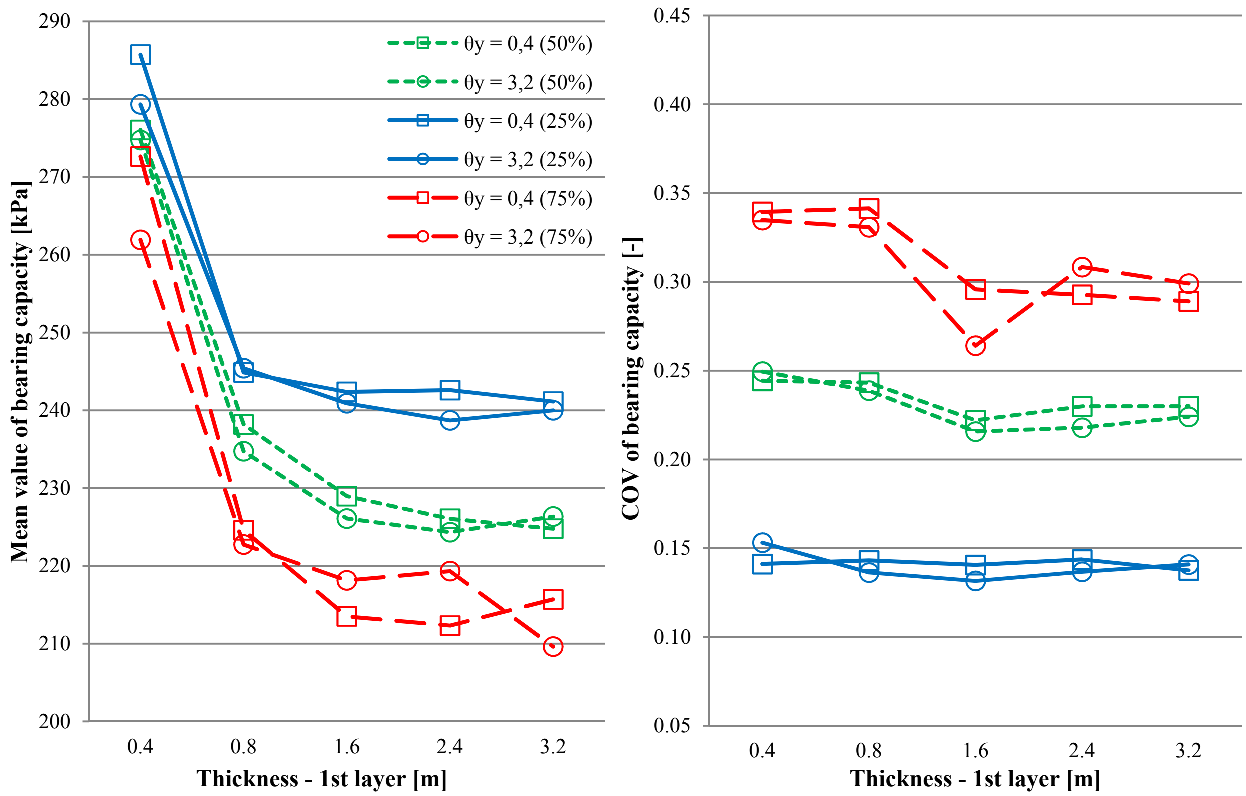


b) Cases A4, B4, C4 (1st layer – weak, 2nd layer – strong)

Figure 3. Mean values and the COV of bearing capacity in dependence of thickness of 1st layer (in legend in brackets are presented COV of cohesion) – θ_{y1} - variable



a) Cases A5, B5, C5 (1st layer – strong, 2nd layer – weak)



b) Cases A6, B6, C6 (1^{st} layer – weak, 2^{nd} layer – strong)

Figure 4. Mean values and the COV of bearing capacity in dependence of thickness of 1^{st} layer (in legend in brackets are presented COV of cohesion) – θ_{y2} - variable

CONCLUSION

The new code for the evaluation of random bearing capacity of two-layered subsoil was elaborated. Considered problem is the generalization of former works mentioned in the introduction of this paper. The constructed code enables to investigate the influence of thickness of layers, soil properties (the cohesion, the friction angle) and their correlation structure on random bearing capacity.

The stabilization of the mean value of bearing capacity on 3*B* level was observed in the case strong-weak configuration and on 1*B* level in the case weak-strong configuration. The above conclusion is no longer valid for COV of bearing capacity.

The strong-weak configuration results in increasing of the expected value of bearing capacity as a function of a vertical fluctuation scale. The opposite trend is observed in the weak-strong case.

REFERENCES

Fenton, G. A., & Griffiths, D. V. (2003). “Bearing-capacity prediction of spatially random c φ soils”. Canadian Geotechnical Journal, 40(1), 54-65.

Fenton, G. A., & Griffiths, D. V. (2008). Risk Assessment in Geotechnical Engineering. New York: John Wiley & Sons.

Fenton, G. A., & Vanmarcke, E. H. (1990). “Simulation of random fields via local average subdivision”. Journal of Engineering Mechanics, 116(8), 1733-1749.

Griffiths, D. V., & Fenton, G. A. (2001). “Bearing capacity of spatially random soil: the undrained clay Prandtl problem revisited”. Geotechnique, 351-359.

Pieczyńska, J., Puła, W., Griffiths, D. V., & Fenton, G. A. (2011). « Probabilistic characteristics of strip footing bearing capacity evaluated by random finite element method”. In Proceedings of the 11th international conference on applications of statistics and probability in soil and structural engineering (ICASP), Zurich.

Pieczyńska-Kozłowska, J. M., Puła, W., Griffiths, D. V., & Fenton, G. A. (2015). „Influence of embedment, self-weight and anisotropy on bearing capacity reliability using the random finite element method”. Computers and Geotechnics, 67, 229-238.

Zaskórski, Ł., & Puła, W. (2016). “Calibration of characteristic values of soil properties using the random finite element method”. Archives of Civil and Mechanical Engineering, 16(1), 112-124.