Consequence Factors for Use in Shallow Foundation Reliability-Based Design

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ABSTRACT
The reliability-based design of shallow foundations is generally implemented via a load and resistance factor design methodology embedded in a limit states design framework. For any particular limit state, the design proceeds by ensuring that the factored resistance exceeds the factor load effects. Load and resistance factors are determined so as to ensure that the resulting design is sufficiently safe. Load factors are typically prescribed in structural codes and take into account load uncertainty. Factors applied to the resistance depend on both uncertainty in the resistance (resistance factor) and the target reliability level (consequence factor). This paper concentrates on how the consequence factor can be defined and specified in order to adjust the target reliability of a shallow foundation.

RÉSUMÉ
La conception de fiabilité-basé de fondations peu profondes est généralement exécutée via un chargement et une méthodologie de conception de facteur de résistance a enfoncé dans un cadre de conception d'états de limite. Pour l'état particulier de limite, la conception procède en garantant qui l'a factorisé la résistance dépasse les effets de chargement de facteur. Les facteurs de chargement et résistance sont si déterminés comme garantir que la conception résultante est suffisamment sûre. Les facteurs de chargement sont typiquement prescrits dans les codes structurels et prennent en compte l'incertitude de chargement. Les facteurs se sont appliqués à la résistance dépend des deux incertitude dans la résistance (le facteur de résistance) et le niveau de fiabilité de cible (le facteur de conséquence). Ces concentrés en papier sur comment le facteur de conséquence peut être défini et peut être spécifié pour ajuster la fiabilité de cible d'une fondation peu profonde.

1 INTRODUCTION

Geotechnical design codes and manuals world wide are starting to migrate away from working stress design towards reliability-based design. In all codes reviewed by the authors (AASHTO, 2007, CHBDC, 2006, NBCC, 2005, Eurocode 7, 2003, AS5100, 2004, DGI, 1985), the design reliability is achieved by making use of a load and resistance factor methodology embedded in a limit state design (LSD) framework. The LSD framework basically involves identifying possible failure modes (e.g. sliding, overturning, and bearing capacity failures) and then ensuring that the factored resistance to the failure mode is greater than or equal to the factored load effects which are trying to cause the failure. This paper will consider only the ultimate bearing capacity limit state for shallow foundations. Thus, for the bearing capacity ultimate limit state, the load and resistance factor design (LRFD) involves dimensioning the shallow foundation so that an equation of the following generalized form is satisfied,

$$\psi_u \varphi_{gu} \hat{R}_u \geq \sum I_i \gamma_i \hat{F}_i \qquad [1]$$

in which $\hat{F}_i$ is the *i* 'th characteristic load effect , γ_i is its corresponding load factor, I_i is an importance factor, $\hat{R}_u$ is the ultimate geotechnical resistance obtained using characteristic geotechnical parameters, φ_{gu} is the ultimate geotechnical resistance factor, and ψ_u is a consequence factor.

The ultimate geotechnical resistance factor, φ_{gu}, reflects uncertainty in the geotechnical parameters used to estimate $\hat{R}_u$ while both the consequence factor, ψ_u, and the importance factor, I_i, are used to adjust the target reliability level. Note that the consequence factor is newly introduced here, for reasons discussed below. It serves the same basic purpose as the importance factor – namely to adjust the target failure probability, which depends on the failure consequence level (lower failure probabilities for higher failure consequences). It is important to avoid double-factoring via these two factors where not warranted.

The reasons a consequence factor is introduced here into Eq. 1 are as follows. First of all, the importance factor is well ensconced in structural engineering codes. It is largely aimed at adjusting the factored characteristic loads to account for failure consequence and is generally based on site specific load distributions (usually snow, wind, and earthquake). It makes sense to apply the importance factor to the load side of Eq. 1 because snow, wind, and earthquake loads, for example, are quite site specific. On the resistance side, structural engineers typically deal with quality controlled materials (e.g. steel, concrete, and wood), whose distributions are well known and relatively constant world-wide. Thus, for structural engineers, the resistance factor alone generally adequately accounts for resistance uncertainty.

