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Probabilistic stability analysis of shallow landslides using random fields

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ABSTRACT: The paper presents probabilistic studies that demonstrate the influence of spatially random soil properties on the stability of shallow landslides using random fields Results indicate that traditional "first order" methods are inherently unconservative when applied to limit analysis problems unless they allow the failure mechanism to "seek out" the most critical location.

1 Introduction

Recent interest in the analysis of shallow landslides (e.g. Turner and Schuster 2007) has led to the work described in this paper, where three different methods of probabilistic analysis are applied to the classical infinite slope equations. The objective of the analyses is to produce estimates of the probability of failure as opposed to the conventional approach involving a factor of safety.

Consider the infinite slope shown in Figure 1 in which a unit area slice of homogeneous soil subjected to gravity and possibly also a pseudo horizontal acceleration.

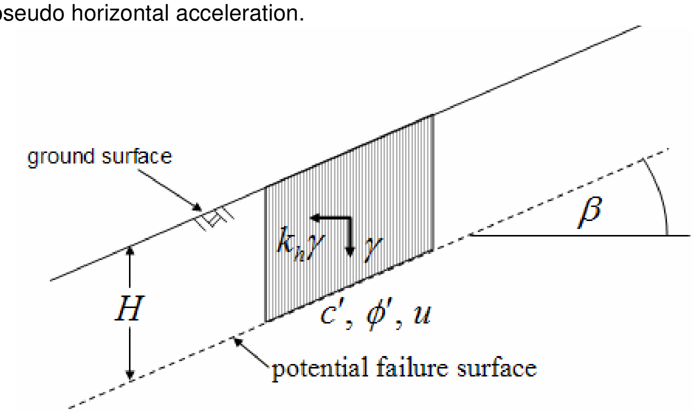

Figure 1. Infinite slope configuration

The infinite slope equation for this case (e.g. Biondi *et al.* 2000) is given by

$$FS = \frac{\left(H\gamma\left(\cos\beta - k_h \sin\beta\right)\cos\beta - u\right)\tan\phi' + c'}{H\gamma\left(\sin\beta + k_h\cos\beta\right)\cos\beta} \tag{1}$$

where the variable names have the following meanings

FS	factor of safety
H	depth of the soil layer to potential failure surface
k_h	horizontal pseudo-acceleration coefficient

u	pore pressure at the base of the slice
β	slope inclination
γ	total unit weight
$\tan\phi'$	tangent of the effective soil friction angle at the base of the slice
c'	effective cohesion at the base of the slice

The main objective of a probabilistic approach to this problem is to estimate the mean and standard deviation of the factor of safety (μ_{FS},σ_{FS}), having defined one or more of the seven input parameters in terms of their means and standard deviations. By fitting a suitable probability density function to the distribution of the factor of safety, the probability of failure is then given by the probability that the factor of safety is less than unity, hence

$$p_f = P[FS<1] \quad (2)$$

Other investigators have studied this problem (e.g. Nadim 2006), but the novelty of the current approach lies in the use of random field theory to describe the vertical spatial distribution of soil properties. This is combined with a Monte-Carlo approach in which each simulation leads to a different factor of safety. The proportion of the factors of safety that fall below unity is then computed as the probability of failure.

This approach allows the inclusion of an additional parameter called the spatial correlation length θ which describes the distance over which properties tend to be correlated (or the log of the property if the field is assumed lognormal). Figure 2 shows a grayscale which portrays a random field of a soil property (e.g. c_u) in which black implies high (strong) values and white implies low (weak). A low spatial correlation length indicates that soil properties are varying rapidly spatially while a high value implies gradually varying properties. A key factor here is that while the fields look quite different from each other, they have the same mean and standard deviation of the property being modeled.

