Resistance and Consequence Factor Calibration for Deep Foundations

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ABSTRACT
This paper presents the results of a resistance factor calibration for the design of deep foundations. Reliability-based design concepts are used to determine the resistance factors for use within a Load and Resistance Factor Design methodology. The theoretical results are verified using the random finite element method. Both the level of site understanding as well as the severity of failure consequences are considered in the resistance factor calibration. The current Canadian Highway Bridge Design Code does not accommodate changes in resistance factor with respect to changes in the level of site understanding and failure consequence. The research reported here will thus provide the code with greater flexibility to allow for improved safety and economy in the serviceability and ultimate limit state design of deep foundations.

RÉSUMÉ
Cet article présente les résultats d'une calibration du facteur de résistance à la conception des fondations profondes. Basés sur la fiabilité des concepts de design sont utilisés pour déterminer les facteurs de résistance pour une utilisation au sein d'une charge et le facteur méthodologie de conception de la Résistance. Les résultats théoriques sont vérifiés à l'aide de la méthode des éléments finis aléatoire. Tant au niveau de la compréhension du site ainsi que la gravité des conséquences de défaillance sont pris en compte dans l'étalonnage du facteur de résistance. Le Code Canadien Sure le Calcul des Ponts Routiers actuel ne s'adapter aux changements dans le facteur de résistance par rapport à des changements dans le niveau de compréhension du site et une conséquence de la panne. La recherche présentée ici sera donc fournir le code avec une plus grande flexibilité pour permettre l'amélioration de la sécurité et de l'économie dans la maintenance et la conception de l'état limite ultime de fondations profondes.

1 INTRODUCTION

By and large, the ground is one of the most highly variable, hence uncertain, engineering materials. Unlike quality controlled materials such as wood, concrete, or steel, whose probability distributions are well known and relatively constant world-wide, geotechnical designs face large resistance uncertainties from site to site, and even within a site. There is a real desire in the geotechnical community to account for site understanding in order to achieve economical yet safe designs. To accomplish this, it makes sense to have a resistance factor which is adjusted as a function of site understanding and that allows maintaining overall safety at a common target maximum failure probability as well as demonstrating the direct economic advantage of increased site understanding. Currently, the Canadian design codes specify a single resistance factor for each limit state which does not accommodate changes in resistance factor with respect to changes in the level of site understanding and failure consequence.

The overall safety level of any design should depend on at least three factors: 1) the uncertainty in the loads, 2) the uncertainty in the resistance, and 3) the severity of the failure consequences. In most modern codes, these three items are assumed independent of one another and are thus treated separately. The load factors handle the uncertainties in the loads and, on the load side, failure consequences are handled by applying an importance factor to the more uncertain of loads (e.g. earthquake, snow, and wind).Uncertainties in resistance are handled by resistance factors that are usually specific to the material used in the design (e.g. φ_c for concrete, φ_s for steel, etc). When dealing with a highly variable material such as the ground, it makes sense to apply a partial safety factor that depends on both the resistance uncertainty and on the consequence of failure. The basic idea is that the overall partial factor applied to the geotechnical resistance varies with both uncertainty and failure consequence such that increased site investigation leads to lower uncertainty and a higher resistance factor (and a more economical design). Similarly, for geotechnical systems with high failure consequences, e.g. failure of the foundation of a major multi-lane highway bridge in a capital city, the resistance factor is decreased to ensure a decreased maximum acceptable failure probability.

Similar to the multiplicative approach taken in structural engineering (where the load is multiplied by both a load factor and an importance factor), the overall safety factor applied to geotechnical resistance consists of two parts;

1. a resistance factor, φ_{gu} or φ_{gs}, which accounts for resistance uncertainty. This factor basically aims to achieve a target maximum acceptable failure probability equal to that used for geotechnical designs for typical failure consequences currently (e.g. lifetime failure probability of 1/5,000 at ultimate limit states or 1/300 at serviceability limit states). The subscript *g* refers to 'geotechnical' (or 'ground'), while the subscripts *u* and *s* refer to ultimate and serviceability limit states, respectively.
2. a consequence factor, Ψ, which accounts for failure consequences. Essentially, $\Psi > 1$ if failure consequences are low and $\Psi < 1$ if failure consequence exceed those of typical geotechnical systems. For typical systems, or where system

importance is already accounted for adequately by load importance factors, $\Psi = 1$. The basic idea of the consequence factor is to adjust the maximum acceptable failure probability of the design down (e.g. to 1/10,000 at ULS and 1/1,000 at SLS) for high failure consequences, or up (e.g. to 1/1,000 at ULS and 1/100 at SLS) for low failure consequences.

This paper will consider limit state design (LSD) of deep foundations (piles, hereafter) within the load and resistance factor design (LRFD) approach. The ultimate bearing capacity and serviceability settlement limit states of piles are studied here.