On the other hand, geotechnical engineers are faced with large resistance uncertainties from site to site, and even within a site, that are quite unrelated to the loading type. There is a real desire in the geotechnical community to account for failure consequence even when the loading consists of just typical dead and live loads. For example, the current Canadian Highway Bridge Design Code (CHBDC, 2006) only provides an importance factor for earthquake loading. Nevertheless, under any loading scenario there is a huge difference between the consequences of failure of a major lifeline highway bridge in Toronto, for example, and a bridge on a minor back road in northern Canada.

The consequence factor proposed in Eq. 1 is aimed at adjusting the factored resistance to account for failure consequences in those cases not covered by the load side importance factor. The authors note that further research needs to be performed in order to establish the interaction between the importance and consequence factors and their combined effect on failure probability. Until such research has been carried out, the authors recommend that the consequence factor be set to 1.0 whenever the importance factor is other than 1.0. In this paper, the importance factors, I_i, will be assumed to have values 1.0. The LRFD equation considered in this paper thus has the form

$$\psi_u \varphi_{gu} \hat{R}_u \geq \sum \gamma_i \hat{F}_i \qquad [2]$$

Three target reliabilities will be considered; high, medium, and low, corresponding to important structures where failure has large consequences (e.g. hospitals, schools, and lifeline highway bridges), typical structures which constitute the majority of civil engineering projects, and low-failure consequence structures (e.g. storage sheds, low use bridges, etc.). Most designs will be aimed at the typical failure consequence level, which in this paper will be assumed to have a maximum lifetime failure probability, p_m, of about 1/5000. This corresponds to a reliability index of about $\beta = 3.5$, (e.g. Meyerhof, 1994). Note that this target failure probability commonly assumes some structural redundancy (as typically required in structural codes), so that the actual system lifetime failure probability is significantly less than the component target failure probability, p_m. The effect of redundancy in geotechnical components on reliability is still in need of further research.

The load factors on the right-hand-side of Eq. 2 will be taken from the National Building Code of Canada (NBCC, 2005). The resistance factor, φ_{gu}, is derived using the theoretical framework presented by Fenton et al (2008), as summarized in the next section, to achieve the medium target reliability ($p_m \simeq 1/5000$). For the medium (typical) target reliability, the consequence factor, ψ_u, is set to 1.0.

The remainder of the paper concentrates on the consequence factor, ψ_u, and how it varies with respect to target failure probability and site uncertainty.

Ideally, the two factors on the resistance side, ψ_u and φ_{gu}, are independent of one another, the consequence factor dependent only on the desired reliability and the resistance factor only dependant on the level of site uncertainty. However, as will be shown in the sequel, the consequence factor does have some secondary dependence on site uncertainty. The dependence is relatively slight, so that code recommendations regarding the consequence factor can still be made considering only target reliability

2 FAILURE PROBABILITY

In order to determine the required resistance and consequence factors, the probability of a shallow foundation reaching its bearing capacity ultimate limit state must be estimated. This probability will depend on the load distribution, the load factors selected, and the resistance distribution. The details of the following mathematical analysis can be found in Fenton et al (2008). Only dead and live loads will be considered, with load factors $\gamma_L = 1.5$ and $\gamma_D = 1.25$ (NBCC, 2005), and the analysis will be carried out using a simple example of a strip footing founded on a weightless $c-\phi$ soil. For this case, the characteristic ultimate bearing capacity is given by Terzaghi's (1987) relationship, which for a weightless soil simplifies to

$$\hat{q}_u = \hat{c}\hat{N}_c \qquad [3]$$

in which $\hat{c}$ is the soil's characteristic cohesion, and $\hat{N}_c$ is the bearing capacity factor, which is a function of the soil's characteristic friction angle, $\hat{\phi}$,

$$\hat{N}_c = \frac{\tan^2\left(\frac{\pi}{4}+\frac{\hat{\phi}}{2}\right)\exp\left(\pi \tan\hat{\phi}\right)-1}{\tan\hat{\phi}} \qquad [4]$$