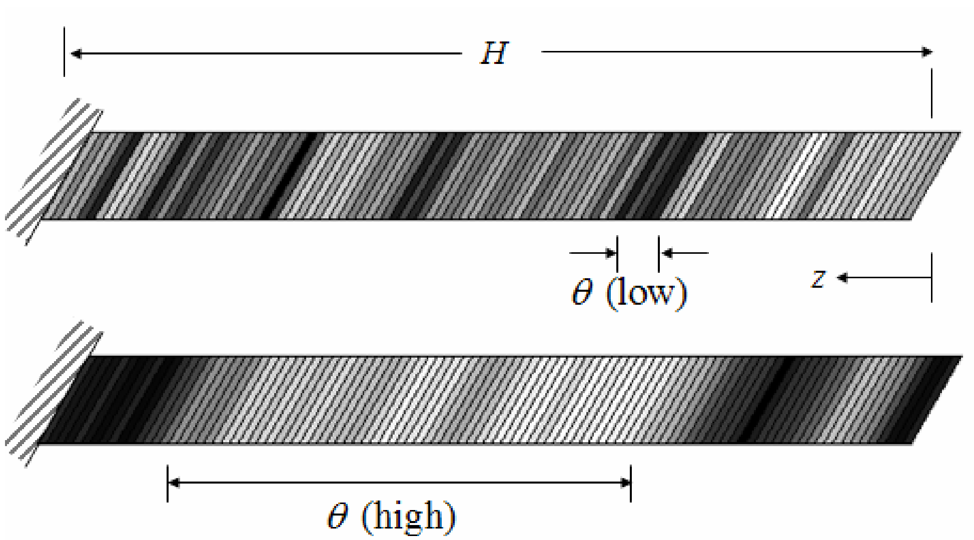

Figure 2. Portrayal of spatial correlation in a random field.

The results presented in Section 2.3 use a dimensionless spatial correlation length Θ that has been normalized with respect to the soil depth, thus $\Theta=\theta/H$ .The authors have developed software to optionally generate random fields relating to the cohesion c', the tangent of the friction angle $\tan\phi'$, the pore pressure u and the total unit weight γ. Other parameters, such as the soil column height H, the slope angle β and the pseudo-acceleration coefficient k_h can be generated as ordinary random variables. Variables that are not treated as random in an analysis are simply fixed to constant values. The soil column is subdivided into 100 equal elements and, following generation of the random variables as shown in Figure 2, the factor of safety of each element is computed using Eq.(1). It should be noted that when implementing Eq.(1), H is replaced by the depth z of each element. This approach will lead to 100 different factors of safety (as it would with a homogeneous soil) however with spatially random properties substituted into Eq.(1), the lowest factor of safety of the set does not necessarily occur at the base of the soil column.

This is a very important result, because simpler probabilistic approaches such as "first order" methods, have no way of explicitly modeling spatial variability, and are locked into the assumption that the critical depth is always at the base of the soil column. The simpler approaches are therefore inherently unconservative. This point will be demonstrated in the next section by comparing first order solutions with the random field approach as applied to an example problem.

2 Example problem solved by different methods

Although there are seven independent variables in Eq.(1), the example calculations presented here consider just one random variable, namely the undrained shear strength c_u. Of the other six parameters, three are set to zero $(\tan\phi_u = 0,\ u = 0,\ k_h = 0)$ and the other three $(\gamma,\ H,\ \beta)$ are set to constant values. Equation (1) therefore simplifies to the form

$$FS = \frac{c_u}{H\gamma \sin\beta \cos\beta} \tag{3}$$

The problem now amounts to finding the mean and standard deviation of the factor of safety (μ_{FS}, σ_{FS}) given the mean and standard deviation of the undrained shear strength (μ_{c_u}, σ_{c_u}). In the random field calculations described later in this section, an additional parameter, the spatial correlation length (θ), will also be provided as input. In the following, units will not be included, but any consistent system of units can be assumed.

The example problem has the following random parameters: $\mu_{c_u} = 25,\ \sigma_{c_u} = 2.5$ with the other deterministic parameters fixed to $\gamma = 20,\ H = 2.5$ and $\beta = 30°$. Values of the spatial correlation length θ will be introduced in the description of the random field solutions in Section 2.3.