The geotechnical design proceeds by ensuring that the factored geotechnical resistance at least equals the effect of factored loads. For example, for ultimate limit states (ULS), this means that in Canada the geotechnical design will soon need to satisfy an equation of the form

$$\Psi_u \varphi_{gu} \hat{R}_u \geq \sum I_i \alpha_i \hat{F}_i \qquad [1]$$

in which Ψ_u is a consequence factor, φ_{gu} is the geotechnical resistance factor at ULS, $\hat{R}_u$ is the characteristic ultimate resistance. The right-hand-side consists of I_i, an importance factor, multiplying the i^{th} factored load effect, $\alpha_i \hat{F}_i$. A similar equation must be satisfied for serviceability limit states (SLS), with the subscript *u* replaced by *s*, i.e.,

$$\Psi_s \varphi_{gs} \hat{R}_s \geq \sum I_i \alpha_i \hat{F}_i \qquad [2]$$

The load factors, α_i, typically account for uncertainty in loads, and are greater than 1.0 for ultimate limit states but usually assumed equal to 1.0 for serviceability limit states. The geotechnical resistance factor, φ_{gu} or φ_{gs}, is typically less than 1.0 and accounts for uncertainties in geotechnical parameters used to estimate the characteristic geotechnical resistance, $\hat{R}_u$ or $\hat{R}_s$, while both the consequence factor, Ψ_u or Ψ_s, and the importance factor, I_i, are employed to adjust the target reliability level. The importance factor is added to the load side of Eq's 1 and 2 in order to account for failure consequence and is generally based on site specific and highly uncertain load distributions (usually snow, wind, and earthquake). Because the ground is also site specific and highly uncertain, it makes sense to add a consequence factor to the resistance side of Eq's 1 and 2 and so adjust the factored resistance to account for failure consequences in those cases not covered by the load side importance factor. Further research needs to be performed to establish the interaction between the importance and consequence factors and their combined effect on failure probability. To avoid double factoring prior to such research, the consequence factor should be set to 1.0 whenever the importance factor is other than 1.0. The focus of this paper is on calibrating resistance factors and studying consequence factors for various limit states. Thus, the importance factors, I_i, will be assumed to have values 1.0. This simplifies the LRFD Eq.'s 1 and 2 to

$$\begin{aligned} \Psi_u \varphi_{gu} \hat{R}_u &\geq \sum \alpha_i \hat{F}_i \\ \Psi_s \varphi_{gs} \hat{R}_s &\geq \sum \alpha_i \hat{F}_i \end{aligned} \qquad [3]$$

Three target reliabilities will be considered; high, typical, and low, corresponding to important structures where failure has large consequences (e.g. hospitals, schools, and lifeline highway bridges), typical structures, which constitute the majority of civil engineering projects, and low-failure consequence structures (e.g. low use storage facilities, low use bridges, etc.). Most designs will be aimed at the typical failure consequence level, which in this paper will be assumed to have a maximum lifetime failure probability, p_m, of about 1/5,000 at ULS and 1/300 at SLS. These correspond to lifetime reliability indices of about $\beta = 3.5$ (e.g. Meyerhof, 1995) and 2.7 at ULS and SLS respectively. Note that these target failure probabilities assume some redundancy (as typically required in structural codes), so that the actual system lifetime failure probability is usually less than the component maximum lifetime failure probability, p_m. The effect of redundancy in geotechnical components on reliability is still in need of further research that may lead to adjustment of the consequence factor.

The theoretical framework to obtain the resistance factor, φ_{gu} or φ_{gu}, is summarized in sections 2 and 3.The factors are targeted to achieve the 'typical' failure probability ($p_m \simeq 1/5,000$ at ULS and $p_m \simeq 1/300$ at SLS) for which the consequence factor, Ψ_u, is set to 1.0. The details of the following mathematical analysis can be found in Fenton and Naghibi (2011) and Naghibi and Fenton (2011) for ULS analysis. For SLS case, however, the interested reader is referred to an upcoming paper by the current authors.

The remainder of the paper concentrates on the consequence factor, Ψ_u or Ψ_s, and how it varies with respect to target failure probability and site uncertainty for each limit state individually. A key question to address is: Do we require two sets of consequence factors (three values each) for ultimate and serviceability limit states since they have quite different maximum acceptable failure probabilities?

2 FAILURE PROBABILITY AT ULS

In this work only live and dead loads are considered, which is a typical assumption in code development under static loading. The load considered in reliability-based design of a pile at ULS has two important values. One is the characteristic total load used in the deterministic pile design, which comes from current code provisions:

$$\hat{F} = \alpha_L \hat{F}_L + \alpha_D \hat{F}_D \qquad [4]$$

where $\hat{F}_L$ is the characteristic live load, $\hat{F}_D$ is the characteristic dead load, and α_L and α_D are the live and dead load factors, respectively. The load factors used in this paper are as given by the National Building Code of Canada: $\alpha_L = 1.5$ and $\alpha_D = 1.25$ (NRC, 2005). The characteristic loads, $\hat{F}_L$ and $\hat{F}_D$, are obtained by applying bias factors to the means of the load distribution: $\hat{F}_L = 1.41\mu_L$ (Becker, 1996) and $\hat{F}_D = 1.18\mu_D$ (Allen, 1975), where μ_L and μ_D are the means of the maximum lifetime dead and live loads, respectively.

