A comparison of numerical algorithms in the analysis of pile reinforced slopes

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Abstract

The paper describes the influence of pile reinforcement on the stability of slopes through numerical analysis. Included in the paper is some discussion of the modifications made to include pile reinforcement in an existing finite element slope stability program that uses the strength reduction method. Then the finite element program developed is compared for accuracy in the solution of the piled slope problem with a popular proprietary code that uses the finite difference method. Finally, parametric studies are presented to assess the influence of pile location and length on the slope stability.

1 Introduction

Piles have been used in geotechnical engineering to stabilize slope for many years and the methodology has been accompanied by a significant bibliography (e.g. Ito and Matsui 1975; Jeong et al. 2003; Won et al. 2005; Chow 1996; Hassiotis et al. 1997; Harry 1995; Ito et al. 1981; Laudeman and Chang 2004; Poulos and Chen 1997). In the past, methods of analysis of pile-reinforced slopes have often used limit equilibrium methods, where soil–pile interaction was not properly considered (e.g. Won et al. 2005). Recently, with rapid development of computer techniques, numerical methods using either finite element or finite difference methods have been widely applied in slope engineering, and have been shown to offer many advantages over limit equilibrium method (Griffiths and Lane, 1999), such as the ability to develop the critical failure surface automatically with fewer assumptions.

In this paper, we will make some modifications for an existing finite element slope stability program that uses the strength reduction method, to include pile reinforcement. Results obtained using the developed finite element program are then compared for accuracy in the solution of the piled slope problem with a popular proprietary code that uses the finite difference method. Finally, parametric studies are presented to assess the influence of pile location and length on slope stability and the factor of safety.

2 Finite element slope stability program including pile reinforcement

The programs used in this paper are based on Program 6.3 in the text by Smith and Griffiths (2004), and have been modified to include the pile reinforcement in slope to form a new program (named p63_s). The program is for two-dimensional plane strain analysis of elastic perfectly plastic soils with a Mohr-Coulomb failure criterion utilizing eight-node quadrilateral elements with reduced integration (four Gauss points per element) in the gravity loads generation, the stiffness matrix generation and the stress redistribution phases of the algorithm. The soil is initially assumed to be elastic and the model generates normal and shear stresses at all Gauss points within the mesh. These stresses are then compared with the Mohr-Coulomb failure criterion. If the stresses at a particular Gauss point lie within the Mohr-Coulomb failure envelope, then that location is assumed to remain elastic. If the stresses lie on or outside the failure envelope, then that location is assumed to be yielding.

The pile is simulated by a beam-rod element, based on Program 4.3 in the text by Smith and Griffiths (2004) which contains three degrees of freedom for each node (two translational and one rotational). The beam-rod element stiffness matrix is formed by superposing the beam and rod stiffness matrices and can sustain axial and transverse loads in addition to moments.

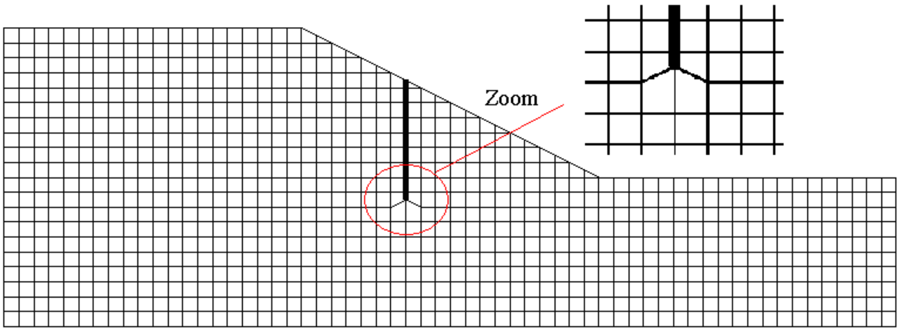

Fig.1 Numerical model for slope with pile reinforcement

In order to add a pile element to the slope, the following modifications were made,

(1) the coordinates of the mesh were adjusted to accommodate the lateral location and length of the pile as shown in Figure 1 ;

(2) the soil stiffness matrix `km` of elements adjacent to the pile were augmented by the pile element stiffness matrix `p_km`. Each slope element will usually be adjacent to two pile elements. For example as shown in Figure 2, `km` for slope

element `iel` is augmented in its upper part.

