Risk Assessment in Geotechnical Engineering:
Stability Analysis of Highly Variable Soils

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ABSTRACT: The paper will review the state-of-the-art in the use of finite element methods for modeling geotechnical engineering problems involving non-typical geometries and highly variable soil properties. Examples will focus on slope stability analyses in which traditional limit equilibrium methods, and even well-established probabilistic methodologies may lead to misleading results.

Keywords: Finite element method, Variable soils, Probability of failure, Random Fields, Risk assessment.

INTRODUCTION

Classical limit equilibrium methods of slope stability analysis have remained essentially unchanged for decades. The finite element method offers a powerful alternative with the following main advantages:

- No assumption needs to be made in advance about the shape or location of the failure surface. The failure mechanism "seeks out" the weakest path through the soil.
- Since there is no concept of slices in the finite element approach there is no need for assumptions about slice side forces. The finite element method preserves global equilibrium until "failure" is reached.
- If realistic soil compressibility data is available, the finite element solutions will give information about deformations at working stress levels.
- The finite element method is able to monitor progressive failure up to and including overall shear failure.

Finite element slope stability analysis can hardly be considered a new technique. The first paper to tackle the subject by Smith & Hobbs (1974) is over 35 years old followed by an important paper on the topic by Zienkiewicz et al. (1975). Both of these papers had a very significant influence on the first author's finite element slope stability software developments over the years. Early publications date back to Griffiths (1980) and the first ever published source code for finite element slope stability appeared in the second edition of the text by Smith & Griffiths (1988, 2004). Readers are also referred to Griffiths & Lane (1999) for a thorough review of how the methodology works.

This paper will focus initially on demonstrating the use of the finite element method as applied to slope examples that would not necessarily be amenable to traditional limit equilibrium methods (LEM). The paper will then go on to discuss risk assessment methods in geotechnical engineering, particularly for slope stability, including the most recent developments that combine random fields with finite element methods in the Random Finite Element Method (RFEM). Examples will be given of slope reliability analysis, where traditional methods may deliver quite misleading results.

LONG SLOPES

How long is "infinite"?

It has been noted previously (e.g. Duncan & Wright 2005) that the infinite slope assumptions can be expected to lead to conservative estimates of the factor of safety. This is primarily due to support provided at the ends of a finite slope that is not accounted for in the infinite slope model. Here we present some finite element slope stability analyses on "long slopes" with uphill and downhill boundary conditions, to assess the range of validity and conservatism of the "infinite slope" assumptions. The main question to be addressed is; Howlong must a slope be for it to be considered "infinite"? A typical finite element mesh of 8-noded quadrilateral elements is shown in Figure 1. Note that H and L are respectively vertical and horizontal measures of the slope geometry.

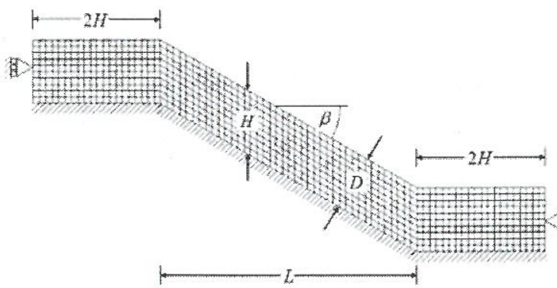

Fig. 1. Mesh of 8-node quadrilateral element for "long slope" analysis.

For simplicity, we have considered an undrained clay slope for which the infinite slope equation would give the following factor of safety

$$FS = \frac{c_u}{\gamma_{sat} H \cos\beta \sin\beta} \tag{1}$$

In this example, the properties shown in the caption of Figure 2 were held constant while L/H was gradually increased. As shown, the computed factor of safety converged on the infinite slope solution of $FS = 1.15$ from equation (1) for L/H greater than about 16. As expected, the infinite slope solution is always conservative.

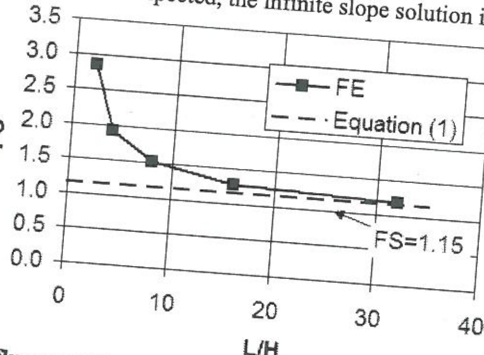

Fig. 2. Influence of length ratio on the computed factor of safety for a slope with $H = 2.5$ m, $\beta = 30°$, $c_u = 25$ kN/m^2 and $\gamma_{sat} = 20$ kN/m^3.

For example, with $L/H = 2$, the computed factor of safety was $FS = 2.86$; more than twice the infinite slope value. A typical failure mechanism for a steeper slope is shown in Figure 3. The figure indicates that as the slope gets longer, the infinite slope mechanism starts to dominate and the "toe" failure at the downhill end becomes less important.

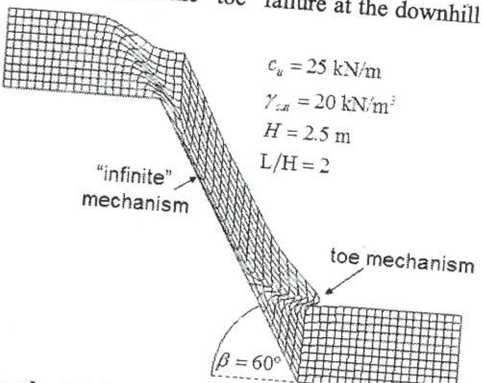

Fig. 3. Deformed mesh at failure corresponding to a slope with $L/H = 2$ and $\beta = 60°$ indicating a toe mechanism with $FS = 1.58$.

Influence of slope angle

A curiosity of the infinite slope equation (1) as shown in Figure 4, is that for constant H, γ_{sat} and c_u, the factor of safety starts to increase as the slope steepens in the range $\beta > 45°$. This result seems counter intuitive since our experience of finite slopes is that the factor of safety always falls as a slope gets steeper.

