Analysis of Infinite Slopes with Spatially Random Shear Strength

D.V. Griffiths [1], F. ASCE, P.E., Giorgia F. deWolfe [2], Jinsong Huang [3], and Gordon A. Fenton [4], M. ASCE, P.Eng.

[1]Professor, Colorado School of Mines University,1610 Illinois Street, Golden, CO 80401; d.v.griffiths@mines.edu
[2]PhD candidate, Colorado School of Mines University, 1610 Illinois Street, Golden, CO 80401; gfulcher@mines.edu
[3]Research Professor, Colorado School of Mines University, 1610 Illinois Street, Golden, CO 80401; jhuang@mines.edu
[4]Professor, Dalhousie University, Halifax, NS, Canada, B3J 1Y9; gordon.fenton@dal.ca

ABSTRACT: The study investigates the role of spatially random soil on the stability of infinite slopes with application to landslides and other geohazards. The influence of the shear strength mean, standard deviation and spatial correlation length on the probability of failure is thoroughly investigated through parametric studies. The results show that the traditional "first order second moment" approach to this problem is inherently unconservative, due to its inability to allow the failure mechanism to "seek out" the critical depth below ground surface, which is frequently not at the base of the soil layer.

INTRODUCTION

One of the main objectives of this work was to create a powerful general framework for modeling statistically described parameters relating to long slopes. The method involves a combination of Random Field theory (e.g. Fenton and Vanmarcke, 1990) with infinite slope theory (e.g. Taylor, 1948; Lambe and Whitman, 1969; Bromhead 1992; Duncan, 1996). The method, applied in a Monte-Carlo framework, takes into account the mean, standard deviation and spatial correlation length of the input parameter. Repeated calculations using the same input statistics of soil parameters (e.g. undrained shear strength) eventually lead to stable output statistics of the design parameters (e.g. Factor of Safety). The paper then compares results from the Monte-Carlo analyses with those obtained using the first order second moment method (FOSM).

ANALYTICAL METHOD

The analytical method considers a slice of soil in the potential failure zone as shown in Figure 1. The slope is homogeneous with a ground water free surface and critical failure surface running parallel to the slope surface. The analytical solution includes the option of different heights of the ground water surface through the slope as well as a horizontal pseudo-acceleration.

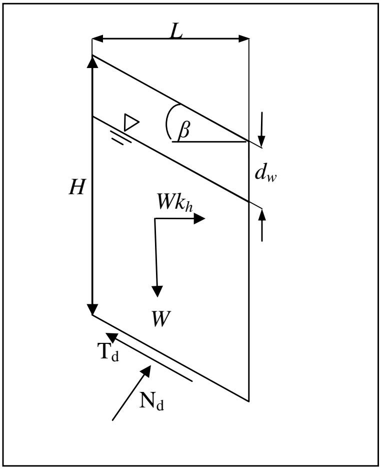

FIG. 1: Representation of forces acting on the infinite slope

Based on the key principles of infinite slope theory the factor of safety *FS* can be calculated as

$$FS = \frac{\tau_f}{\tau_d} \qquad (1)$$

$$FS = \frac{c'}{\left((H-d_w)\gamma_{sat} + (d_w\gamma_m)\right)(\sin\beta\cos\beta + k_h\cos^2\beta)} + \frac{\left(\left((H-d_w)\gamma' + d_w\gamma_m\right)(\cos^2\beta - k_h\sin\beta\cos\beta)\right)(\tan\phi')}{\left((H-d_w)\gamma_{sat} + (d_w\gamma_m)\right)(\sin\beta + k_h\cos\beta)} \qquad (2)$$

Assuming $\gamma_m = \gamma_{sat}$ we can obtain a simplified form of Eq. 2:

$$FS = \frac{c'}{H\gamma_{sat}(\sin\beta\cos\beta + k_h\cos^2\beta)} + \frac{\left((H\gamma_{sat} - (H-d_w)\gamma_w)(\cos\beta - k_h\sin\beta)\right)(\tan\phi')}{H\gamma_{sat}(\sin\beta + k_h\cos\beta)} \qquad (3)$$

The following symbols are used in the analytical solution of this problem:

c'	soil cohesion	β	slope inclination
d_w	depth of the water table	γ_{sat}	saturated unit weight
E'	Young's modulus	γ_w	unit weight of water
FS	factor of safety	γ_m	unit weight of material
H	depth of the soil layer	σ	normal total stress
k_h	horizontal pseudo acceleration coefficient	σ'	normal effective stress
L	width of slice	τ_d	developed shear stress
N_d	normal force component	τ_f	shear strength
T_d	shear force component	υ	Poisson's ratio
u	pore pressure	ϕ'	soil friction angle
W	weight of slice		