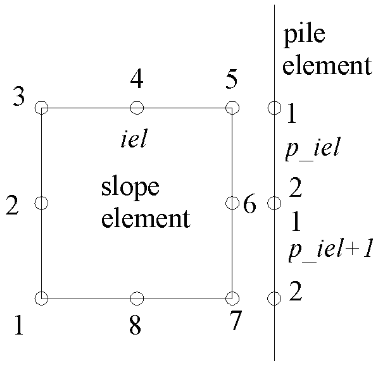

Fig.2 Local node numbering for soil and pile elements.

3 Validation for the program

3.1 Slope model

In order to validate the program p63_s, its calculated results are compared with those obtained using FLAC2D. Firstly, the same homogenous slopes are formed by two programs (p63_s and FLAC2D) as shown in Figures 3 and 4. The height of the slope is 10m, with a slope angle of 26.56° (2:1 gradient). Parameters of the slope are 20.0 kN/m^3 for unit weight, 1×10^5 kPa for elastic modulus, 0.3 for Poisson' ratio, 15.0kPa for cohesion, and 20.0° for friction angle. Parameters of pile are 0.62m for diameter *D* and 25×10^6 kPa for elastic modulus *E*. Then axial rigidity EA and bending stiffness EI for the beam-rod elements can be formed by,

$$EA = E\cdot\frac{1}{4}\pi D^2 = 7.55\times10^6 \text{ kN}$$

$$EI = E\cdot\frac{\pi D^4}{64} = 1.81\times10^5 \text{ kNm}^2$$

In the actual situation, piles are driven periodically in the third direction, which means that some kind of averaging of pile properties must be accounted for when performing a 2D (plane strain analysis). We do not have the space in the current paper to address this in detail, although strategies for property averaging have been discussed elsewhere (see e.g. Donovan et al. 1984).

The slope model is fixed on the bottom boundary with vertical rollers on the side boundaries. The factor of safety (*F*) of a soil slope is defined as the number by which the original shear strength parameters must be divided in order to bring the slope to the point of failure. This method is referred to as the 'shear strength reduction technique' (Zienkiewicz et al. 1975, Griffiths 1980, Matsui and San (1992), Ugai and Leshchinsky (1995), Griffiths and Lane 1999).

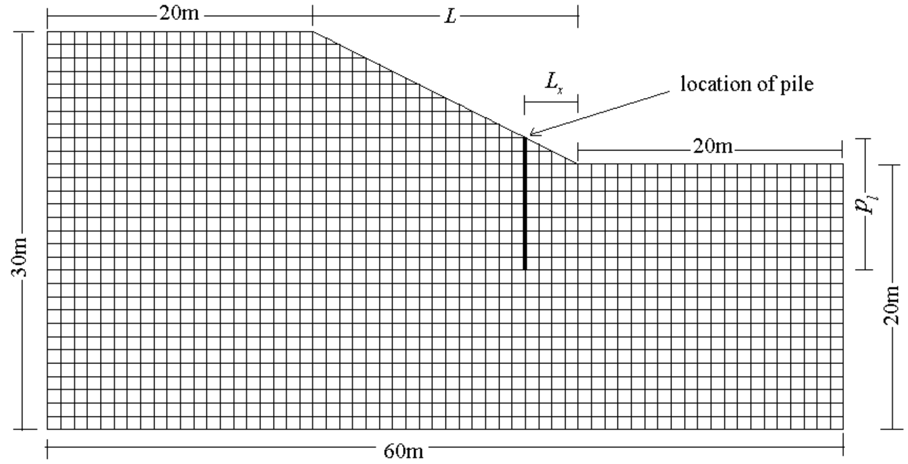

Fig.3 FE model for p63_s with 1510 elements and 4711 nodes.