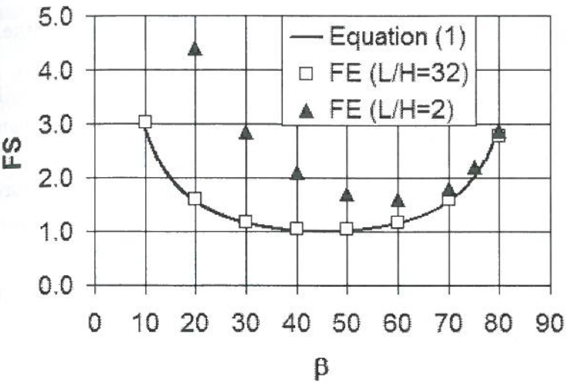

Fig 4. Influence slope angle on the factor of safety for an undrained clay slope with $H = 2.5\,\mathrm{m}$, $c_u = 25\,\mathrm{kN/m^2}$ and $\gamma_{sat} = 20\,\mathrm{kN/m^3}$.

An explanation of this effect for infinite slopes comes from the fact that as the slope becomes steeper, the length of the potential failure surface available to resist sliding is increasing at a faster rate than the down-slope component of soil weight trying to cause sliding. Even a short slope analysis with $L/H = 2$ demonstrates this effect as shown in Fig. 4 (Griffiths et al. 2011a).

STRATIFIED SLOPES

James Bay Dike

The James Bay Dike slope shown in Figure 5 has a terraced cross-section with four different soil types consisting of cohesionless soil in the embankment and undrained clays in the foundation. This profile has attracted considerable interest (see e.g., El Ramly et al. 2002, Duncan et al. 2003) because published LEM solutions that assumed circular failure mechanisms (e.g. Bishop's method), led to unconservative estimates of the factor of safety. Although limit equilibrium procedures are available for estimating the factor of safety associated with non-circular surfaces, it is still hard to guarantee that the critical surface corresponding to the minimum factor of safety has been found.

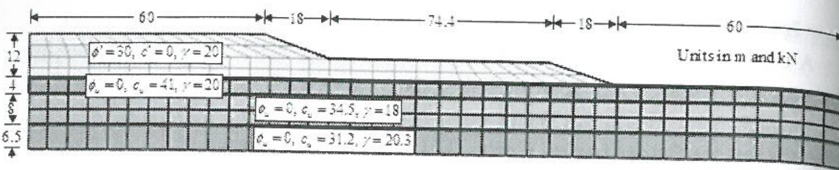

Fig. 5. FE geometry and soil properties assigned to the James Bay dike.

The benefits of the FE slope stability approach are even more striking in an example such as this in which the factor of safety can be accurately estimated, and the corresponding failure mechanism observed. The sudden displacement increase shown in Figure 6 indicates that $FS \approx 1.27$ and the deformed mesh at failure given in Figure 7 clearly shows the anticipated non-circular critical failure mechanism.

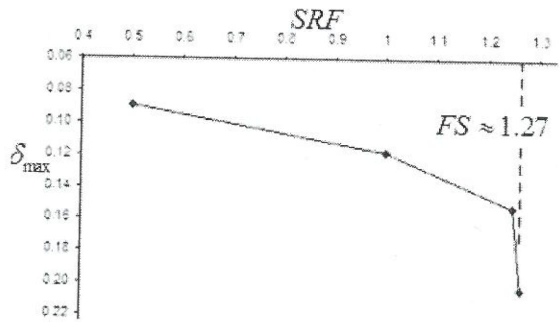

Fig. 6. FE solution of James Bay Dike by strength reduction indicating $FS = 1.27$.

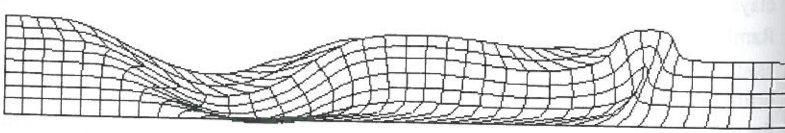

Fig. 7. Deformed mesh at failure demonstrating a non-circular failure mechanism.

Multiple failure mechanisms

As mentioned in the Introduction, the finite element method "seeks out" the most critical failure path through the soil, and unlike many LEM approaches, does not require the user to anticipate in advance where the critical failure mechanism might lie. The example shown in Figures 8 makes this point quite clearly by demonstrating multiple mechanisms, which all have the same factor of safety of $FS = 1.38$. A traditional approach could easily miss one or more of these surfaces, which could lead to an unsafe design if the goal of the analysis, for example, was to identify locations for possible soil reinforcement.

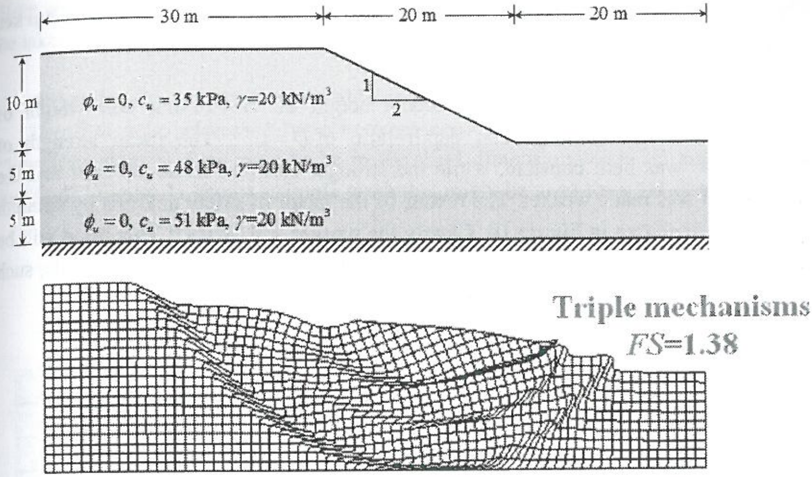

Fig. 8. Multiple failure mechanisms of an undrained slope.

Checkerboard slope stability analysis.

Soils and rocks are the most variable of all engineering materials, so when an engineer chooses "characteristic values" of the soil shear strength for a slope analysis, it is very likely that some parts of slope consist of soil that is stronger than the characteristic values, and other parts that are weaker.