The other important value is the "true", but random, total load applied to the pile, F . It is assumed that the total load is equal to the sum of the maximum life time live load, F_L , and the relatively static dead load, F_D , i.e.

$$F = F_L + F_D \qquad [5]$$

where F_L and F_D are each assumed to be lognormally distributed. The mean and variance of total load, F , assuming live and dead loads are independent, are given by

$$\mu_F = \mu_L + \mu_D,\ \sigma_F^2 = \sigma_L^2 + \sigma_D^2 \qquad [6]$$

The estimation of the resistance factor, φ_{gu} , derived using the theoretical framework presented by Naghibi and Fenton (2011), is summarized in this section.

In order to determine the required resistance and consequence factors, the probability of a deep foundation reaching its bearing capacity ultimate limit state must be estimated. This probability will depend on the load distribution, the resistance distribution and the load and resistance factors selected. If an axially loaded pile founded within purely cohesive soils is considered, the characteristic ultimate bearing resistance, using the α -method is given by Das(2000) to be,

$$\hat{R}_u = pH\alpha\hat{c} \qquad [7]$$

for pile length H , in which p is the effective perimeter length of the pile section, α is an empirical adhesion factor, typically in the range from 0.5 to 1 (CGS, 2006), and $\hat{c}$ is the characteristic cohesion which is commonly estimated from a set of m observations $\hat{c}_1, \hat{c}_2, ..., \hat{c}_m$ of soil cohesion taken at the site (each of which in turn may be estimated via some indirect measurement, such as SPT or CPT). In this paper, an arithmetic average of the observations will be used to define the characteristic cohesion,

$$\hat{c} = \frac{1}{m}\sum_{j=1}^{m}\hat{c}_j \qquad [8]$$

and measurement error is ignored (so that the obtained resistance factors are actually upper bounds).

The required minimum design pile length, H , is then obtained by substituting Eq's 4 and 7 into Eq. 3, giving for the ULS case,

$$H = \frac{\hat{F}}{\varphi_{gu}p\alpha\hat{c}} \qquad [9]$$

The probability of failure involves determining the probability that the actual lifetime extreme load acting on the pile, F , exceeds the actual ultimate resistance, $R_u = pH\alpha\overline{c}$ (where $\overline{c}$ is the equivalent cohesion as 'seen' by the pile over its entire length). In other words, the probability of failure is computed as

$$p_f = P\left[F > R_u\right] = P\left[\frac{F\hat{c}}{\overline{c}} > \frac{\hat{F}}{\varphi_{gu}}\right] \qquad [10]$$

All three quantities on the left hand side of the inequality, i.e. $F, \hat{c}, \text{and } \overline{c}$, are random. See Naghibi and Fenton (2011) for the details of their joint distribution and how the probability in Eq. 10 is computed.

Once the probability of failure is computed via Eq. 10, it can be compared to the maximum acceptable failure probability, p_m . If p_f exceeds p_m , then the resistance factor and/or the consequence factor need to be reduced (specifically, the product $\Psi_u\varphi_{gu}$ needs to be reduced). The determination of required consequence factors then proceeds in two steps;

1. Consider first the typical consequence level and set $\Psi_u = 1$. For a variety of different levels of variability in soil properties, degrees of spatial correlation between soil properties, and distance between pile and sample location (which is taken as a proxy for site understanding), estimate the probability of pile failure using Eq. 10. For each case, adjust the resistance factor, φ_{gu} , until $p_f = p_m$. This then is the required resistance factor.
2. Using the required resistance factor(s) determined in step 1 in Eq. 10, repeat the procedure of step 1 except now at the high (reduced p_m) and low (increased p_m) consequence levels and adjust the consequence factor, Ψ_u , until $p_f = p_m$. This then yields the required consequence factor.

3 FAILURE PROBABILITY AT SLS

The problem considered here is of an individual pile subjected to a random vertical load and supported by a spatially random soil without end bearing. Similar to the ULS case, only dead and live loads have been considered, except that the load factors are typically taken as $\alpha_L = \alpha_D = 1.0$ for serviceability limit states, i.e.

$$\hat{F} = \hat{F}_L + \hat{F}_D \qquad [11]$$

The characteristic dead and live loads are defined as

$$\hat{F}_L = k_L / \mu_L, \hat{F}_D = k_D / \mu_D \qquad [12]$$

where μ_L and μ_D are the means of the maximum lifetime live and dead loads, and k_L and k_D are the bias factors estimated by Bartlett et. al. (2003) and Ellingwood et al. (1980) to be 0.9 and 1.05 respectively.

The characteristic settlement of a pile is given by Poulos in Rowe (2001) to be,

$$\hat{\delta} = \frac{\hat{F}}{\hat{E}d}I_p \qquad [13]$$

where $\hat{\delta}$ is the characteristic pile settlement, $\hat{F}$ is the characteristic load, $\hat{E}$ is the estimated characteristic soil elastic modulus, d is the pile width, and I_p is a settlement influence factor which includes the effect of Poisson's ratio (assumed here to be $\nu = 0.3$) as well as the pile slenderness ratio H/d (H being the pile length) and the pile to soil stiffness ratio, $k = E_p / \hat{E}$, E_p being the pile elastic modulus. A function of form

$$I_p = a_0 + \frac{1}{(H/d + a_1)^{a_2}} \qquad [14]$$

has been found by regression which well fits the I_p values obtained using 3-D finite element analysis.