For variable soil strength profiles the classical infinite slope equation for homogeneous frictionless soil,

$$FS = \frac{c_u}{H\gamma\cos\beta\sin\beta} \tag{4}$$

should be written as:

$$FS = \frac{c_u}{z}\frac{1}{\gamma\cos\beta\sin\beta} \tag{5}$$

noting that the critical failure surface occurs at a depth where $\frac{c_u}{z}$ is a minimum.

In the random field approach, the input undrained shear strength is defined by its mean (μ_{c_u}), standard deviation (σ_{c_u}) and correlation length (θ). The spatial correlation length recognizes that soil samples "close" together are more likely to have similar properties than if they are "far apart". A visual example of a case of low and high correlation lengths is given in Figure 2.

In the results presented later in the paper, the spatial correlation length θ is expressed as a dimensionless parameter with respect to the soil depth as follows

$$\Theta = \frac{\theta}{H} \tag{6}$$

The issue of how many Monte-Carlo simulations are needed is addressed in Figure 3 for the case of $\Theta = 0.2$ and $V_{c_u} = 0.1$, where V_{c_u} is the coefficient of variation of the undrained shear strength. The probability of failure represents the proportion of Monte-Carlo realizations for which $FS \le 1$. Five thousands simulations appear to give stable results.

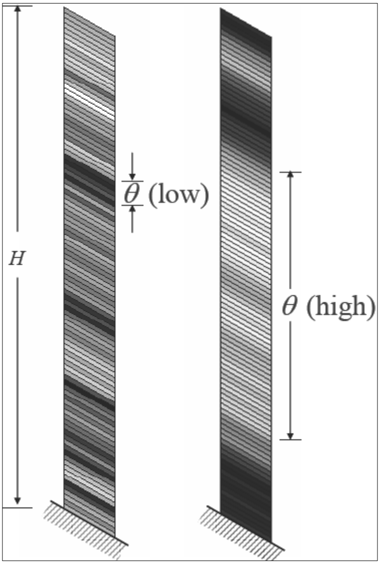

FIG.2: The grayscale represents varying shear strength values, with the light sections showing low strength areas. Both images represent a slope with same mean and standard deviation

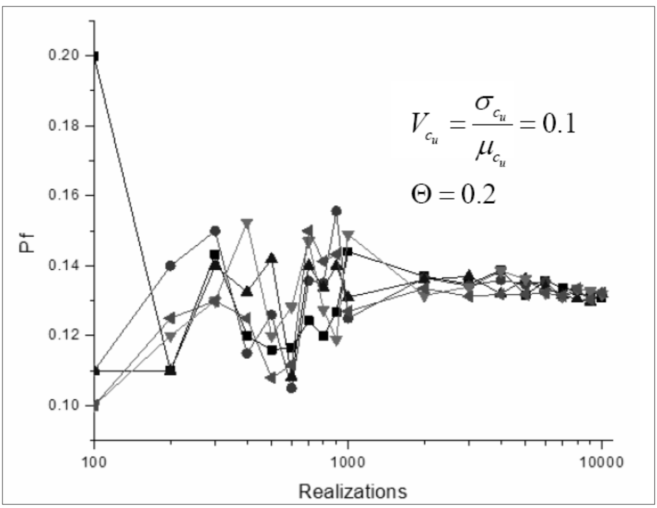

FIG. 3: Number of realizations vs. probability of failure.

FIRST ORDER SECOND MOMENT METHOD (FOSM)

The FOSM method for a single random variable is easily applied. For example, from Eq. 4 we get

$$\mu_{FS} = \frac{1}{\gamma H \sin\beta \cos\beta}\mu_{c_u} \tag{7}$$

and

$$\sigma_{FS} = \frac{1}{\gamma H \sin\beta \cos\beta}\sigma_{c_u} \tag{8}$$

It should be noted however that since no spatial variability is accounted for in this method (the soil is assumed to be variable but homogeneous) the failure mechanism is always assumed to act at a depth H.