In this section we take a simple 2D undrained clay slope and assign the slope two different properties arranged in a checkerboard pattern as shown in Figure 9.

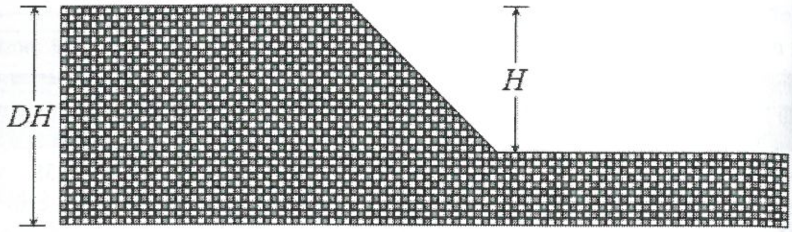

Fig. 9. Slope stability analysis with checkerboard strength pattern. The darker zones are stronger.

The 45° undrained clay slope has a height of $H = 10$ m a unit weight of $\gamma_{sat} = 20$ kN/m^3 and a foundation depth ratio of $D = 1.5$. The mean strength of $c_u = 50$ kPa was held constant, while the stronger soil was made stronger and the weaker soil was made weaker. The results of the factor of safety analysis by strength reduction are shown in Figure 10. Clearly the weaker soil "wins"! This trend will be repeated when wider ranges of strength values are incorporated into an analysis, such as later in this paper when we discuss random field modeling of soils.

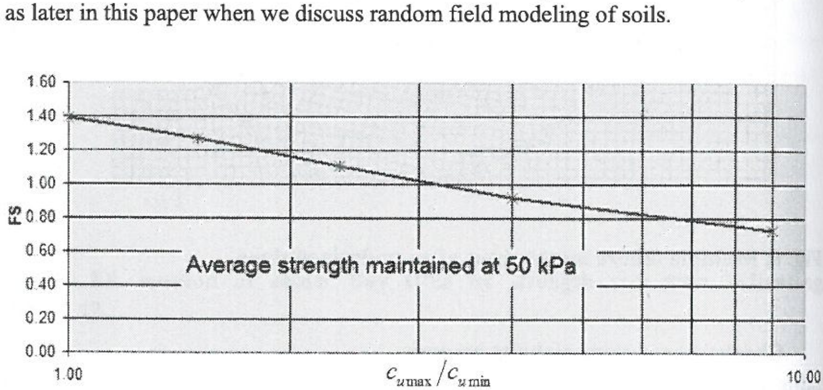

Fig. 10. Influence of strength ratio in checkerboard slope analysis.

3D SLOPE STABILITY ANALYSIS

3D slope stability analysis has received considerable attention in the literature (e.g. Hungr 1987, Seed et al. 1990, Duncan 1996, Stark and Eid 1998, Chen et al. 2005, Griffiths and Marquez 2007, Michalowski 2010), yet the vast majority of slope stability analyses in research and practice, are still performed in 2D under the assumption of plane strain conditions. Even when 2D conditions are not appropriate, 3D analysis is rarely performed. There are a number of reasons for this. The majority of work on this subject has shown that the 2D factor of safety is conservative (e.g. lower than the "true" 3D factor of safety), and existing methods of 3D slope stability analysis are often complex, and not well established in practice. A further disadvantage of some 3D LEM approaches, is that being based on extrapolations of 2D "methods of slices" to 3D "methods of columns", they are complex, and not readily modified to account for realistic boundary conditions in the third dimension. The advantages of FE slope stability methods become even more attractive in 3D. Here we demonstrate when 3D may be justified, and also show that great care must be taken in subscribing to the received wisdom that "2D is always conservative".

When is plane strain a reasonable approximation?

The first issue addressed for a homogeneous slope, is to consider the question "how long does a slope need to be in the third dimension for a 2D analysis to be justified?" Figure 11 shows a simple mesh that might be used for a 3D slope analysis.

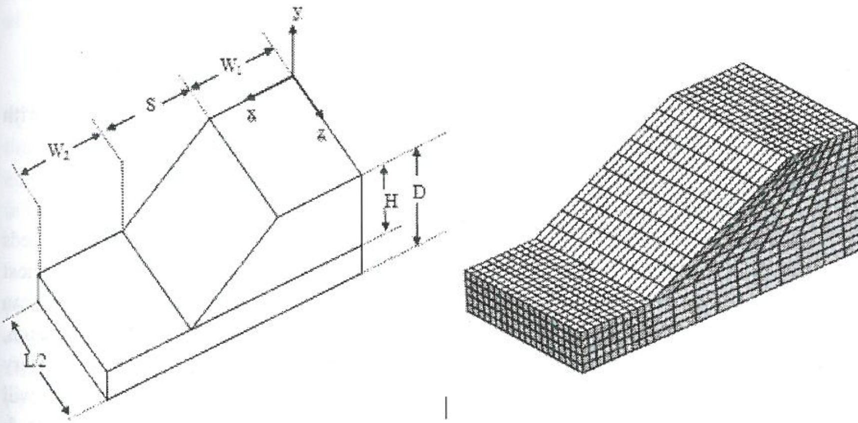

Fig. 11. FE mesh for 3D slope stability analysis using 20-node hexahedral elements.

The boundary conditions are such that one side $(z=0)$ is fully fixed and the other $(z=L/2)$ allows vertical movement only implying a plane of symmetry. The bottom $(y=D)$ of the slope is fully fixed, while the back $(x=0)$ and front-side $(x=W_1+W_2+S)$ of the slope allow vertical movement only. The results from a series of FE analyses with different depth ratios (L/H) while keeping all other parameters constant are shown in Figure 12. It can be seen that the factor of safety in

3D is always higher than in 2D, but tends to the plane strain solution of $FS = 1.25$ for depth ratios of the order $L/H > 10$. It is shown that results of the same slope with a coarser mesh gave slightly higher values of FS.