The design pile length, H, can now be determined as follows; i) the random soil is sampled at some location over a column of length D (as would occur if, say, a CPT sounding were taken) to obtain a series of 'observations' of the soil's elastic modulus, ii) the characteristic elastic modulus used in design, $\hat{E}$, is estimated from the soil sample, and iii) the required design pile length H is obtained via the LRFD requirement of Eq. 3 for the SLS case.

The reliability-based design goal is to determine the required pile length, H, such that the probability of exceeding a specified maximum tolerable settlement, δ_{max}, is acceptably small, i.e. to find H such that

$$p_f = \mathrm{P}\left[\delta > \delta_{max}\right] \le p_m \qquad [15]$$

in which δ is the actual (random) pile settlement. Design failure occurs if the actual pile settlement, δ, exceeds the maximum tolerable settlement, δ_{max}, which is taken as 0.025 m in this study.

The design pile length can be determined by returning to Eq. 13 and replacing $\hat{\delta}$ with the maximum tolerable settlement, δ_{max}, I_p with Eq. 14, introducing a resistance factor, φ_{gs}, and finally solving for H as,

$$H = d\left[\left(\frac{1}{(\delta_{max}\varphi_{gs}\hat{E}d/\hat{F}) - a_0}\right)^{1/a_2} - a_1\right] \qquad [16]$$

Now that the pile has been designed, attention can be turned to its actual (random) pile settlement, δ. It is hypothesized that δ can be determined using Eq. 13 by replacing the characteristic load with the true (random) load, $F = F_L + F_D$, and the characteristic elastic modulus, $\hat{E}$, with the (random) effective elastic modulus, E_{eff}, giving:

$$\delta = \frac{F}{E_{eff}d} I_p \qquad [17]$$

Using Eq. 14 in Eq. 17, and replacing H with Eq. 16, the actual (random) pile settlement can be estimated to be,

$$\delta = \frac{\delta_{max}\varphi_{gs}\hat{E}}{E_{eff}}\left(\frac{F}{\hat{F}}\right) \qquad [18]$$

which means that the design requirement of Eq. 15 becomes,

$$p_f = \mathrm{P}\left[\delta > \delta_{max}\right] = \mathrm{P}\left[\frac{\varphi_{gs}\hat{E}}{E_{eff}}\left(\frac{F}{\hat{F}}\right) > 1\right] = P\left[F\left(\frac{\hat{E}}{E_{eff}}\right) > \frac{\hat{F}}{\varphi_{gs}}\right] \qquad [19]$$

All three quantities on the left hand side of the inequality, i.e. $F, \hat{E}, \text{ and } E_{eff}$, are random. The effective elastic modulus as seen by the pile, E_{eff}, is assumed to be the geometric average of the soil's elastic modulus over the pile depth, H

$$E_{eff} = \exp\left\{\frac{1}{H}\int_0^H \ln E(z)dz\right\} \qquad [20]$$

where $E(z)$ is the elastic modulus of the soil surrounding the pile at depth z.

The characteristic elastic modulus, $\hat{E}$, is estimated by sampling the soil over a single column somewhere in the vicinity of the pile to obtain a series of elastic modulus samples, $E_1^o, E_2^o, \ldots, E_m^o$. The value of $\hat{E}$ is determined as a geometric average of the observed sample,

$$\hat{E} = \left(\prod_{j=1}^{m} E_j^o\right)^{1/m} = \exp\left\{\frac{1}{m}\sum_{j=1}^{m} \ln E_j^o\right\} \qquad [21]$$

A more complete discussion, along with simulation-based validation of the theory will be published by the authors shortly.

Once the probability of failure is computed, via Eq. 19, the 2-step approach mentioned in Section 2 can be followed to obtain the required resistance and consequence factors of piles at SLS.

4 THEORETICAL CONSEQUENCE FACTORS AT ULS

Consequence factors were determined for a particular example problem with parameters as follows;

1. The mean lifetime maximum live load acting on the pile is assumed to be $\mu_L = 20$ kN/m with coefficient of variation $v_L = 0.3$. The mean dead load is assumed to be $\mu_D = 60$ kN/m with coefficient of variation $v_D = 0.15$. The mean values assumed here are not particularly important, since the design (see Eq. 9) takes the separation between the load and resistance distributions into account in the design process (e.g. the higher the load is relative to the resistance, the larger the required pile length). Both live and dead loads are assumed to be lognormally distributed.
2. The mean cohesion is assumed to be $\mu_c = 50$ kN/m^2 with coefficient of variation $v_c = 0.1, 0.2, 0.3, 0.4,$ and 0.5. As mentioned above, the mean value is expected to have little influence on the results, but the coefficient of variation definitely affects the resistance factor and has a slight influence on the consequence factor, as will be shown. The cohesion is assumed to be lognormally distributed.
3. This paper looks specifically at the case where the soil is frictionless ($\phi = 0$) and the cohesion, $c = s_u$, is the undrained shear strength.
4. The correlation length, θ, which measures the distance within which soil properties are significantly correlated, is varied from a low of 0.1 m to a high of