2.1 First Order Second Moment (FOSM) Method

Details of this classical method are described elsewhere (e.g. Harr 1987, Baecher and Christian 2003)
The right hand side of Eq.(2) involves a single random variable equation, hence

$$\mu_{FS} = \frac{\mu_{c_u}}{\gamma H \cos\beta \sin\beta} \quad \text{and} \quad \sigma_{FS} = \frac{\sigma_{c_u}}{\gamma H \cos\beta \sin\beta} \tag{4}$$

In order to compute the probability of failure, we must assume a distribution for FS. Since FS is always a positive quantity, we could assume that FS is lognormal (that is, $\ln FS$ is normal). The probability of failure is then given by

$$p_f = \mathrm{P}[FS<1] = \mathrm{P}[\ln FS < \ln 1] = \Phi\left[\frac{\ln 1 - \mu_{\ln FS}}{\sigma_{\ln FS}}\right] \tag{5}$$

where the mean and standard deviation of the underlying normal distribution of $\ln FS$ are given by

$$\mu_{\ln FS} = \ln \mu_{FS} - \frac{1}{2}\ln\left\{1 + v_{FS}^2\right\} \quad \text{and} \quad \sigma_{\ln FS} = \sqrt{\ln\left\{1 + v_{FS}^2\right\}} \tag{6}$$

and $\Phi(.)$ is the cumulative standard normal function.

Using the parameters of the example problem, Eq.(6) gives $\mu_{\ln FS} = 0.13887$ and $\sigma_{\ln FS} = 0.09975$.
Which after substitution into Eq.(5) gives

$$p_f = \Phi\left[\frac{-0.13887}{0.09975}\right] = \Phi[-1.39212] = 1 - \Phi[1.39212] = 1 - 0.918 = 0.082 \tag{7}$$

hence the FOSM method predicts a probability of failure of 8.2%.

2.2 First Order Reliability Method (FORM)

A drawback of the FOSM method is that is can lead to non-unique probabilities of failure for the same problem when stated in equivalent, but different ways (e.g. Ditlevsen 1973, Madsen *et al.* 1986, Nadim *et al.* 2005) . What the FOSM method is doing is computing the distance from the mean point to the failure surface *in the direction of the gradient at the mean point*. Hasofer and Lind (1974) solved the non-uniqueness problem in the FORM by looking for the overall minimum distance between the mean point and the failure surface, rather than looking just along the gradient direction. FORM is essentially an optimization problem, which is easily coded in widely available software such as Excel (see e.g. Low and Tang 1997, Griffiths *et al.* 2007).

Unlike FOSM, the user must decide on the form of the input probability density function(s) *before* using FORM. If c_u is assumed to be normally distributed in the example considered above, FORM gives $p_f = 0.090$ (9.0%) whereas if c_u is assumed to be lognormally distributed, $p_f = 0.082$ (8.2%). Clearly in this case both the first order methods are in close agreement.

2.3 Random Field (RF) Method

In this section we present results obtained using the authors' RF infinite slope analysis model as described in Section 1. All results presented in this section used 5000 Monte-Carlo simulations. Figure 3 shows the computed probability of failure for the test problem assuming a lognormally distributed c_u for spatial correlation lengths varying in the range $0.04 < \Theta < 2.56$

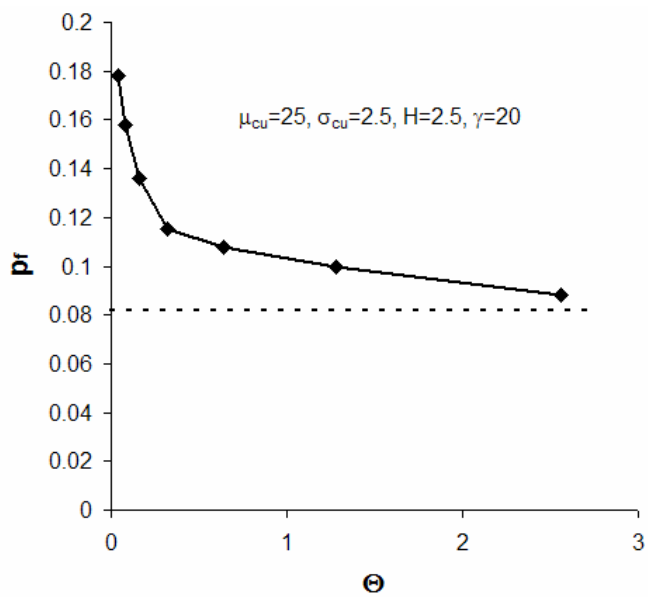

Figure 3. Comparison of FORM and Random Field approach for example problem showing the importance of the spatial correlation length Θ

The result clearly demonstrates the unconservative nature of the first order approaches. The RF solutions give a higher probability of failure for all reasonable correlation lengths and converge asymptotically on the first order solution as the correlation length becomes large ($\Theta \to \infty$). This convergence emphasizes the fact that first order approaches are "single random variable" methods in which the soil column is always assumed to be homogeneous, albeit with a shear strength that varies randomly.