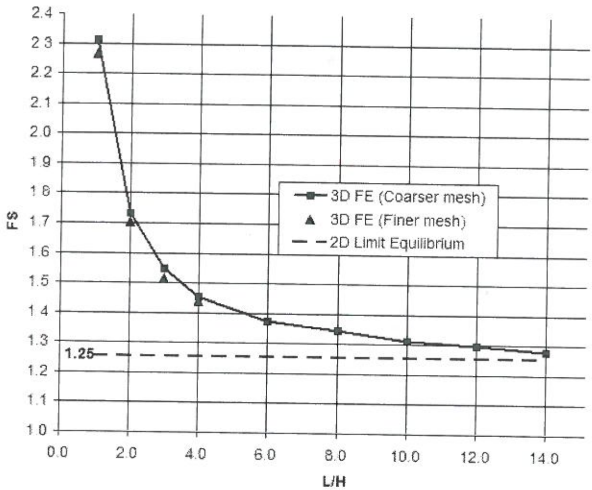

Figure 12: Comparison of 3D and 2D solutions for a $\phi_u = 0°$ slope with $c_u/(\gamma H) = 0.20$.

Is plane strain conservative?

The assumption that 2D analyses lead to conservative factors of safety needs some qualification. Firstly, a conservative result will only be obtained if the "most pessimistic" section in the 3D problem is selected for 2D analysis (see e.g., Duncan 1996). In a slope that contains layering and strength variability in the third dimension, this conservative 2D section may not be intuitively obvious. Secondly, the corollary of a conservative 2D slope stability analysis is that back analysis of a failed slope will lead to an unconservative overestimation of the soil shear strength (e.g. Arellano & Stark 2000). Bromhead & Martin (2004) argued that some landslide configurations with highly variable cross-sections could lead to failure modes in which the 3D mechanism was the most critical. Other investigators have also indicated situations where more critical 3D factors of safety were observed (e.g., Chen & Chameau 1982 and Seed et al. 1990).

Finite element slope stability analysis offers us the opportunity to perform objective comparisons in which 2D and 3D factors of safety are compared for variable soil conditions. This point is highlighted in the 3D example shown in Figure 13 which represents a 2:1 slope of height 10 m, foundation depth 5 m and a length in the out-of-plane direction of 60 m with smooth boundary conditions.

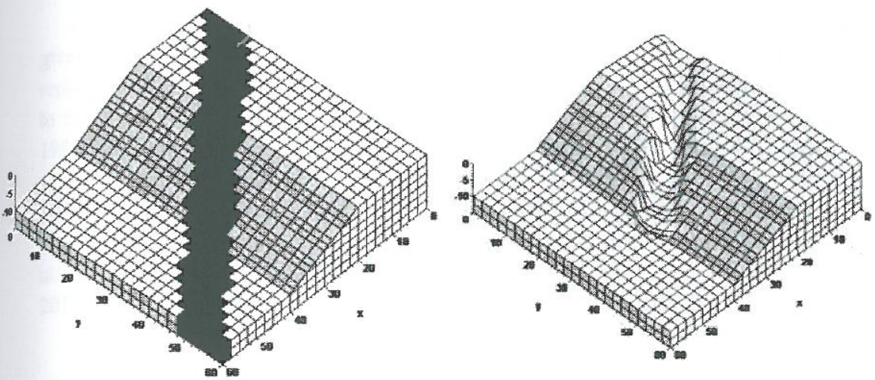

Fig. 13. Three-dimensional slope mesh and at failure including an oblique layer of weak soil.

An oblique zone of weak soil (shaded black) with undrained strength $c_u = 20\ \text{kN/m}^2$ has been introduced into the slope with the surrounding soil four times stronger with $c_u = 80\ \text{kN/m}^2$. The 3D factor of safety was found to be approximately 1.5 and the mechanism clearly follows the weak zone as also shown in Figure 13.

When 2D stability analyses are then performed on successive slices in the $x - z$ plane moving from $y = 0$ m to $y = 60$ m, the result shown in Figure 14 is obtained. As a check, the 2D analyses were performed both by finite elements and by a standard LEM. It can be seen that towards the boundaries of the 3D slope ($y < 21$ m and $y > 34$ m) where the majority of soil in the sections is strong, the 2D results led to higher and therefore unconservative estimates of the factor of safety. On the other hand, at sections towards the middle of the slope ($21\text{ m} < y < 34\text{ m}$) where there is a greater volume of weak soil, the 2D results led to lower, and therefore conservative estimates of the factors of safety. The 2D factor of safety closely approached unity at $y = 29$ m. An even more critical 2D plane however, is the oblique one that runs right down the middle of the weak soil. This 2D plane has a 2.5:1 slope and is flatter than the $x - z$ planes considered previously. A 2D slope stability analysis on this plane gives an even lower factor of safety of about 0.7. This result, also shown on Figure 14, is less than half of the factor of safety given by the 3D analysis, and would be considered excessively conservative, even by geotechnical design standards.

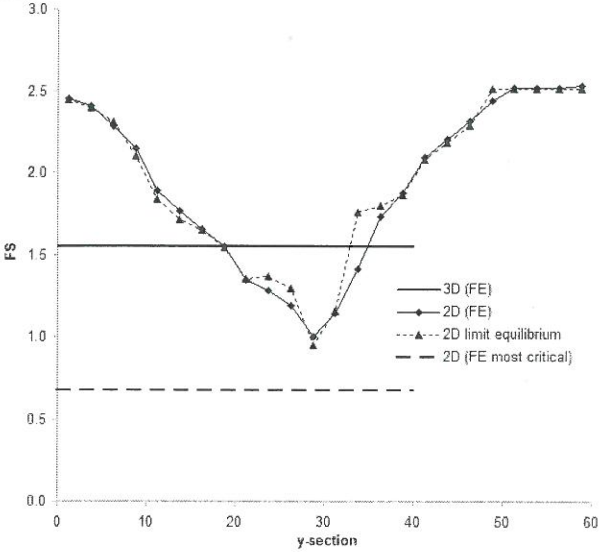

Figure 14: Factors of safety from 3D analysis and various 2D sections.