The characteristic dead and live loads are defined in terms of the means of the dead and live load distributions according to

$$\begin{aligned}\hat{F}_D &= k_D \mu_D \\ \hat{F}_L &= k_L \mu_L\end{aligned} \qquad [5]$$

where the bias factors, k_D and k_L, are estimated by Allen (1975) and Becker (1996b) to be 1.18 and 1.41, respectively. For a strip footing, the characteristic ultimate geotechnical resistance is $\hat{R}_u = B\hat{c}\hat{N}_c$, for footing width B. Using these results in Eq. 2 leads to a design relationship for the required footing width,

$$B = \frac{\gamma_L \hat{F}_L + \gamma_D \hat{F}_D}{\psi_u \varphi_{gu} \hat{c}\hat{N}_c} \qquad [6]$$

Once the footing width has been determined, the footing is constructed and loaded. The probability of failure involves determining the probability that the true lifetime extreme load acting on the footing, F, exceeds the true soil resistance, $B\bar{c}\bar{N}_c$, where the overbars indicate that these parameters are the equivalent soil parameters as 'seen' by the footing. In other words, the probability of failure is computed as

$$p_f = \mathrm{P}\left[F > B\bar{c}\bar{N}_c\right] = \mathrm{P}\left[F\,\frac{\hat{c}\hat{N}_c}{\bar{c}\bar{N}_c} > \frac{\gamma_L\hat{F}_L + \gamma_D\hat{F}_D}{\psi_u\varphi_{gu}}\right] \quad [7]$$

All five quantities on the left hand side of the inequality, i.e. $F, \hat{c}, \bar{c}, \hat{N}_c, \text{ and } \bar{N}_c$, are random. See Fenton et al (2008) for the details of their joint distribution and how the probability in Eq. 7 is computed. The characteristic soil parameters, $\hat{c}$ and $\hat{N}_c$, are obtained by sampling the soil at some distance, r, from the footing location estimating the soil's cohesion and friction angle from the sample, and then using some average measurement as the characteristic value. In particular, it is assumed here that cohesion is lognormally distributed and so the characteristic cohesion value used is the geometric average of the observations (since this is also lognormally distributed). For example, suppose that m soil samples are taken at a distance $r = 4.5$ m from the footing centerline. Then the characteristic cohesion is computed as

$$\hat{c} = \left(\prod_{i=1}^{m} c_i\right)^{1/m} = \exp\left(\frac{1}{m}\sum_{i=1}^{m}\ln c_i\right) \quad [8]$$

which is somewhat low-value dominated (i.e. a somewhat conservative estimate of the mean – it is actually an estimate of the median cohesion value). The distance that the sample is taken from the footing location affects how strongly the characteristic value is expected to match the actual cohesion under the footing. The farther the sample is taken away from the footing, the less likely it is to accurately predict conditions under the footing.

The friction angle is assumed to follow a bounded tanh type distribution (see Chapter 1 of Fenton and Griffiths, 2008, for details) which is symmetric about its mean and so its characteristic value is taken to be the arithmetic average,

$$\hat{\phi} = \frac{1}{m}\sum_{i=1}^{m}\phi_i \quad [9]$$

and this value is used in Eq. 4 to compute the characteristic bearing capacity value.

Once the probability of failure is computed via Eq. 7, it can be compared to the maximum acceptable failure probability, p_m. If p_f exceeds p_m, then the resistance factor and/or the consequence factor need to be reduced (specifically, the product $\psi_u\varphi_{gu}$ needs to be reduced). The determination of required consequence factors proceeds in two steps;

1. Consider first the medium (typical) consequence level and set $\psi_u = 1$. For a variety of different levels of variability in soil properties, degrees of spatial correlation between soil properties, and distance between footing location at sample location, estimate the probability of footing failure using Eq. 7. For each case, adjust the resistance factor, φ_{gu}, until $p_f = p_m$. This then is the required resistance factor.
2. Using the required resistance factor(s) determined in step 1 in Eq. 7, repeat the procedure of step 1 except now at the high (reduced p_m) and low (increased p_m) consequence levels and adjust the consequence factor, ψ_u, until $p_f = p_m$. This then is the required consequence factor.