Consider the particular case where $H = 2.5$ m, $\beta = 30°$, $\gamma = 20$ kN/m^3 with $\mu_{c_u} = 25$ kN/m^2 and $\sigma_{c_u} = 5$ kN/m^2 $(V_{c_u} = 0.2)$

From equations (7) and (8), FOSM gives $\mu_{FS} = 1.155$ and $\sigma_{FS} = 0.231$ $(V_{FS} = 0.2)$

If we assume a lognormal distribution of FS, then the mean and standard deviation of the underlying normal distribution of $\ln FS$ are given by

$$\mu_{\ln FS} = \ln \mu_{FS} - \frac{1}{2}\ln\left\{1+V_{FS}^2\right\} = 0.124 \tag{9}$$

$$\sigma_{\ln FS} = \sqrt{\ln\left\{1+V_{FS}^2\right\}} = 0.198 \tag{10}$$

To estimate the probability of failure, we need to estimate the probability that $FS < 1$, or in log-space, that $\ln FS < 0$

This is given by

$$\begin{aligned} P\left[FS<1\right] &= \Phi\left[\frac{\ln 1 - \mu_{\ln FS}}{\sigma_{\ln FS}}\right] \\ &= \Phi\left[-\frac{0.124}{0.198}\right] = \Phi\left[-0.626\right] = 1-\Phi\left[0.626\right] \\ &= 0.265 \end{aligned} \tag{11}$$

A similar procedure with an input $V_{c_u} = 0.4$ led to $P\left[FS<1\right] = 0.428$.

If the distribution of FS is assumed to be normal in the above examples, FOSM gives $P\left[FS<1\right] = 0.251$ and $P\left[FS<1\right] = 0.369$ for input of $V_{c_u} = 0.2$ and $V_{c_u} = 0.4$ respectively.

RANDOM FIELD STUDIES WITH MONTE-CARLO

Numerous parametric studies have been performed, but in the interests of brevity only two of them are summarized in this paper. An undrained clay slope with $H = 2.5$ m, $\beta = 30°$, $\gamma = 20$ kN/m^3, and $\mu_{c_u} = 25$ kN/m^2 was considered with two different standard deviations of $\sigma_{c_u} = 5$ kN/m^2 and $\sigma_{c_u} = 10$ kN/m^2 corresponding to coefficients of variation of $V_{c_u} = 0.2$ and 0.4.

In both cases the dimensionless correlation length was varied in the range $0.1 < \Theta < 4$. This study used 5000 Monte-Carlo simulations which was sufficient to give statistically reproducible results for all the parametric combinations considered. The proportion that gave $FS < 1$ was calculated as the probability of failure p_f.

Figure 4 illustrates a typical histogram of FS values for $V_{c_u} = 0.2$ and $\Theta = 0.1$ generated by the Monte-Carlo simulations, together with both normal and lognormal fitted functions. The curve fits were based on the statistics of the factor of safety coming out of the Monte-Carlo analysis, which for this case were $\mu_{FS} = 0.92$ and $\sigma_{FS} = 0.11$

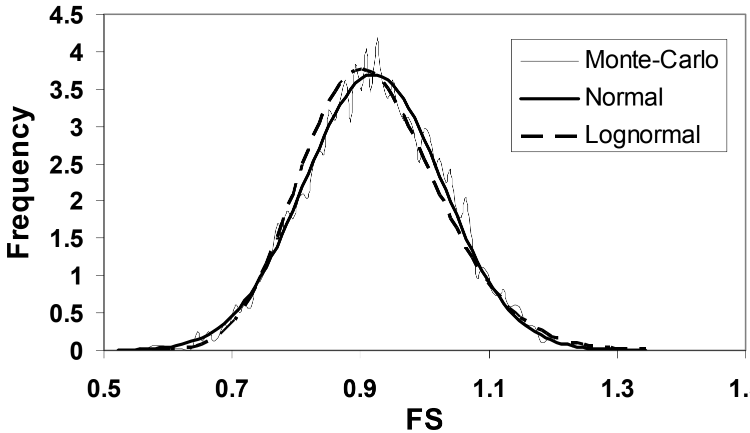

FIG.4: FS distribution from Monte-Carlo compared with normal and lognormal functions ($V_{c_u} = 0.2$, $\Theta = 0.1$)

Figure 5 and 6 gives plots of p_f vs. Θ from the Monte-Carlo simulations. The horizontal line in each case gives the probability of failure predicted by FOSM leading to $p_f = 0.264$ for $V_{c_u} = 0.2$ as computed earlier, and $p_f = 0.433$ for $V_{c_u} = 0.4$