Even in the rather simple problem considered here, the results have shown a quite complex relationship between 2D and 3D factors of safety. The results confirm that 2D analysis will deliver conservative results, but only if the most pessimistic plane in the 3D problem is selected. Even so, this result may lie well below the "true" 3D factor of safety. More importantly however, it has also been shown that selection of the "wrong" 2D plane could lead to an unconservative result.

RISK ASSESSMENT IN GEOTECHNICAL ENGINEERING

Soils and rocks are the most variable of all engineering materials, yet this is often coupled with inadequate site data. These factors combine to make geotechnical engineering one of the most appropriate areas for the application of probabilistic tools.

Risk assessment and probabilistic analyses in geotechnical engineering are rapidly growing areas of importance and activity for practitioners and academics (e.g. Baecher and Christian 2003, Fenton and Griffiths 2008). At a recent G-I specialty conference called *Georisk 2011* for example, several important state of practice papers were presented (e.g. Christian and Baecher 2011, Lacasse and Nadim 2011, Scott 2011) and in this *GeoCongress 2012*, Lacasse et al. 2012 have presented a comprehensive review of the state of risk assessment and mitigation in geo-practice. It is now commonplace for major geotechnical conferences to include sessions on risk assessment in geotechnical engineering.

Of all areas of geotechnical engineering, slope stability analysis has received greater attention using risk assessment tools than any other, since the concept of replacing a "factor of safety" by a "probability of failure" is immediately appealing to many engineers (see e.g. Alonso 1976, Catalan and Cornell 1976, Li and Lumb 1987, Oka and Wu 1990, Chowdhury and Xu 1992, Mostyn and Soo 1992, Juang et al. 1992, Mostyn and Li 1993, Lacasse 1994, Lacasse and Nadim 1996, Liang et al 1999, Malkawi et al. 2000, Griffiths and Fenton 2000,2004, Duncan 2000, El Ramly et al. 2002, Bhattacharya et al. 2003, Babu and Mukesh 2004, Jiminez-Rodriguez et al. 2006, Low and Tang 2007, Hong and Roh 2009, Griffiths et al. 2009a, Huang et al. 2010, Ching et al. 2010, Mbarka et al. 2010, Wang et al. 2011).

The Random Finite Element Method (RFEM)

The goal of a probabilistic slope stability analysis is to estimate the probability of slope failure as opposed to the ubiquitous factor of safety used in conventional analysis. Several relatively simple tools exist for performing this calculation that include the First Order Second Moment (FOSM) method and the First Order Reliability Method (FORM). The FORM method in particular has now been developed to a quite significant level of sophistication to tackle correlation and system slope reliability (e.g. Low et al. 2007, Low et al. 2011).

A legitimate criticism of these first order methods however, is that they are unable to properly account for spatial correlation in the 2D or 3D random materials, and are inextricably linking with "old fashioned" slope stability methods that involve simple shapes for the failure surfaces (typically circular).

To overcome these deficiencies, a method called the Random Finite Element Method (RFEM) that combines random field theory with deterministic finite element analysis was developed by the authors in the early 1990's and has been applied to a wide range of geotechnical applications (e.g. Griffiths and Fenton 2007, Fenton and Griffiths 2008). In a stability analysis, input to RFEM is provided in the form of the mean, standard deviation and spatial correlation length of the soil strength parameters which may consist of several layers with different statistical input parameters. In the absence of site specific information, there is an increasing number of publications presenting typical ranges for the standard deviation of familiar soil properties (e.g. Lee et al. 1983).

In RFEM, local averaging is fully accounted for at the element level indicating that the mean and standard deviation of the soil properties are statistically consistent with the mesh density. Since the finite element method of slope stability allows mechanisms to "seek out" the most critical path through the soil, the method offers great promise for more realistic reliability assessment of slopes and other geotechnical applications. The flow chart for a typical RFEM slope stability analysis is shown in Figure 15.

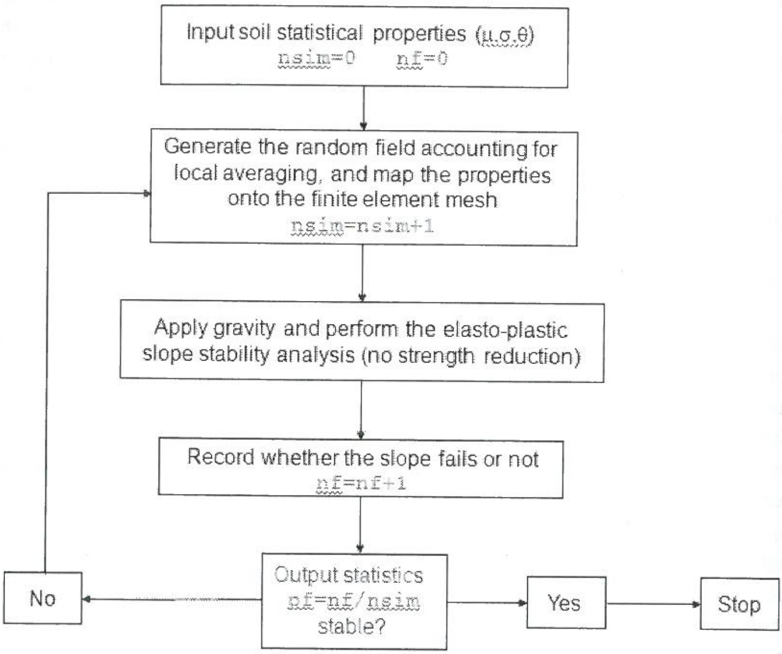

Fig. 15. Flow chart for a typical RFEM slope stability analysis.

The RFEM codes developed by Griffiths and Fenton for a range of geotechnical applications are freely available in source code from the authors' web site at `www.mines.edu/~vgriffit/rfem`. The 2D slope stability program is called `rslope2d`. A couple of failure mechanisms computed using this program for slopes with quite different spatial correlation lengths but with the same mean and standard deviation of strength parameters are shown in Figure 16. The spatial correlation length is expressed in dimensionless form relative to the height of the embankment, e.g. $\Theta_C = 0.5$ means the spatial correlation length is $0.5H$ etc. It is seen that the slope with the higher spatial correlation length in the lower figure gives a quite smooth failure mechanism more like the classical "mid-point" circle. The soil with a lower spatial correlation length in the upper figure however, displays a quite complex system of interacting mechanisms which would defy analysis by any traditional LEM.