3 Discussion of results from example problem

3.1 Location of critical failure plane

Consider once more Eq.(3) with the column depth H replaced by the depth coordinate z to give

$$FS = \frac{c_u}{z\gamma \sin\beta \cos\beta} \tag{8}$$

The minimum factor of safety will clearly occur where c_u/z is a minimum (e.g. Duncan and Wright 2005). For homogeneous soil with a constant c_u, the critical failure surface will always occur at the base of the column where $z = H$. When c_u is treated as a spatially random variable in the RF approach, the minimum value of c_u/z will tend to be near the bottom of the column, but will frequently occur further up.

An additional set of analyses have been performed on the infinite slope problem using the random finite element method (RFEM) (Griffiths and Fenton 1993, Fenton and Griffiths 1993). These analyses have the ability to compute failure deformations as well as factors of safety for each Monte-Carlo simulation (Griffiths *et al.* 2007). Figure 4 shows some deformed meshes for simulation that were deliberately chosen because they gave mechanisms that were well removed from the base. In addition, some of the simulations display simultaneous failure mechanisms at different depths, implying multiple elevations exhibiting the same minimum factor of safety.

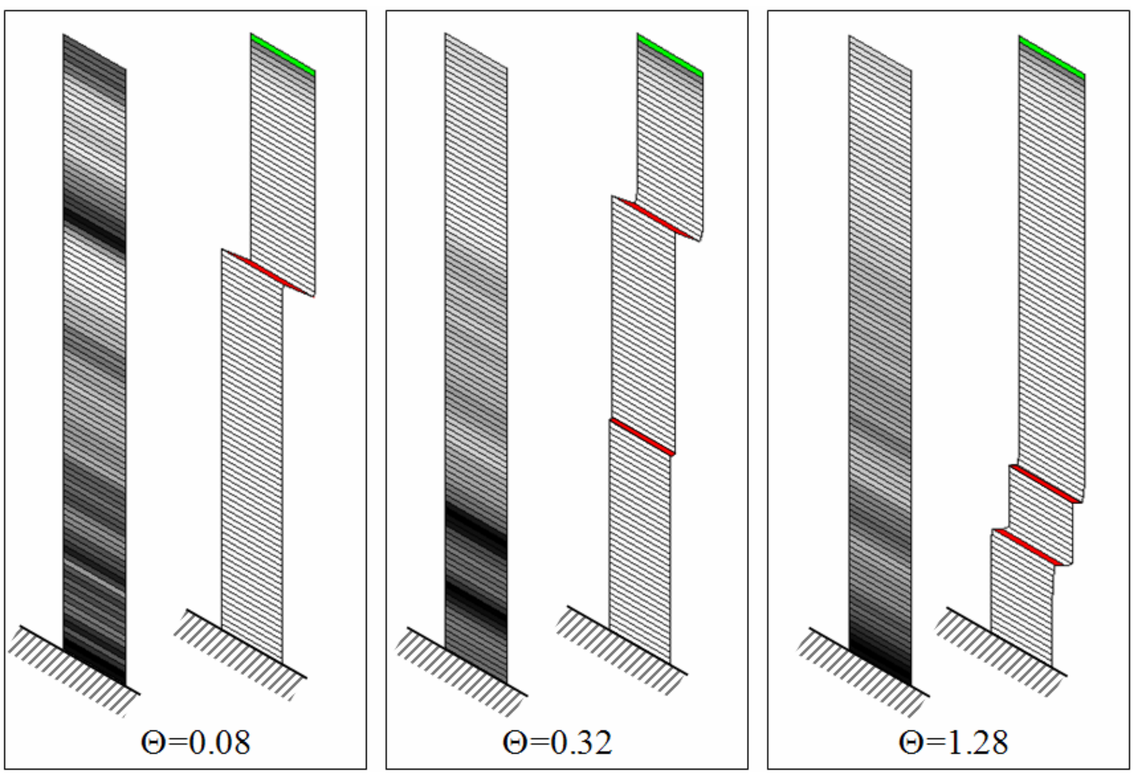

Figure 4. Failure modes of RFEM analyses showing mechanisms well removed from the base of the column.