50 m. Low values of θ lead to soil properties varying rapidly spatially, while high values mean that the soil properties vary only slowly with position. A large correlation length, of say $\theta = 50$ m, means that soil samples taken well within 50 m from the pile location will (e.g. at $r = 10$ m) will be quite representative of the soil properties along the pile. Lower failure probabilities are expected when the soil is sampled well within the distance θ from the pile. Interestingly, because the characteristic value derived from the soil sample is generally some form of average, when θ is very small (say, 0.01 m) then the sample will again accurately reflect the average conditions along the pile regardless of the sampling location. The worst case correlation length occurs when θ is approximately equal to the distance from the pile to the sampling location.

5. Three soil sampling locations are considered; directly along the pile location ($r = 0$), corresponding to good site understanding, $r = 4.5$ m, corresponding to moderate site understanding, and $r = 9$ m, corresponding to lower site understanding.
6. Three consequence levels are considered; high failure consequence, typical failure consequence, and low failure consequence. Maximum acceptable failure probabilities have been assigned to these consequence levels; $p_m = 1/1{,}000$, $1/5{,}000$, and $1/10{,}000$ for low, typical, and high failure consequence levels, respectively. These failure probabilities correspond to reliability indices of 3.1, 3.5, and 3.7, respectively, which is in the range of foundation reliabilities suggested in the literature (see, e.g., Meyerhof, 1995, and Becker, 1996a).

Figure 1 illustrates the worst case correlation length. The figure presents failure probabilities for the case where the soil is sampled at $r = 4.5$ m from the pile. Clearly, the worst case failure probability (highest) occurs for values of correlation length near 4.5 m. The Figure is shown for the typical consequence case ($p_m = 1/5{,}000$), for which the consequence factor $\Psi_u = 1.0$ was selected. It can be seen that the worst case probability of failure, p_f, is only slightly less than the acceptable maximum probability when the coefficient of variation of the soil properties is at a moderate level ($v_c = 0.3$). However, if the soil property variability exceeds $v_c = 0.3$, the worst case probability of failure becomes unacceptable. See, for example, the $v_c = 0.5$ curve, which reaches a failure probability of 0.08 at the worst case correlation length. The unacceptable failure probabilities that occur for higher soil variabilities emphasize the need to perform enough site investigation to reduce the residual variability to no more than moderate levels. The theoretical results shown in Figure 1 have also been validated by simulation, as shown by the plotted circles and squares.

As mentioned in the Introduction, the consequence factor should ideally depend only on the target failure probability, p_m, and not on soil variability, correlation length, and sampling location. Variations in the latter three parameters should ideally be entirely handled by the resistance factor, φ_{gu}. Figures 2 and 3 illustrate the effect of correlation length and sampling location on the consequence factor for low consequence level (Figure 2) and for high consequence level (Figure 3). Both figures are shown for a moderate variability ($v_c = 0.3$).

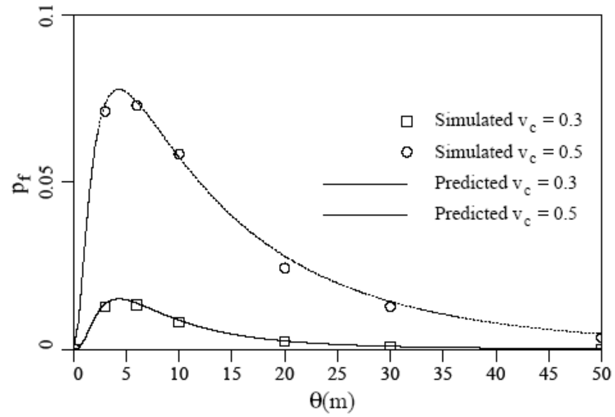

Figure 1. Failure probability versus correlation length for $\Psi_u = 1.0$ (typical consequence), $\varphi_{gu} = 0.8$, and $r = 4.5$ m at ULS.

The overall change in the consequence factor in Figure 2 with respect to correlation length and sampling location ranges from about 1.06 to 1.18, which is about a 10% relative change. Similarly, in Figure 3, the overall change in Ψ_u is from about 0.936 to 0.975, which is about a 4% relative change. These two plots demonstrate that the consequence factor is little affected by soil parameters, at least for $v_c \leq 0.3$, particularly when soil properties are well understood ($r = 0$ m), and is mostly dependant on the target acceptable failure probability alone.