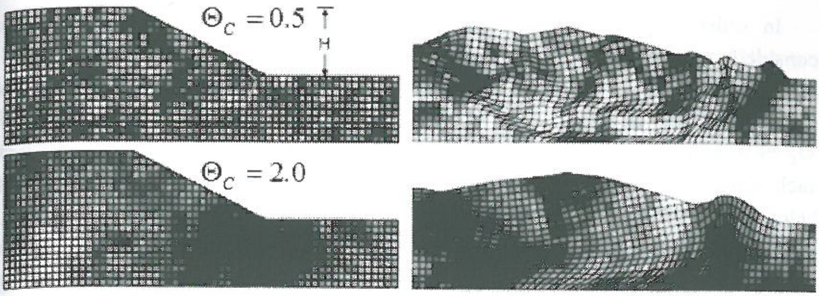

Fig. 16. Typical failure mechanisms from an RFEM analysis with two different spatial correlation lengths.

Following the results of Griffiths and Fenton (2004), the RFEM results for an undrained clay slope with a spatially random, lognormally distributed dimensionless undrained strength given by $C = c_u/(\gamma_{sat}H)$ is shown in Figure 17. The computed probability of failure by RFEM (p_f) is given as a function of the spatial correlation length $(\Theta_C = \theta_{\ln C}/H)$ and the coefficient of variation $(V_C = \sigma_C/\mu_C)$. It can be seen that an increasing correlation length may either increase or decrease the slope failure probability depending on the input coefficient of variation V_C.

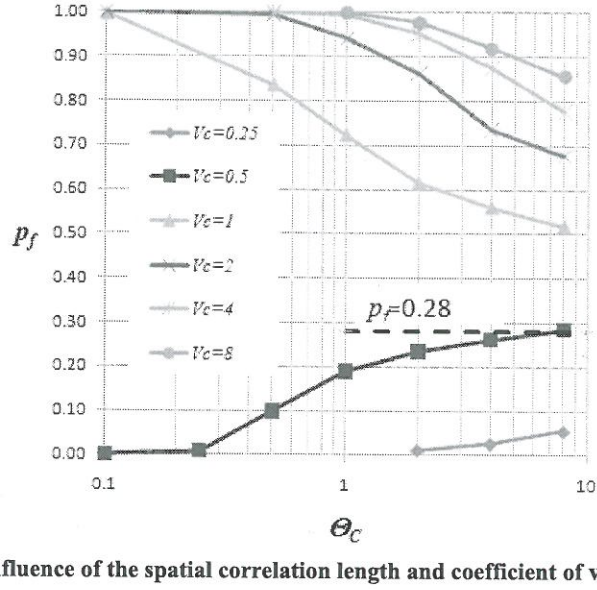

Fig. 17. Influence of the spatial correlation length and coefficient of variation on the probability of failure of an undrained slope $(\mu_C = 0.25)$.

In order to interpret these results, a couple of key deterministic solutions considering a homogeneous soil, should be kept in mind. (i) if $C = 0.25$, $FS = 1.47$ and (ii) if $C = 0.17$, $FS = 1.0$. The diverging results from the probabilistic studies shown in Figure 17 can then be explained by considering the limiting cases of $\Theta_C \to 0$ and $\Theta_C \to \infty$. As $\Theta_C \to 0$, the slope becomes essentially homogeneous at each simulation, with a constant strength given by its median. If the median falls below 0.17, all simulations fail and $p_f \to 1$, but if the median is greater than 0.17, none of the simulations fail and $p_f \to 0$. On the other hand, as $\Theta_C \to \infty$, each simulation involves a homogeneous soil with the property varying from one simulation to the next, so $p_f \to \mathrm{P}[C < 0.17]$.

For example, in the case of $\mu_C = 0.25$, $V_C = 0.5$, the parameters of the underlying normal distribution of $\ln C$ are given as

$$\begin{aligned} \mu_{\ln C} &= \ln \mu_X - \frac{1}{2}\ln\left\{1 + V_C^{\,2}\right\} = -1.498 \\ \sigma_{\ln C} &= \sqrt{\ln\left\{1 + V_C^{\,2}\right\}} = 0.472 \end{aligned} \tag{2}$$

hence

$$p_f = \Phi\left(\frac{\ln 0.17 - \mu_{\ln C}}{\sigma_{\ln C}}\right) = 0.28 \tag{3}$$

which is shown as the asymptotic trend of the line corresponding to $V_C = 0.5$ as $\Theta_C \to \infty$ in Figure 17.

On the other hand, as $\Theta_C \to 0$, the median of the shear strength is given by

$$\mathrm{Median}_C = \exp(\mu_{\ln C}) = \exp(-1.498) = 0.22 > 0.17 \tag{4}$$

hence $p_f \to 0$.

First order methods and single random variable Monte-Carlo methodologies that treat each simulation as a homogeneous material can be considered special cases of RFEM with $\Theta_C \to \infty$ but cannot be guaranteed to deliver conservative results.

Influence of Mesh Refinement

A commonly asked question of any finite element analysis, including RFEM, is the extent to which mesh refinement and discretization errors affect the results. As mentioned previously, the statistics of the random field mapped onto the finite element mesh are adjusted in a consistent way to account for element size.

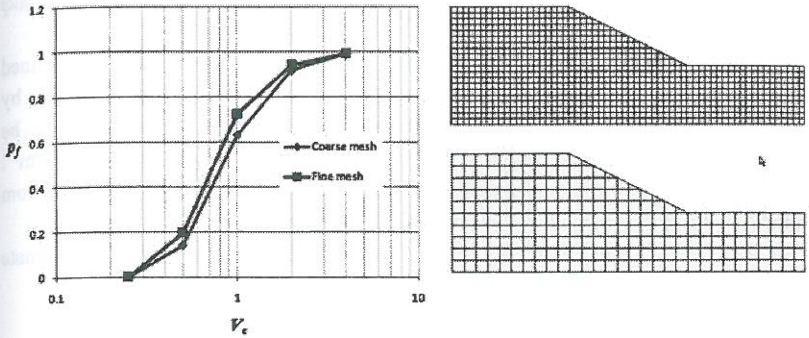

Fig. 18. Influence of mesh density on p_f for an undrained slope.