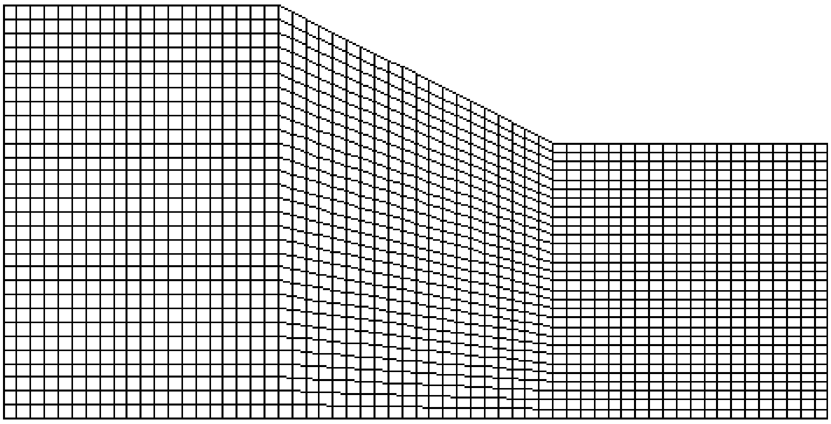

Fig.4 FD model for FLAC2D with 1800 zones and 1891 grid points.

3.2 Comparison

Comparisons are done for slopes reinforced by the pile with maximum length, results are shown in Tables 1, where L_x is the horizontal distance between pile location and the slope toe. It can be seen that the factor of safety *F* values from p63_s are similar to those from FLAC2D with p63_s giving slightly lower (conservative) values. When taking into consideration the CPU time required by each of the models on the same computer, both p63_s and FLAC2D take about 3 minutes per run.

Table 1. Comparison of results obtained by p63_s and FLAC2D for a slope reinforced by pile with maximum length $\left(p_l = 25 \text{ m}\right)$

L_x/L	FLAC2D F_1	P63_s F_2	$(F_1-F_2)/F_2\times100\%$ / %
No pile	1.61	1.58	1.898
0.0	1.64	1.59	3.145
0.1	1.72	1.67	2.994
0.2	1.83	1.78	2.809
0.3	1.97	1.89	4.233
0.4	2.16	2.06	4.854
0.5	2.41	2.28	5.702
0.6	2.23	2.19	1.826
0.7	2.03	2.00	1.500
0.8	1.89	1.86	1.613
0.9	1.78	1.75	1.714
1.0	1.68	1.67	0.599

4 Parametric study

Initial studies indicated that the soil elastic modulus, pile elastic modulus and diameter had little effect on computed slope factor of safety so long as the pile elements were significantly stiffer than the soil modeling an essentially “rigid” pile.

Parametric studies were performed to assess the influence of pile location and length. The pile was assumed to be driven at varying distances from the slope toe, with L_x / L varied from 0 to 1, with the pile length varied from 6 m to 16 m at each location. The calculation model is the same as Figure 3.

The computed slope factor of safety by program p63_s are plotted in Figure 5 indicating that as L_x / L increases, the factor of safety initially rises and then falls. For shorter piles, e.g. $6\text{m} \le p_l \le 8\text{m}$, the slope factor of safety reached its maximum value at $L_x/L \approx 0.3$ which is in the lower part of slope surface. For longer piles, e.g. $p_l \ge 10\text{m}$, the slope factor of safety reached its maximum value at $L_x/L \approx 0.5$

which is in the middle of slope surface. Ideally it appears the most effective location for the pile would be in the lower half of the slope, although this may not be a practical location for access.

