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DISCUSSION

Three-dimensional slope stability analysis by elasto-plastic finite elements

D. V. GRIFFITHS and R. M. MARQUEZ (2007). *Géotechnique* **57**, No. 6, 537–546

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The authors present three-dimensional (3D) finite element analyses of slopes and give a bibliography of such work. They show that computer hardware has become sufficiently powerful in the last two or three decades to allow analysis in three dimensions to be considered. The method they adopt is to reduce the strength progressively until the predicted deformations are very large. They then deduce a strength reduction factor, which is something like the safety factor determined in conventional limit equilibrium stability analysis. The advantage of continuum analysis in simulating actual behaviour is thereby lost. They discuss whether analysis in three dimensions indicates higher stability than analysis in two dimensions, as is often thought. They arrive at no definite conclusions. They say that the usefulness of analysis depends on the extent to which it predicts and gives understanding of a physical mechanism involved, with which the discusser agrees. This can only be established by comparison with actual behaviour in the field.

Probably the most well-documented slide, which involves strongly 3D behaviour, was the failure of the upstream side of Carsington Dam embankment in 1984, just before it was raised to full height. A full account of this and a bibliography is given by Skempton & Vaughan (1993). The slide occurred through the central core of the dam of rather plastic clay and through the ‘boot’ where the core had been extended upstream to blanket the foundation. Numerous piezometers showed that there had been little consolidation of the core and boot. Upstream of the boot the slide passed through a narrow layer of plastic clay (the ‘yellow clay’) on top of the foundation on the sides of the valley and through the top of the mudstone fill at the valley centre where the ‘yellow clay’ was absent. Both the fill and foundation clay contained shears, the effect of which could not be measured representatively by test.

Skempton & Vaughan (1993) concluded that the slide involved progressive failure on the cross-section where it first occurred and also in the lateral direction (along the length of the dam). This latter they termed ‘lateral load transfer’. This caused much of the slide to fail at a factor of safety lower than that predicted using the conventional plane-strain analysis (Fig. 16). The stability analysis was adjusted to give a factor of safety of 1·0 where it was at a minimum (see point A in Fig. 16). The analysis of safety factor was dependent to some extent on the estimation of the effect of shears on shear strength of both the fill and foundation. This had to be estimated.

The lateral extent of the slide was large in relation to its depth, as shown in Fig. 16 (Vaughan, 1991; Skempton & Vaughan, 1993). It passed through two different foundations and varied in depth along its length by a factor of more than two. It was observed to develop progressively from the position of initial movement, and did not become unstable along its whole length at the same time. Thus some additional agent causing instability was indicated.

It was concluded that this extra cause of instability was lateral load transfer occurring according to a mechanism illustrated in Fig. 17 (Vaughan, 1991). It is postulated that a length A, which fails, initially exerts an out-of-slope destabilising force on the slope B outside the initial failure. Either the slope fails and the 3D effect is towards stabilising the slide or the initial failure A ‘drags off’ the next length B of slope, which joins the slide. In turn it may be joined by length C and so on.

Lateral load transfer involves large strains as well as strain-softening, which affects the whole of the slide (i.e. in-plane and out-of-plane). This type of behaviour is indicated if a slide moves to a new flatter slope. This behaviour is common. Finite element analysis of this scenario is probably impractical at the present time.

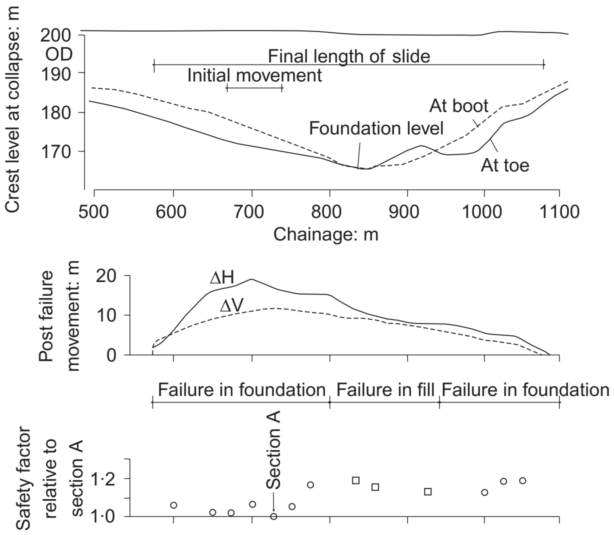

Fig. 16. Carsington slide longitudinal section and extent of slide with final post failure movements, horizontal at toe, vertical at crest. Safety factor by limit equilibrium adjusted to unity at initial failure (after Vaughan, 1991)