From Figure 3, the first order methods were most unconservative when the spatial correlation length was relatively low. The histogram shown in Figure 5 for $\Theta = 0.04$ indicates that only about 23% of the critical mechanisms occurred at the base with 77% occurring higher up the soil column. Figure 6 shows a similar plot for $\Theta = 1.28$. For the higher spatial correlation length, the percentage of mechanisms occurring at the base ($\approx 51\%$) is considerably increased as the RF solutions tend to the spatially uniform soil implied by first order.

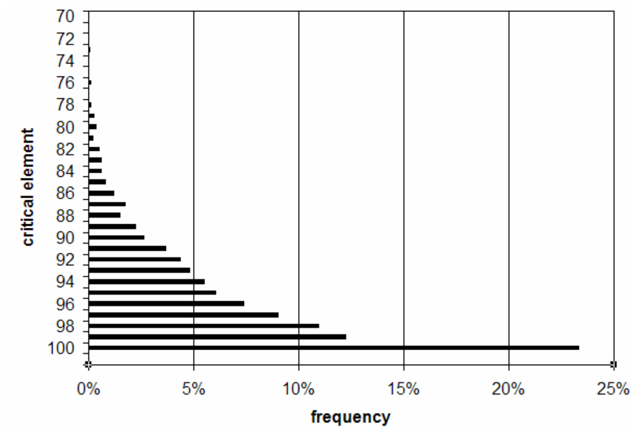

Figure 5. Histogram showing the frequency with which the critical depth occurs at different depths throughout the 100 element column in RF analysis ($\Theta = 0.04$)

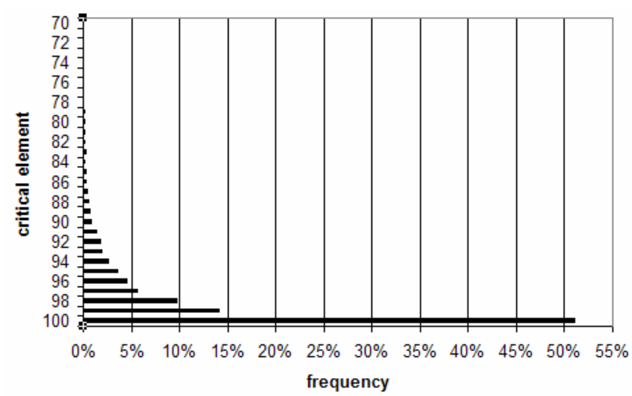

Figure 6. Histogram showing the frequency with which the critical depth occurs at different depths throughout the 100 element column in RF analysis ($\Theta = 1.28$)

3.2 Distribution of FS values

In the previous sub-section, the probability of failure p_f was computed simply as the proportion of the total number of Monte-Carlo simulations that resulted in $FS < 1$. Since each random field simulation computes a different factor of safety, the full probability density function of FS values can be plotted and used for further analysis. For example, Figure 7 shows a probability density plot of FS values computed for the example problem when $\Theta = 0.16$. Included on the figure are analytical normal and lognormal fits to the random field results based on the computed mean and standard deviation values of $\mu_{FS} = 1.102$ and $\sigma_{FS} = 0.093$. Both analytical curves agree well with the random field results, although this might be expected in view of the rather low coefficient of variation $v_{FS} \approx 0.084$.

Figure 8 shows similar fits to random field results for the same correlation length, but with a much higher input coefficient of variation of $v_{c_u} = 1.0$. In this case $\mu_{FS} = 0.307$ and $\sigma_{FS} = 0.170$ and the distribution of FS is clearly much better fitted to the lognormal curve. This might be expected considering that the input random variable on the right hand side of Eq.(3) is also lognormal. Certainly as $\Theta \to \infty$ the distribution of FS will theoretically tend to the lognormal in this case.