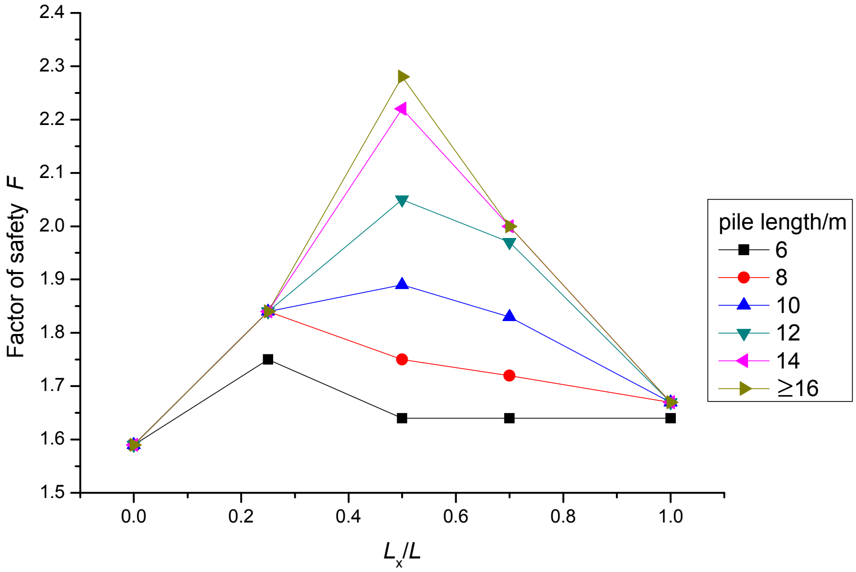

Fig.5 Effect of pile location and length on the slope factor of safety

The influence of pile length depends on its location. For the case considered, if the pile is driven at the slope vertex or toe its length has little effect on the slope factor of safety. If the pile is driven at the middle of slope surface ($L_x/L = 0.5$) however, its length has a considerable influence as shown in Figure 6. For pile lengths over a critical value (e.g. $p_l \geq 16\text{m}$), the factor of safety will remain constant,

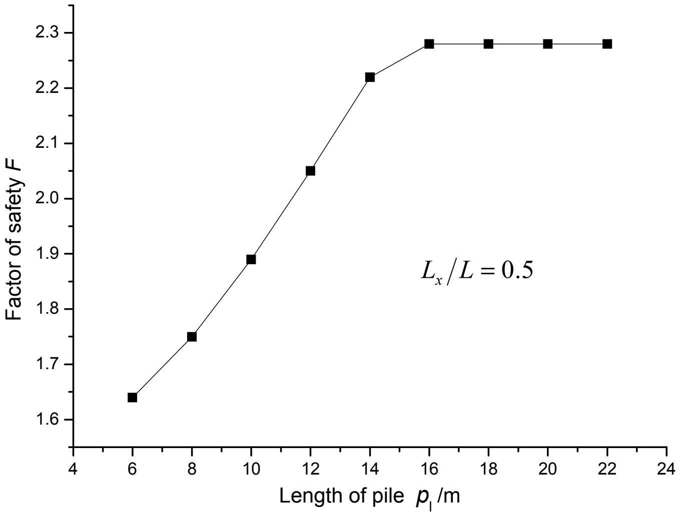

Fig.6 Relationship between slope factor of safety and pile length $\left(L_x/L=0.5\right)$

In order to further study the effect of pile length on the potential slip plane when it is driven in the middle of slope surface, we obtained the potential slope slip surface from the graphical output of displacement vectors from p63_s as shown in Figure 7. The effect of pile length on the potential slip surface is shown in Figure 8 indicating how the surface is forced to run beneath the bottom of the pile. With no pile at all, the surface corresponds to a classical "toe" failure mechanism, but as the pile length is increased, the surface is forced ever deeper into the soil mass, with a corresponding increase in the factor of safety. When the pile length is greater than 14 m however, the potential slope failure surface radically relocates to a very shallow location just uphill of the pile tip. The change in location presumably occurs because the shallow mechanism requires less energy to develop than the much longer path navigating its way beneath the pile.