3 THEORETICAL CONSEQUENCE FACTORS

Consequence factors were determined for a particular example problem with parameters as follows;

1. The mean lifetime extreme live load along the strip footing is assumed to be $\mu_L = 200$ kN/m with coefficient of variation $v_L = 0.3$. The mean dead load is assumed to be $\mu_D = 600$ kN/m with coefficient of variation $v_D = 0.15$. The mean values assumed here are not particularly important, since the design equation (see Eq. 6) takes the distance between the load and resistance distributions into account through the load and resistance factors. Both live and dead loads are assumed to be lognormally distributed.
2. The mean cohesion is assumed to be $\mu_c = 100$ kN/m^2 with coefficient of variation $v_c = 0.1, 0.2, 0.3$ and 0.5. As mentioned above, the mean value is expected to have little influence on the results, but the coefficient of variation definitely affect the resistance factor and has a slight influence on the consequence factor, as will be shown. The cohesion is assumed to be lognormally distributed.
3. The friction angle is assumed to follow a bounded distribution with $\phi_{\min} = 10^\circ, \phi_{\max} = 30^\circ$ and mean $\mu_\phi = 20^\circ$. Its coefficient of variation ranges from $v_\phi = 0.07, 0.14, 0.20,$ and 0.29 in step with the cohesion (i.e. as the cohesion variability increases, so does the friction angle variability).
4. The correlation length, θ, which measures the distance within which soil properties are significantly correlation, is varied from a low of 0.1 m to a high of 50 m. Low values of θ lead to soil properties varying rapidly spatially, while high values mean that the soil properties vary only slowly with position. A large correlation length, of say $\theta = 50$ m, means that soil samples taken well within 50 m from the footing location will (e.g. at $r = 10$ m) will be quite representative of the soil properties under the footing. Lower failure probabilities are expected when the soil is sampled well within the distance θ from the footing. Interestingly, because our soil sample is generally some sort of average, when θ is very small (say, 0.01 m), then our sample will again accurately reflect the average conditions under the footing regardless of the sampling location. The worst case correlation length occurs when θ

is approximately equal to the distance from the footing to the sampling location.

5. Three soil sampling locations are considered, directly under the footing ($r = 0$), corresponding to good site understanding, $r = 4.5$ m, corresponding to moderate site understanding, and $r = 9$ m, corresponding to lower site understanding.
6. Three consequence levels are considered: high failure consequence, medium (typical) failure consequence, and low failure consequence. Maximum acceptable failure probabilities have been assigned to these consequence levels; $p_m = 1/1000, 1/5000,$ and $1/10000$ for low, medium, and high failure consequence levels, respectively. These failure probabilities correspond to reliability indices of 3.1, 3.5, and 3.7, respectively, which is in the range of foundation reliabilities suggested in the literature (see, e.g., Meyerhof, 1995, and Becker, 1996a).

Figure 1 illustrates how the probability of bearing capacity failure changes with the consequence factor. It can be seen that fairly moderate changes in ψ_u can make large differences in p_f.

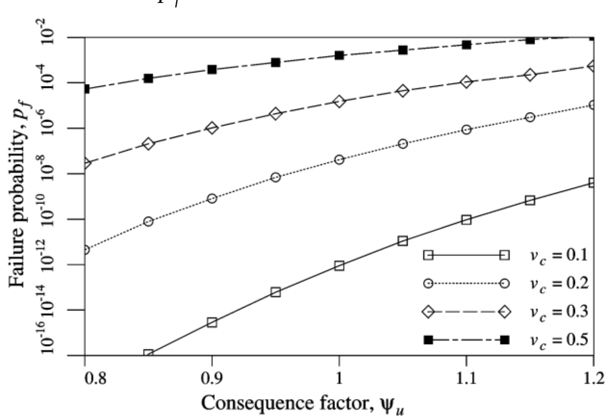

Figure 1. Failure probability versus consequence factor for $\theta = 1$ m, $r = 0$ m, and $\varphi_{gu} = 0.55$.