This is an integral part of the Local Average Subdivision method (Fenton and Vanmarcke 2000). As for the overall discretization issue, Figure 18 shows the influence of mesh refinement for two different cases with $\Theta_C = 1$ and $\mu_C = 0.25$. It can be seen that the finer mesh gives somewhat higher values of p_f, which is to be expected, since more paths are available for failure to occur.

IMPORTANCE OF SPATIAL VARIABILITY

In the following section we present two short examples that emphasize the importance of proper modeling of spatial variability in slope risk assessment.

Infinite Slope Example

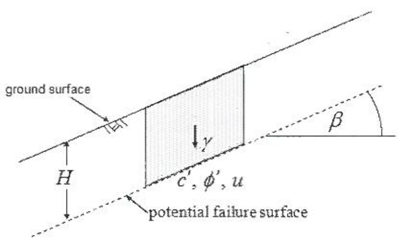

Fig. 19. Geometry and parameters of an infinite slope

This is one of the oldest and simplest types of slope problem in which the failure mechanism is assumed to be purely translational with the failure plane at the base of the layer. In the absence of pore pressures $(u=0)$, the factor of safety can be expressed explicitly by the equation

$$FS=\frac{c'}{\gamma H\sin\beta\cos\beta}+\frac{\tan\phi'}{\tan\beta} \tag{2}$$

In this example (Griffiths et al. 2011b) the cohesion is defined by $\mu_{c'}=10\,\text{kN/m}^2$ and $\sigma_{c'}=3.0\,\text{kN/m}^2$ and the tangent of the friction angle by $\mu_{\tan\phi'}=0.5774$ and $\sigma_{\tan\phi'}=0.1732$. The remaining parameters are assumed to be deterministic with values given by $H=5.0$ m, $\beta=30°$, and $\gamma=17.0\text{ kN/m}^3$. Substitution of these deterministic parameters and the mean values of the random variables into Eq. (2) leads to a deterministic factor of safety of $FS=1.27$.

From Eq. (2), and assuming c' and $\tan\phi'$ are uncorrelated, we can estimate the mean and standard deviation of FS by the FOSM as

$$\mu_{FS}\approx\frac{\mu_{c'}}{\gamma H\sin\beta\cos\beta}+\frac{\mu_{\tan\phi'}}{\tan\beta} \tag{3}$$

$$\sigma_{FS}\approx\sqrt{\left(\frac{1}{\gamma H\sin\beta\cos\beta}\right)^2\sigma_{c'}^2+\left(\frac{1}{\tan\beta}\right)^2\sigma_{\tan\phi'}^2} \tag{4}$$

which gives $\mu_{FS}=1.27$ and $\sigma_{FS}=0.311$

Assuming that FS is lognormal, the probability of failure is then given by

$$p_f=\text{P}[FS<1]=\text{P}[\ln(FS)<\ln(1)]=\Phi\left[-\frac{\mu_{\ln FS}}{\sigma_{\ln FS}}\right] \tag{5}$$

where the mean and standard deviation of the underlying normal distribution of $\ln(FS)$ are given by $\mu_{\ln(FS)}=0.2113$ and $\sigma_{\ln(FS)}=0.2409$. After substitution

$$p_f=\Phi\left[-\frac{0.2113}{0.2409}\right]=\Phi[-0.8772]=1-\Phi[0.8772]=1-0.810=0.19 \tag{6}$$

hence the probability of failure is approximately 19.0%. It should be noted that this result, being based on the deterministic Eq.(2), assumes failure always occurs at the base of the layer.

The same problem was then solved using RFEM by including lognormal and uncorrelated c' and $\tan\phi'$ and a range of spatial correlation lengths defined in dimensionless form as $\Theta=\theta/H$ (assumed in this example to be the same for both

c' and $\tan\phi'$). The results shown in Figure 20 indicate that the FOSM results are consistently unconservative, but less so as $\Theta \to \infty$. This is because in RFEM, failure takes place along the weakest path, which doesn't necessarily occur at the base of the layer. For shorter values of Θ, the critical plane is more likely to occur above the base and p_f is higher. The figure also shows a typical random field and failure plane from the RFEM Monte-Carlo analyses.

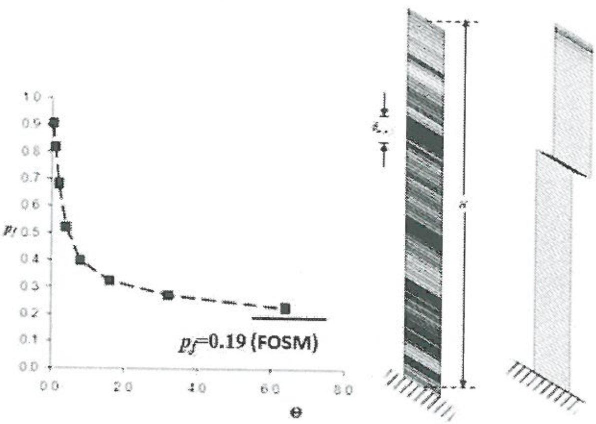

Fig. 20. Comparison of RFEM and FOSM results for an infinite slope analysis.

Three dimensional slope reliability

Since the 2D factor of safety is generally considered to be conservative, practitioners are reluctant to invest in the more time-consuming 3D approaches. A key question to be addressed is, under what circumstances will the probability of failure of a slope predicted by a full 3D analysis be higher than that obtained from an equivalent 2D analysis?