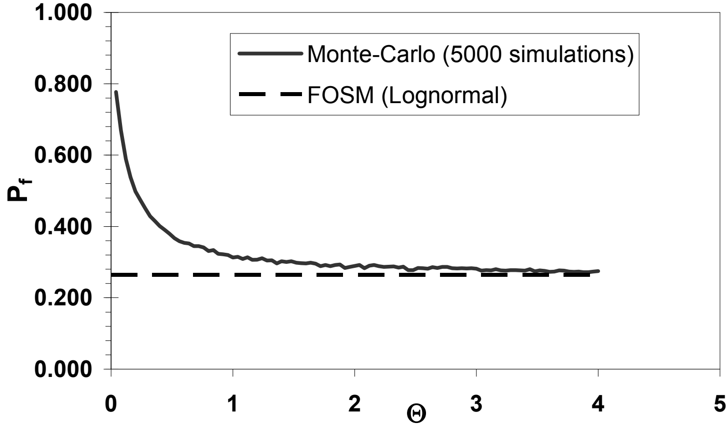

FIG. 5: Probability of failure vs. Spatial correlation length for $V_{c_u} = 0.2$

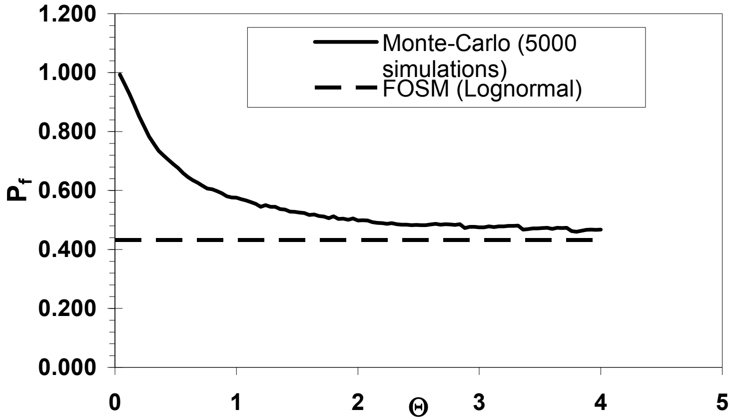

FIG. 6: Probability of failure vs. Spatial correlation length for $V_{c_u} = 0.4$

Clearly, FOSM is unconservative and is shown to be a special case of the random field results as $\Theta \to \infty$. The key reason for this is that the random field approach allows the slope to fail at its weakest point, while the FOSM method, being based on a classical formula, assumes the failure mechanism is at the base of the column which is not necessarily critical. Although not presented in this paper, similar conclusions are reached if c_u and FS are assumed to be normally distributed.

CONCLUSIONS

A novel analytical solution based on random properties has been developed and validated for the analysis of "infinite slopes" with the ability to model many different parametric variations. The classical analytical solution from infinite slope theory has been combined with random field theory to perform probabilistic infinite slope analyses in a Monte-Carlo framework. The method was compared with results obtained using the First Order Second Moment (FOSM) method. The FOSM lead in all cases to unconservative results because it is locked into the assumption that failure must occur at the base of the column. The random field approach has the key advantage that it "seeks out" the critical mechanism and is therefore a proper model of a spatially random soil. This phenomenon is also present in more conventional probabilistic studies of finite slope stability problems. Methodologies that resort to classical slope stability methodologies that do not allow the failure mechanism to "seek out" the most critical path are almost inevitably going to lead to unconservative results.

ACKNOWLEDGEMENT

The authors wish to acknowledge the support of NSF grant CMS-0408150 on "Advanced probabilistic analysis of stability problems in geotechnical engineering".

REFERENCES

Bromhead, E. N. (1992) "The Stability of Slopes." 2nd edition, Blackie Academic & Professional, New York: 135-137.
Duncan, J.M. (1996) "Slope stability analysis." Chapter 13 in "*Landslide Investigation and Mitigation.*", (Special report 247), TRB, National Research Council. National Academy Press, Washington, D. C.
Fenton, G.A. and Vanmarcke, E. H. (1990), "Simulation of random fields via local average subdivision." *J Eng Mech, ASCE,* Vol. 116, No. 8, pp. 1733-1749.
Lambe, T. W. and Whitman R. V. (1969), "Soil Mechanics." Wiley, New York: 553.
Taylor, D.W. (1948) "Fundamentals of Soil Mechanics." Wiley, Hoboken , New Jersey.