Figure 2 illustrates clearly the presence of a worst case correlation length. The figure presents failure probabilities for the case where the soil is sampled at $r = 4.5$ m from the footing centerline. Clearly, the worst case failure probability (highest) occurs for values of correlation length near 4.5 m. The Figure is shown for the low consequence case ($p_m = 1/1000$), for which the consequence factor $\psi_u = 1.1$ was selected. It can be seen that the worst case probability of failure, p_f, is just slightly less than the acceptable maximum probability when the coefficient of variation of the soil properties is at a moderate level ($v_c = 0.3, \quad v_\phi = 0.2$). However, if the soil property variability exceeds $v_c = 0.3, \quad v_\phi = 0.2$, then the probability of failure becomes unacceptable. See, for example, the $v_c = 0.5$ curve, which reaches a failure probability of 1/100 at the worst case correlation length. The unacceptable failure probabilities that occur for higher soil variabilities emphasizes the need to perform enough site investigation to reduce the residual variability to no more than moderate levels.

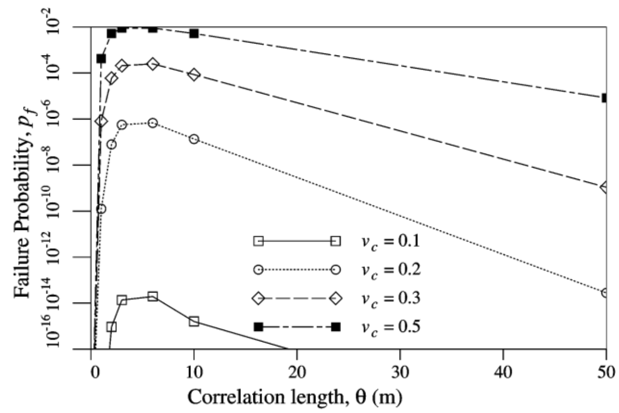

Figure 2. Failure probability versus correlation length for $\psi_u = 1.1$ (low consequence), $\varphi_{gu} = 0.35$, and $r = 4.5$ m.

As mentioned in the Introduction, the consequence factor should be ideally depend only on the target failure probability, p_m, and not on soil variability, correlation length, and sampling location. Variations in the latter three parameters should ideally be entirely handled by the resistance factor, φ_{gu}. Figures 3 and 4 investigate the effect of correlation length and sampling location on the consequence factor for low consequence level (Figure 3) and for high consequence level (Figure 4). Both figures are at a moderate variability ($v_c = 0.3, \quad v_\phi = 0.2$).

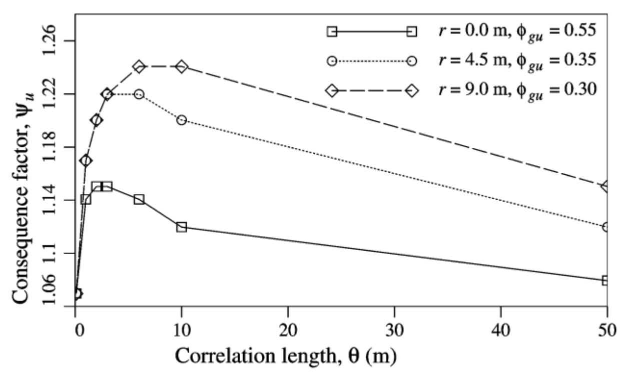

Figure 3. Consequence factor versus correlation length for various sampling locations at low consequence level ($p_m = 1/1000$)for $v_c = 0.3$.

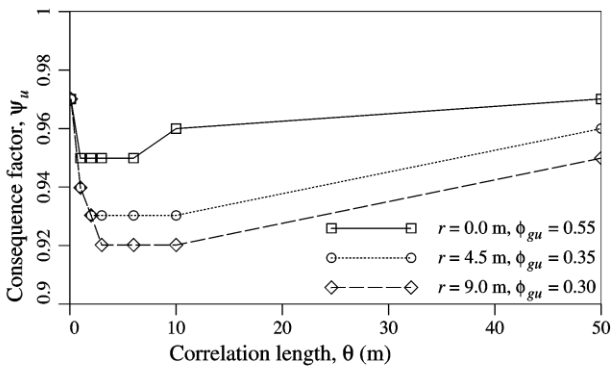

Figure 4. Consequence factor versus correlation length for various sampling locations at high consequence level ($p_m = 1/10000$) for $v_c = 0.3$.