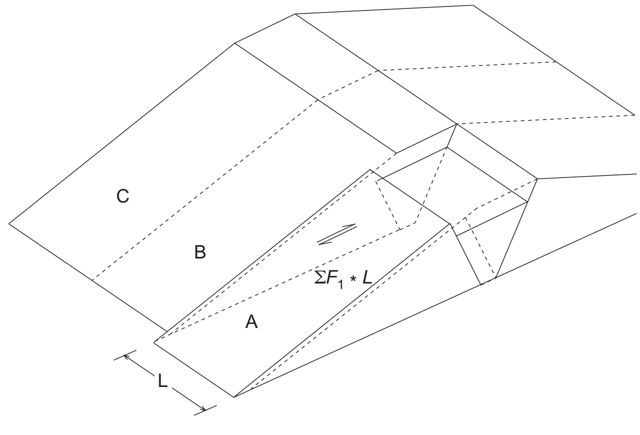

Fig. 17. Mechanism of lateral load transfer (after Vaughan, 1991)

Vaughan (1991) considers the lateral behaviour of the Carsington slide and presents an approximate independent analysis that combines the limit equilibrium analysis of forces generated as the slide moves and the loss of resistance of the sliding surface as it slides. It involves large strains and indicates that the forces that could be generated by lateral load transfer could be greater than those required by the alternative analysis from which the safety factors shown in Fig. 16 are derived.

In this analysis the increase and decrease in downstream/upstream force as the slide moves are estimated by limit equilibrium methods. The embankment is simulated as a thick elastic beam. This analysis suggests that, if the initial failure is longer than about twice the slope height, the loss of shearing resistance of the moving mass exceeds the reduction in driving force and results in the slide extending as described above. The slide progresses laterally. The approximate analysis shows that the stresses involved are credible and so gives support to the concept and mechanisms of lateral load transfer.

Thus the behaviour of the Carsington Dam and the effect of lateral load transfer that it showed indicate the following.

(*a*) When a deep-seated slide exhibits brittle behaviour after its first yield, the real factor of safety should be less when it is assessed in three dimensions rather than two. There is no justification for assuming it increases. This may give a false sense of security.
(*b*) The slide may spread laterally for long distances.
(*c*) It may affect lengths of slope which have been previously stable for some time and seem as if they should remain stable when analysed in two dimensions.
(*d*) It may affect lower parts of a slope which have previously remained stable.

The authors should be congratulated in drawing attention to the importance of 3D effects in stability of slopes and our modern ability to probe them by finite element analysis. It would be informative if finite element analysis could be employed to illuminate the effect of lateral load transfer in slopes which are potentially liable to this effect.

Authors' reply

The authors thank the discusser for his interest in this paper. The purpose of the paper under discussion was not to model a particular case history; the methodology could, however, easily be applied in this way. Displacements and stresses at working stress levels for a 3D configuration such as the Carsington embankment, could certainly be modelled by performing a finite element analysis using the actual shear strength and deformation parameters (essentially a strength reduction factor of unity).

Regarding the difference between two-dimensional (2D) and 3D factors of safety, the finite element studies we presented in the paper only served to confirm the usual assumption that 2D is conservative. This is intuitively reasonable if one thinks that 2D (plane strain) slopes can derive no support at all from adjacent soil in the third direction.

This point is highlighted in the 3D example shown in Fig. 18, which represents a 2:1 slope of height 10 m, foundation depth 5 m and a length in the out-of-plane direction of 60 m with smooth boundary conditions. An oblique zone of weak soil (shaded black) with undrained strength $c_u = 20$ kN/m^2 has been introduced into the slope with the surrounding soil four times stronger at $c_u = 80$ kN/m^2. Using the program described in the paper, the 3D factor of safety is found to be approximately 1·5 and the mechanism clearly follows the weak zone as shown in Fig. 19. When 2D stability analyses are then performed on successive slices in the x–z plane moving from $y = 0$ m to $y = 60$ m, the result shown in Fig. 20