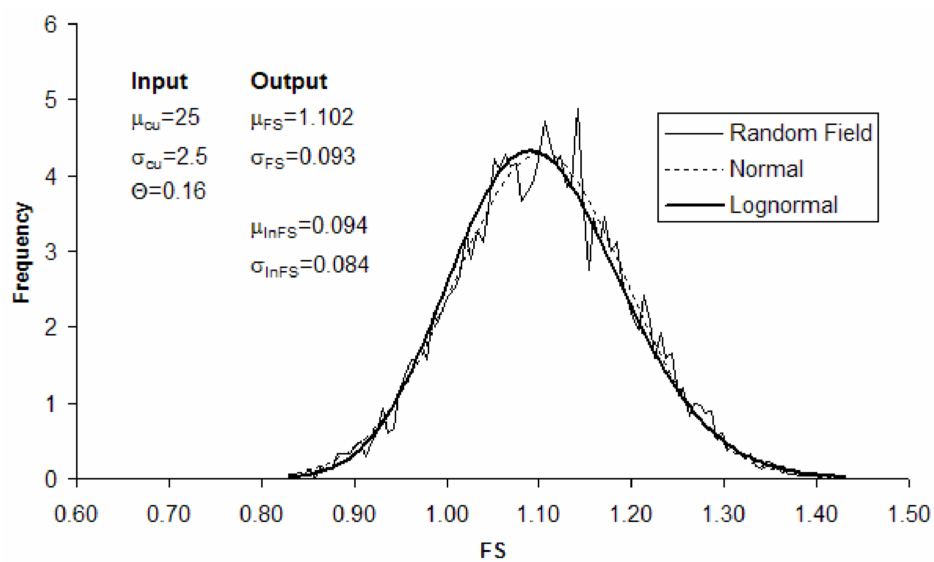

Figure 7. Random field histogram of FS frequency distribution for the example problem with $v_{c_u} = 0.1$ together with normal and lognormal fits based on the computed mean and standard deviation of FS

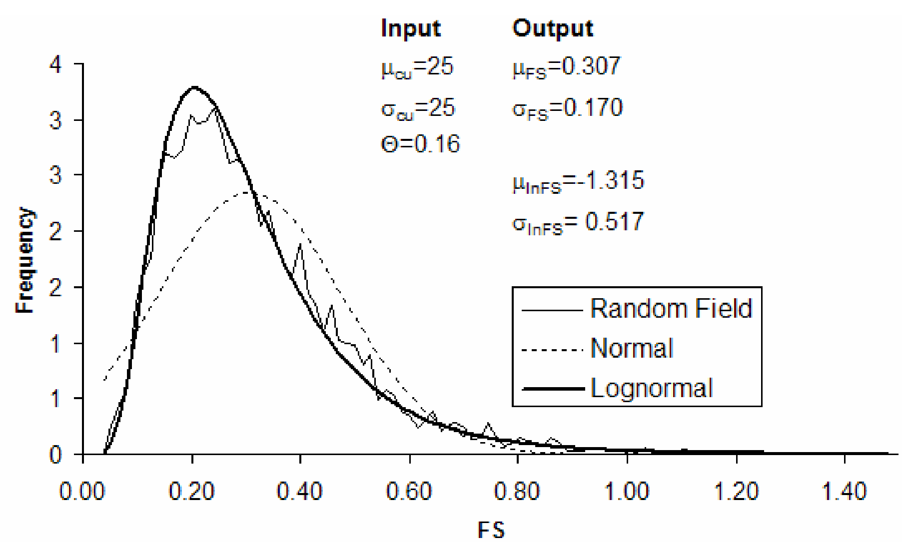

Figure 8. Random field histogram of FS frequency distribution for the example problem with $v_{c_u} = 1.0$ together with normal and lognormal fits based on the computed mean and standard deviation of FS

4 Conclusions

Any probabilistic method of geotechnical limit analysis that does not have the ability to "seek out" the critical failure plane will lead to unconservative results. This follows from classical upper bound limit analysis theory, which dictates that the wrong failure surface will always overestimate the failure load. This conclusion applies to all probabilistic geotechnical limit analysis applications, and particularly methods of slope stability that assume classical circular failure mechanisms (e.g. Bishop's method). The infinite slope example offers a particularly clear demonstration of this effect however, since the system exhibits no progressive failure and is essentially "brittle' in that the first component to fail results in overall system failure. The first order applications (FOSM and FORM) described in this paper, properly accounted for probabilistic input variables, but assumed *a priori* that the failure surface was always at the base of the soil column. The RF analyses indicated that for reasonable spatial correlation lengths, a significant proportion of failures in a Monte-Carlo analysis occurred above the base, where the factor of safety is lower. The first order methods always underestimated the probability of failure but were shown to be special cases of the RF analysis as the spatial correlation length tended to infinity.

5 Acknowledgements

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