The overall change in the consequence factor in Figure 3 with respect to correlation length and sampling location is from about 1.07 to 1.24, which is about a 14% relative change. Similarly, in Figure 4, the overall change in ψ_u is from about 0.92 to 0.97, which is about a 5% relative change. These two plots demonstrate that the consequence factor is little affected by soil parameters, at least for $v_c \le 0.3$. The relative change did climb somewhat for $v_c = 0.5$, reaching 30% at the low consequence level and 9% at the high consequence level. Nevertheless, for moderate levels of soil variability, which should be ensured by suitable investigation, the consequence factor can be deemed to be largely independent of soil parameters and mostly dependent on the target acceptable failure probability alone.

4 RECOMMENDED CONSEQUENCE FACTORS

By carrying out the determination of consequence factors, over the various parameter ranges considered in this paper, according to the methodology suggested in Section 2, Table 1 presents the recommended values for the moderate sampling location case ($r = 4.5$ m). The other sampling location cases gave similar numbers.

Table 1. Consequence factors for $r = 4.5$ m

θ	v_c	Consequence Factor, ψ_u	
		$p_f = 1/1000$ (low)	$p_f = 1/10000$ (high)
1	0.1	1.08	0.97
1	0.2	1.12	0.95
1	0.3	1.17	0.94
1	0.5	1.26	0.91
3	0.1	1.09	0.97
3	0.2	1.15	0.95
3	0.3	1.22	0.93
3	0.5	1.34	0.89
10	0.1	1.09	0.97
10	0.2	1.14	0.95
10	0.3	1.20	0.93
10	0.5	1.31	0.90

For the low consequence level, the consequence factor is seen to range from 1.08 to 1.31. Lower consequence factors are conservative, in that they lead to lower failure probabilities (see Figure 1), and so a value of 1.1 would be reasonably conservative. For the high consequence level, the factor ranges from 0.89 to 0.97. A conservative consequence factor for the high consequence level would thus be about 0.9.

It is instructive to consider the values used by other codes to handle failure consequences. Most codes include an importance factor, I, which is the inverse of the consequence factor since it is applied to the load side of the LRFD equation (see Eq. 1). Table 2 compares the conservatively recommended consequence factors recommended above (0.9 for high consequence and 1.1 for low consequence levels) to a variety of other codes.

Table 2. Comparison of consequence factors recommended in this paper to equivalent ($1/I$) values recommended in other codes.

Source	Consequence Level		
	Low	Medium	High
Recommended in this paper	1.10	1.00	0.90
AASHTO (2004)	1.25	1.00	0.91
AS5100 (2004)	--	1.00	0.83
Eurocode EN 1990*	1.11	1.00	0.91
NBCC (2005, snow and wind)	1.25	1.00	0.87
NBCC (2005, earthquake)	1.25	1.00	0.77

* from Gulvanessian (2002)

5 CONCLUSIONS

The consequence factors recommended in this paper, which are 0.9 for high failure consequence, 1.0 for medium failure consequences, and 1.1 for low failure consequence levels, are in basic agreement with the importance factors employed by other codes world-wide. These values appear reasonable and are generally conservative, except perhaps for high levels of soil variability. More detailed values can be obtained from Table 1 assuming a moderate sampling distance.

Although the results presented here are mathematically rigorous, it is to be noted that a number of simplifying assumptions were made in the model. These are as follows;

1. The analysis considered only a strip footing. This allowed use of a simpler 2-dimensional model. It is not expected that a full 3-D model would make much difference to the probabilistic results presented here.
2. To restrict attention to the most important random soil properties (i.e. cohesion and friction angle), the soil was assumed to be weightless. It is believed that this is a conservative assumption.

3. Only dead and live loads were considered. This is a typical code development assumption.
4. The random soil properties were assumed to be isotropic (i.e. not layered) and stationary (same mean and variance everywhere). Soil layering tends to be a site specific phenomenon. For code development, this simplifying assumption was deemed appropriate.
5. The load factors used were from NBCC (2005). It is expected that different load factors will primarily result in different resistance factors and will not have a significant effect on the consequence factor.

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