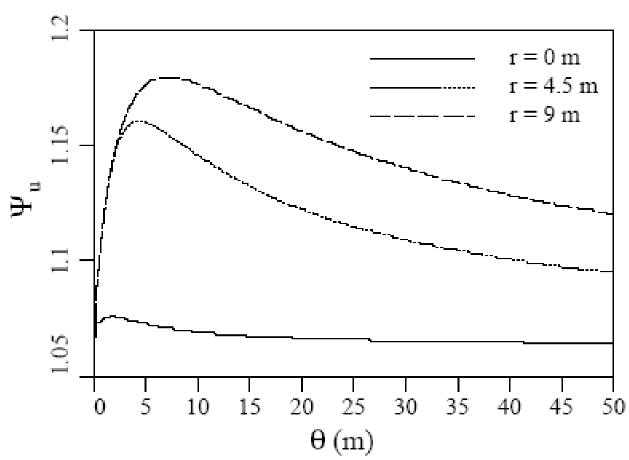

Figure 2. Consequence factor versus correlation length for various sampling locations at low consequence level ($p_m = 1/1{,}000$) for $v_c = 0.3$ at ULS.

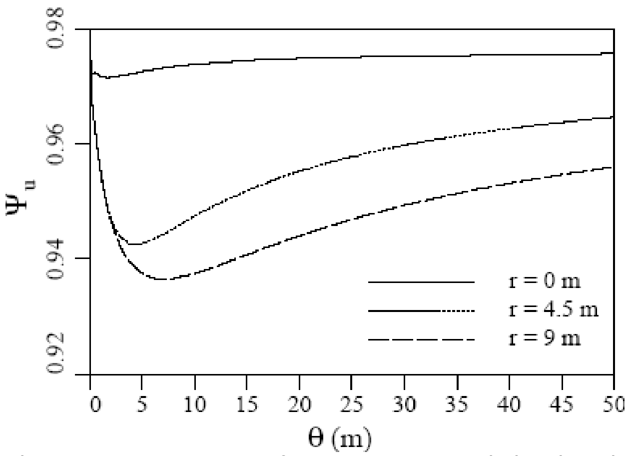

Figure 3. Consequence factor versus correlation length for various sampling locations at high consequence level ($p_m = 1/10{,}000$) for $v_c = 0.3$ at ULS.

5 THEORETICAL CONSEQUENCE FACTORS AT SLS

Consequence factors for SLS design were determined for a particular example problem with parameters as follows;

1. The mean lifetime maximum live load acting on the pile is assumed to be $\mu_L = 400$ kN/m with coefficient of variation $v_L = 0.27$. The mean dead load is assumed to be $\mu_D = 1{,}600$ kN/m with coefficient of variation $v_D = 0.10$. Both live and dead loads are assumed to be lognormally distributed.
2. The mean soil elastic modulus is assumed to be $\mu_E = 30$ MPa with coefficient of variation $v_E = 0.1, 0.2, 0.3, 0.4,$ and 0.5, also assumed to be lognormally distributed.
3. The correlation length, length, θ, is varied from a low of 0.1 m to a high of 100 m.
4. Three soil sampling locations are considered; directly along the pile ($r = 0$ m), corresponding to good site understanding, $r = 5$ m, corresponding to moderate site understanding, and $r = 10$ m, corresponding to lower site understanding.

Similar to ULS case, three consequence levels considered are high failure consequence, typical failure consequence, and low failure consequence. However, the maximum acceptable failure probabilities are much higher than for the ULS case, i.e. $p_m = 1/100$, $1/300$, and $1/1{,}000$ for low, typical, and high failure consequence levels, respectively. These failure probabilities correspond to reliability indices of 2.3, 2.7, and 3.1, respectively, which is in the range of foundation reliabilities suggested in the literature at SLS (see, e.g., Phoon et al. 1995, and Eurocode 2002).

Figure 4 illustrates the effect of consequence factor on the probability of settlement failure. It can be seen that small changes in Ψ_g can make large differences in p_f, especially for larger v_E values.

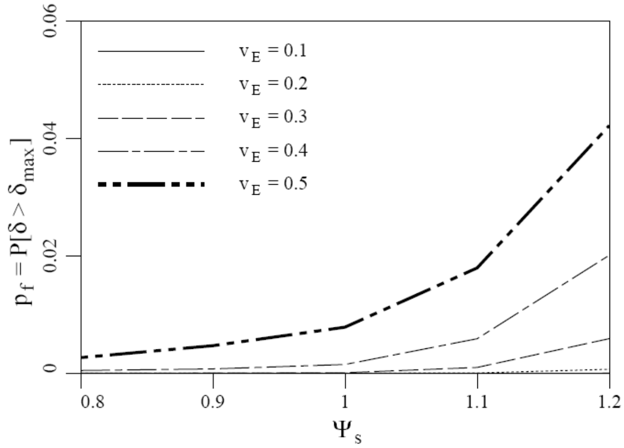

Figure 4. Failure probability versus consequence factor for $\theta = 1$ m, $r = 5$ m, and $\varphi_{gu} = 0.5$ at SLS.

Figure 5 presents failure probabilities estimated by theory, via Eq. 19, and compared to simulation, for $v_E = 0.3$ and various resistance factors, φ_{gs}, when the soil is sampled at $r = 5$ m from the pile location. The agreement between simulation and theory is considered very good given all the approximations made in the theory. It is observed from Figure 5 that the probability of failure, p_f, increases with resistance factor, as expected.