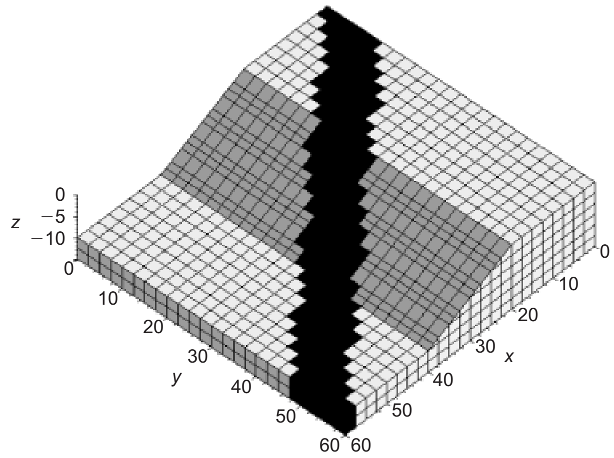

Fig. 18. Three-dimensional slope including an oblique layer of weak soil

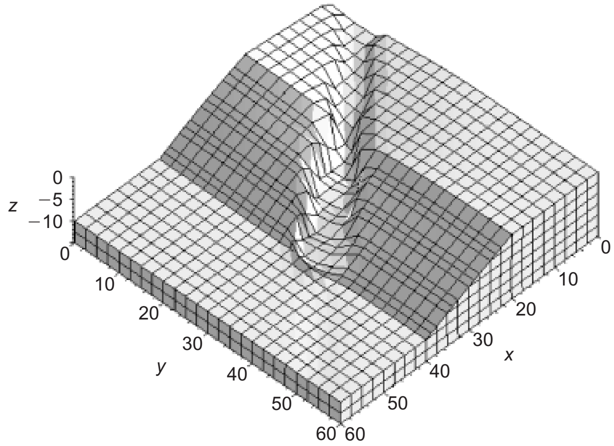

Fig. 19. Three-dimensional failure mechanism

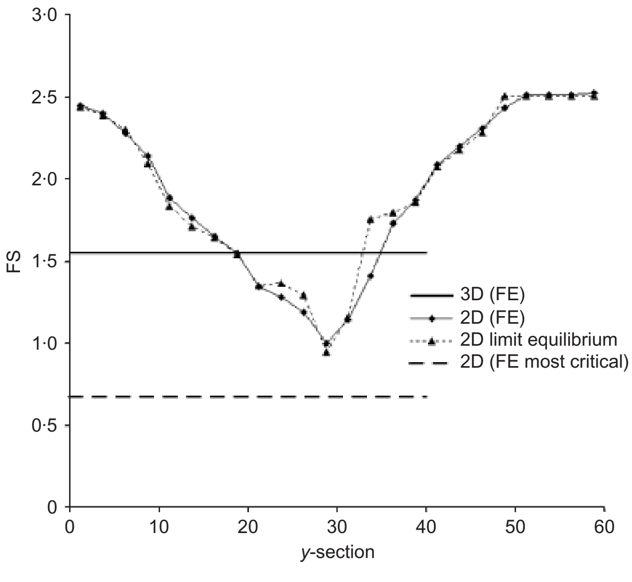

Fig. 20. Factors of safety from 3D analysis and various 2D sections

is obtained. As a check, the 2D analyses were performed both by finite elements and by a standard limit equilibrium program. It can be seen that towards the boundaries of the 3D slope ($y < 21$ m and $y > 34$ m) where the majority of soil in the sections is strong, the 2D results led to higher and therefore unconservative estimates of the factor of safety. On the other hand, at sections towards the middle of the slope (21 m $< y <$ 34 m) where there is a greater volume of weak soil, the 2D results led to lower, and therefore conservative, estimates of the factors of safety. The 2D factor of safety closely approached unity at $y = 29$ m. An even more critical 2D plane, however, is the one that runs right down the middle of the weak soil. This 2D plane gives a 2·5:1 slope which is flatter than the x–z planes considered previously; it is, however, homogeneous and consists entirely of the weaker soil. A 2D slope stability analysis on this plane gives an even lower factor of safety of about 0·7. This result, also shown in Fig. 20, is less than half of the factor of safety given by the 3D analysis and would be considered excessively conservative by geotechnical design standards.

Even in the rather simple problem considered here, the results have shown a quite complex relationship between 2D and 3D factors of safety. The results confirm that 2D analysis will deliver conservative results if a pessimistic plane in the 3D problem is selected; this may, however, lie well below the 'true' 3D factor of safety. It has also been shown, however, that selection of the 'wrong' 2D plane could lead to an unconservative result.

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