In all the RFEM analyses that follow (Griffiths et al. 2009b), the bottom of the mesh $(y=-H)$ is fully fixed and the back of the mesh $(x=0)$ is allowed to move only in a vertical plane. Both "rough" and "smooth" boundary conditions have been considered at the ends in the out-of-plane direction $(z=0 \text{ and } L)$. In the rough cases the ends are fully fixed and in the smooth case, they are allowed to move only in a vertical plane. It is noted that unlike the deterministic study shown previously, there is no symmetry in the RFEM analyses due to the spatial varying soil properties. In this study, it was determined that 2000 realizations of the Monte-Carlo process for each parametric group, was sufficient to give reliable and reproducible estimates of the probability of failure p_f.

The undrained clay slope shown in Figure 21 demonstrates an important characteristic in 3D slope analysis called the "preferred" failure mechanism width W. This is the width of the failure mechanism in the z – direction that the finite element analysis "seeks out". Over a suite of Monte-Carlo simulations the average preferred failure mechanism width is called W_{crit}. It will be shown that this dimension has a significant influence on 3D slope reliability depending on whether the length of the slope L is greater than or less than W_{crit}.

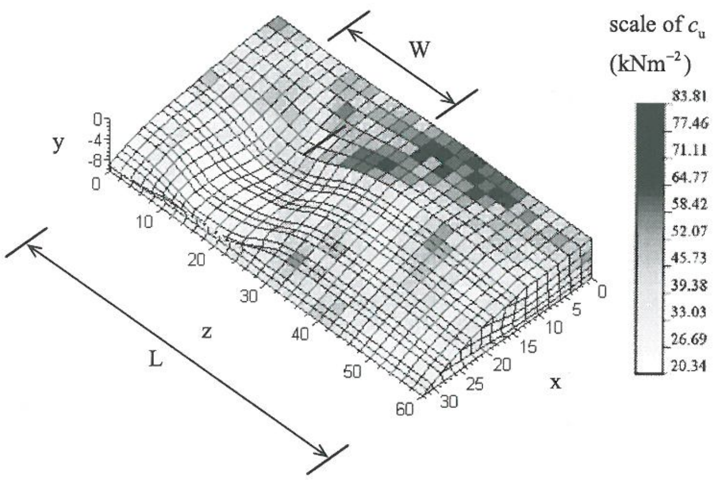

Fig. 21. Slope failure with (isotropic) $\Theta_C = 2.0$ and rough boundary condition

With the same definition of spatial correlation used earlier in the paper for 2D slope analysis $(\Theta_C = \theta_{\ln C}/H)$, the length ratio was varied in the range $0.2 < L/H < 16$ to investigate the influence of three-dimensionality, with results presented in Figure 22.

In the case of smooth boundary conditions, the p_f of one slice $(L/H = 0.2)$ in the 3-d analysis is equivalent to that given by a 2D RFEM analysis since the 3D analysis is essentially replicating plane strain.

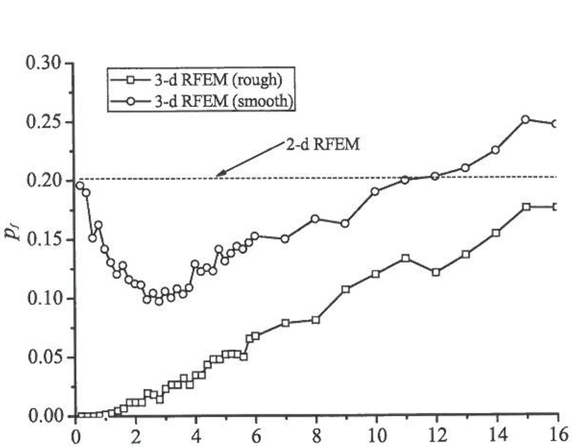

Fig. 22. Probability of failure versus slope length ratio L/H

$$\left(V_{c_u} = 0.5, \Theta_C = 1.0,\ FS = 1.39, \text{slope angle 2h : 1v}\right)$$

It is also shown in the smooth case that as L/H is increased, p_f initially decreases, reaching a minimum before rising to eventually exceed the 2D value. In the rough case, p_f is close to zero for a narrow slice and increases steadily as L/H is increased due to a gradual reduction in the supporting influence of the rough boundaries in the 3D case. As the length ratio is increased in both the rough and smooth cases, the 3-d p_f eventually exceeds the 2D value, indicating that 2D analysis will be always give unconservative results if the slope is long enough. It may also be speculated that $p_f \to 1$ as $L/H \to \infty$ regardless of boundary conditions.

For the case of smooth boundary conditions, let us define the critical slope length L_{crit} and the critical slope length ratio $(L/H)_{crit}$ as being that value of L/H for which the slope is safest and its probability of failure p_f a minimum. It will be shown that this minimum probability of failure in the smooth case occurs when $L_{crit} \approx W_{crit}$. If we reduce the slope length ratio below this critical value $(L < L_{crit})$, the slope finds it easier to form a global mechanism spanning the entire width of the mesh with smooth end conditions, so the value of p_f increases, tending eventually to the plane strain value. However, if we increase the slope length ratio above this critical value $(L > L_{crit})$, the slope finds it easier to form a local mechanism. Since $L > W_{crit}$ the mechanism has more opportunities to develop somewhere in the $z-$direction hence p_f again increases.

CONCLUDING REMARKS

The paper has demonstrated the power and advantages of the finite element method for both deterministic and probabilistic slope stability analysis in highly variable soils. Results were presented indicating the limitations of 2D analysis in infinite slope and 3D slope analysis. It was shown that 2D slope analysis is only conservative if the most pessimistic plane in the 3D geometry is chosen. Even then, the result may be excessively conservative. More seriously however, poor selection of the 2D plane for analysis could lead to unconservative results.

Examples of slope risk analysis were presented using the random finite element method (RFEM) developed by the authors. It was shown that single random variable approaches can give unconservative results compared with RFEM using 2D random fields. The key benefit of RFEM is that it does not require any *a priori* assumptions related to the shape or location of the failure mechanism. In an RFEM analysis, the failure mechanism has freedom to "seek out" the weakest path through the random soil, which generally leads to more simulations reaching failure. The importance of spatial variability was further demonstrated in two examples involving an infinite slope and a 3D slope. In both cases, failure to account for spatial variability could lead to unconservative results.

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