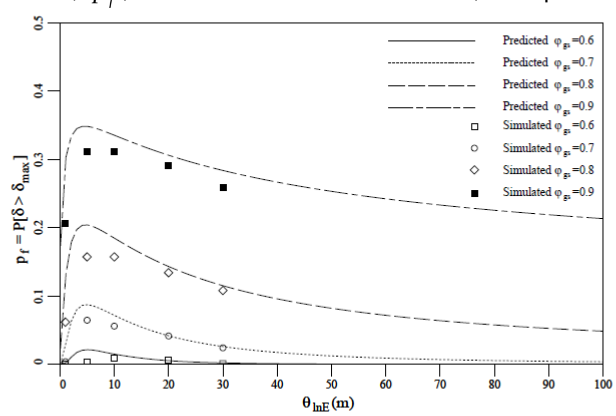

Figure 5. Failure probability versus correlation length for $\Psi_u = 1$ (typical consequence), $r = 5$ m, and $v_E = 0.3$ at SLS.

Figure 6 presents theoretical failure probabilities for the case where the soil is sampled $r = 5$ m away from the pileand for the low consequence case ($p_m = 1/100$), for which the consequence factor $\Psi_g = 1.1$ was selected. Similar to Figure 1, the presence of a worst case correlation length is evident in Figure 5. In addition, it is observed that the worst case probability of failure, p_f, is only slightly less than the acceptable maximum probability when the coefficient of variation of the soil properties is at a moderate level ($v_E = 0.3$). However, the failure probability becomes unacceptable if the soil property variability exceeds $v_E = 0.3$. See, for example, the $v_E = 0.5$ curve, which reaches a failure probability of 0.06 at the worst case correlation length.

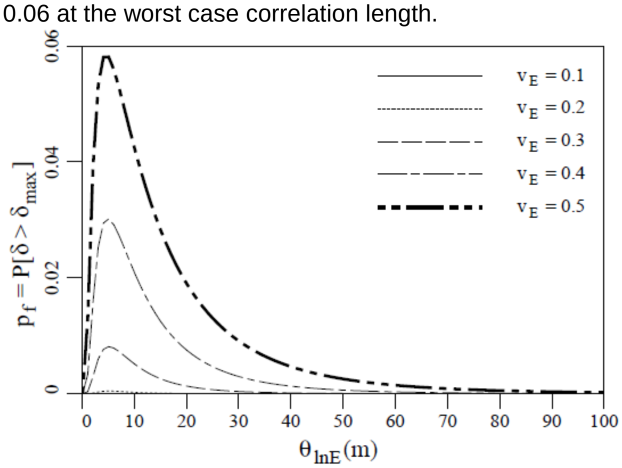

Figure 6. Failure probability versus correlation length for $\Psi_u = 1.1$ (low consequence), $\varphi_{gu} = 0.5$, and $r = 5$ m at SLS.

The effect of correlation length and sampling location on the consequence factor is illustrated in Figures 6 and 7 for low consequence level (Figure 7) and for high consequence level (Figure 8). Both figures are shown for a moderate variability ($v_E = 0.3$).

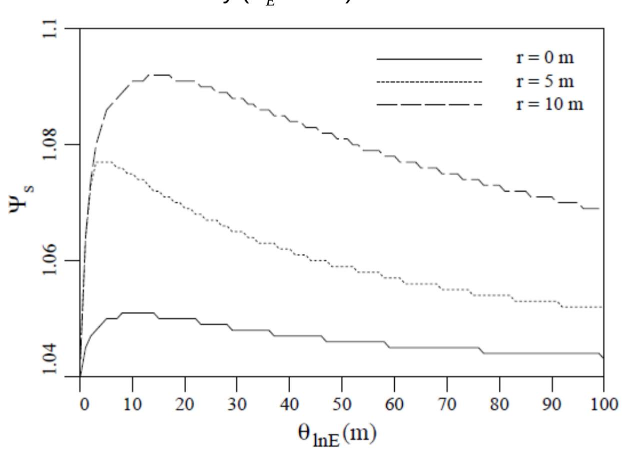

Figure 7. Consequence factor versus correlation length for various sampling locations at high consequence level ($p_m = 1/100$) for $v_E = 0.3$ at SLS.

The overall change in the consequence factor in Figure 7 with respect to correlation length and sampling location ranges from about 1.045 to 1.09, which is about a 4% relative change. Similarly, in Figure 8, the overall change in Ψ_g is from about 0.926 to 0.96, which is about a 3.5% relative change. Similar to the ULS case, these two plots demonstrate that the consequence factor is little affected by soil parameters, at least for $v_E \leq 0.3$, particularly when $r = 0$ m.