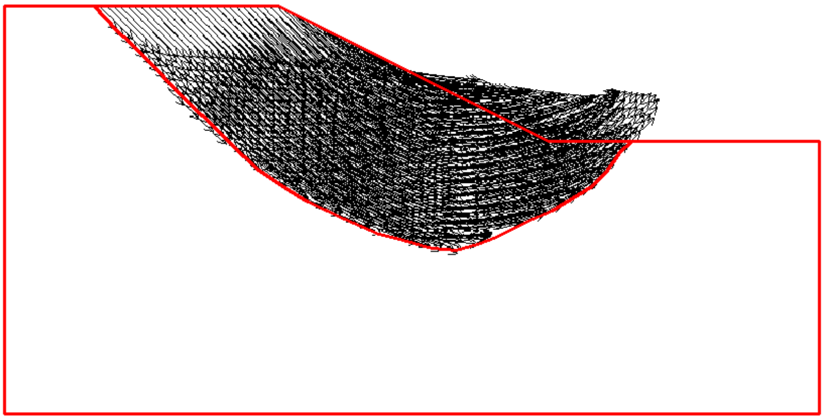

Fig.7 Displacement vectors of slope at failure.

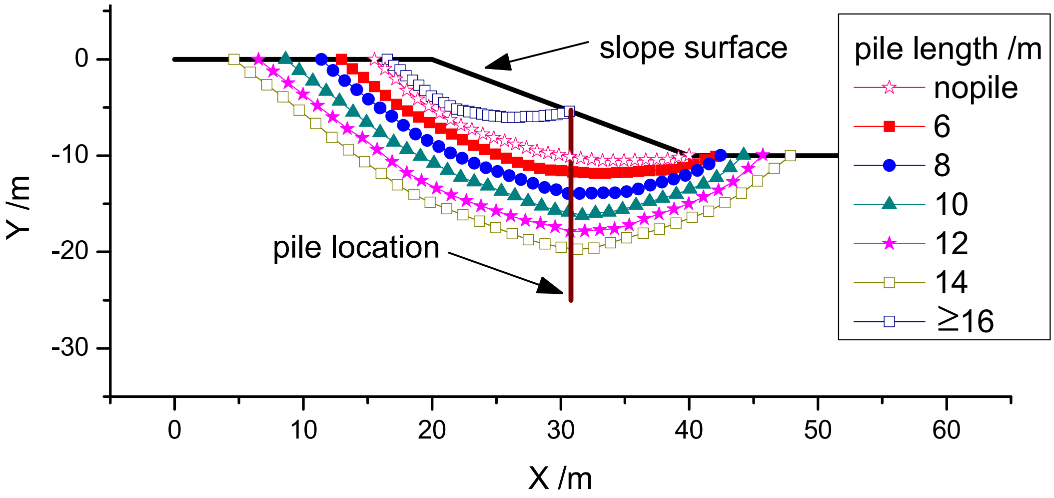

Fig. 8 Effect of pile length on the location of potential slip surfaces $\left(L_x/L=0.5\right)$

5 Conclusions

Parametric studies were performed to assess the influence of pile location and length on slope factor of safety. Although not necessarily a practical location for installation purposes, the optimal location of the pile was found to be approximately half way down the slope. For a pile at this optimal location, it was observed that the factor of safety increased almost linearly with pile length until a critical depth was reached after which the factor of safety remained constant. This result was explained by studying the failure surface locations for different pile lengths. As the pile length was increased, the surface took an ever longer path as it passed below the pile tip causing the factor of safety to increase. A point was reached however as the pile length was further increased, when the energy required for the failure surface to pass below the pile tip became excessive, at which point the surface rapidly transformed to a much shallower location. Once this happened, further lengthening of the pile had not influence. Using strength reduction, a brief comparison between analyses performed using an FE program developed by the authors from the Smith and Griffiths (2004) system called p63_3 and FLAC2D indicated broadly similar results and run-times.

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