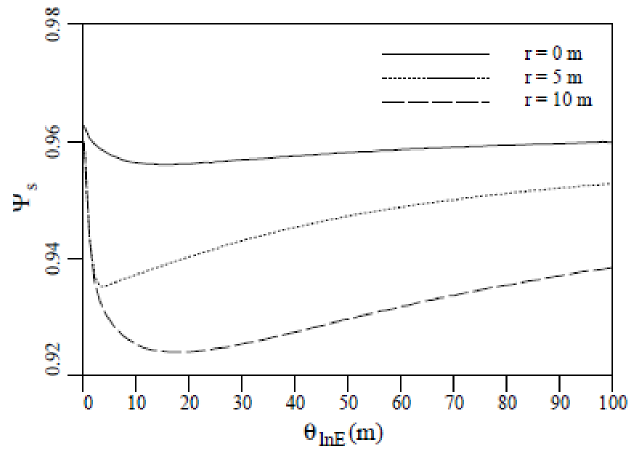

Figure 8. Consequence factor versus correlation length for various sampling locations at high consequence level ($p_m = 1/1,000$) for $v_E = 0.3$ at SLS.

6 RECOMMENDED CONSEQUENCE FACTORS

Tables 1 and 2 present the recommended consequence values for design of deep foundations at ULS and SLS respectively, determined according to the methodology suggested in Sections 2 and 3 over the various parameter ranges considered in this paper using the worst case correlation lengths and a moderate sampling distance.

For the low consequence level, it is evident that the consequence factors range from 1.06 to 1.18 at ULS and from 1.04 to 1.08 at SLS. Lower consequence factors lead to lower failure probabilities (see Figure 4, for example) and therefore are more conservative, hence a value of 1.1 would be reasonably conservative for ULS and possibly slightly unconservative for SLS. For the high consequence level, the consequence factor ranges from 0.94 to 0.98 at ULS and from 0.93 to 0.96 at SLS. A conservative consequence factor for the high consequence level would thus be about 0.9 for both ULS and SLS

Table 1. Consequence factors at ULS

θ (m)	v_c	Consequence Factor, Ψ_u	
		$p_f = 1/1,000$ (low)	$p_f = 1/10,000$ (high)
0.0	0.1	1.06	0.98
0.0	0.2	1.06	0.98
0.0	0.3	1.06	0.98
0.0	0.4	1.06	0.98
0.0	0.5	1.06	0.98
5.0	0.1	1.07	0.97
5.0	0.2	1.08	0.97
5.0	0.3	1.1	0.96
5.0	0.4	1.12	0.96
5.0	0.5	1.14	0.95
10.0	0.1	1.07	0.97
10.0	0.2	1.09	0.97
10.0	0.3	1.12	0.96
10.0	0.4	1.15	0.95
10.0	0.5	1.18	0.94

Table 2. Consequence factors at SLS

θ (m)	v_E	Consequence Factor, Ψ_g	
		$p_f = 1/100$ (low)	$p_f = 1/1{,}000$ (high)
0.0	0.1	1.04	0.96
0.0	0.2	1.05	0.95
0.0	0.3	1.06	0.95
0.0	0.4	1.07	0.94
0.0	0.5	1.07	0.94
5.0	0.1	1.05	0.95
5.0	0.2	1.07	0.94
5.0	0.3	1.08	0.94
5.0	0.4	1.08	0.93
5.0	0.5	1.09	0.77
10.0	0.1	1.05	0.96
10.0	0.2	1.07	0.94
10.0	0.3	1.08	0.94
10.0	0.4	1.08	0.94
10.0	0.5	1.08	0.8

It is instructive to consider the values used by other codes to handle failure consequences. Most codes include an importance factor, I, which is (at least mathematically) the inverse of the consequence factor since it is applied to the load side of the LRFD equation (see Eq. 1). Table 3 compares the conservatively recommended consequence factors recommended above (0.9 for high consequence and 1.1 for low consequence levels) to the inverse of the importance factor from a variety of other codes.

Table 3. Comparison of consequence factors recommended in this paper to equivalent ($1/I$) values recommended in other codes.

Source	Consequence Level		
	Low	Typical	High
Recommended in this paper	1.10	1.00	0.90
AASHTO (2007)	1.25	1.00	0.91
AS5100 (2004)	--	1.00	0.83
Eurocode 1 (Gulvanessian 2002)	1.11	1.00	0.91
NBCC (2005, snow and wind)	1.25	1.00	0.87
NBCC (2005, earthquake)	1.25	1.00	0.77

7 CONCLUSIONS

The consequence factors recommended in this paper, which are 0.9 for high failure consequence, 1.0 for typical failure consequences, and 1.1 for low failure consequence levels, are in basic agreement with the importance factors employed by other codes world-wide. These values appear reasonable and are generally conservative, except perhaps for SLS and high levels of soil variability. More detailed values can be obtained from Tables 1 and 2, which were developed assuming a moderate sampling distance and worst case correlation length.

Interestingly, the investigation shows that consequence factors are largely independent of limit states even though the target maximum failure probabilities for serviceability limit states are greater than those for ultimate limit states. For example, a typical geotechnical system might have a target maximum lifetime failure probability of 1/5,000 for ultimate limit states, but only 1/300 for serviceability limit states. If the geotechnical system has high failure consequences, the lifetime maximum acceptable failure probability might decrease by the same fraction, i.e. to 1/10,000 for ULS and to 1/1,000 for SLS. Therefore the same (or similar) consequence factor can be used to scale the target maximum acceptable failure probability for both ULS and SLS designs since the probabilities scale by the same amount.

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