Reliability Analysis of a Strip Footing Designed Against Settlement

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ABSTRACT: The traditional design of a strip footing for settlement on a sandy soil proceeds by first testing the soil at a limited number of locations to assess its elastic modulus (for example by CPT). Then a simplified settlement design relationship is commonly employed to determine the required footing width. In a real soil, the soil tests may or may not be representative of the average elastic modulus under the footing due to spatial variability. This paper reports on a Monte Carlo investigation in which a spatially variable soil mass is simulated and virtual samples taken at a small number of locations are used to determine the required footing width. The resulting footing is then placed on the simulated soil mass and the actual settlement computed using the finite element method. By repeating this process a large number of times, the reliability of a simplified design approach, based on a limited soil sample, against excess settlement can be assessed. In particular, the paper reports on the reliability of a modified Janbu design approach with respect to the soil's variance and scale of fluctuation. The overall goal is to investigate the reliability of existing design methodologies with a view towards reliability-based code development.

1 INTRODUCTION

Foundation settlement, if excessive, can lead to unsightly cracking of structural and non-structural elements of the supported building. For this reason most geotechnical design codes limit the settlement of footings to some reasonable amount, typically 25 to 50 mm (US Army Corps of Engineers, 1994, Canadian Geotechnical Society, 1992). Since the design of a footing is often governed by settlement it would be useful to evaluate the reliability of typical 'traditional' design methodologies.

In this paper, the reliability of one particular design method is investigated when applied to a soil with spatially random effective elastic modulus. The effective modulus could be either just the initial elastic modulus field or the initial elastic modulus combined with an effective elastic modulus field representing consolidation settlement. In either case, the soil elastic modulus field $E_s(\underset{\sim}{x})$, where $\underset{\sim}{x}$ is spatial position, is modeled as a stationary finite-scale spatially varying two-dimensional random field. Poisson's ratio is assumed deterministic and held constant at $\nu = 0.35$.

A two-dimensional analysis is performed here on a strip footing assumed to be of infinite length out-of-plane. Spatial variation in the out-of-plane direction is ignored, which is equivalent to saying that the out-of-plane scale of fluctuation is infinite. Although settlement of real footings generally depends on both plan footing dimensions, the full three-dimensional model is not yet completed by the authors. This study is designed to provide a methodology to assessing the reliability of traditional design methods as well as to identify problems in doing so. A typical finite element mesh showing a footing founded on a spatially random elastic modulus field, where light regions correspond to lower values of $E_s(\underset{\sim}{x})$, is shown in Figure 1.

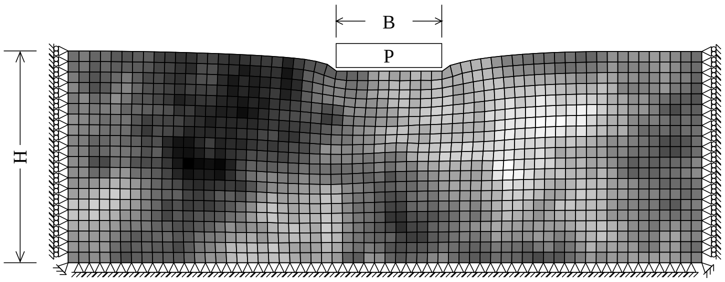

Figure 1. Deformed finite element mesh with sample elastic modulus field.

2 SETTLEMENT DESIGN METHODOLOGY

The design method under consideration is due to Janbu (1956), who expresses settlement under a strip footing in the form

$$\delta = \mu_0 \cdot \mu_1 \cdot \frac{qB}{E_s^*} \tag{1}$$

where q is the vertical stress, in kN/m^2, applied by the footing to the soil, B is the footing width, E_s^* is some equivalent measure of the soil elastic modulus underlying the footing, μ_0 is an influence factor for depth D of the footing below the ground surface, and μ_1 is an influence factor for the footing width B and depth of the soil layer H. A particular case study will be considered here for simplicity and clarity, rather than non-dimensionalizing the problem. The particular case considered is of a footing founded at the surface of a soil layer ($\mu_0 = 1.0$) underlain by bedrock at a depth $H = 6$ m. The footing load is assumed to be deterministic and equal to $P = 1250$ kN per metre length of the footing in the out-of-plane direction. In terms of P, Eq. (1) can be rewritten as

$$\delta = \mu_0 \cdot \mu_1 \cdot \frac{P}{E_s^*} \tag{2}$$

Since the research goal is to compare Janbu's settlement predictions to those obtained by linear finite element analysis, it was decided to calibrate Janbu's relationship against the finite element results obtained using deterministic and spatially constant elastic modulus $E_s^* = 30$ MPa for various ratios of H/B. Figure 2 illustrates how the influence factor μ_1 varies with $\ln(H/B)$. As can be seen, it is very nearly a straight line which is well approximated by

$$\mu_1 = a + b \ln\left(\frac{H}{B}\right) \tag{3}$$

where, for the case under consideration with Poisson's ratio of 0.35, the line of best fit has $a = 0.4294$ and $b = 0.5071$, as shown fitted in Figure 2. The settlement equation now can be written as

$$\delta = \mu_0 \left[a + b \ln\left(\frac{H}{B}\right)\right] \cdot \frac{P}{E_s^*} \tag{4}$$

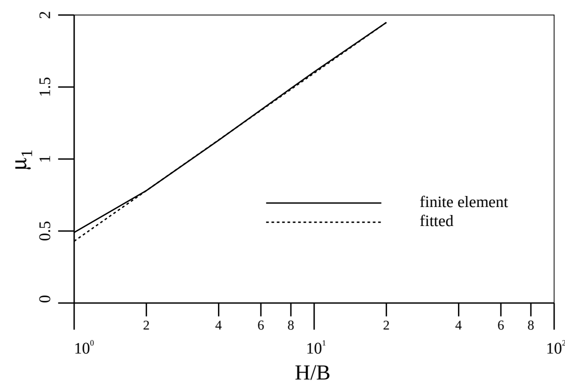

Figure 2. Effect of ratio H/B on settlement influence factor μ_1.

The case where E_s^* is estimated by sampling the soil at a few locations below the footing is now considered. Letting $\hat{E}_s$ be the estimated elastic modulus, one possible estimator is

$$\hat{E}_s = \frac{H_1E_1 + H_2E_2 + \ldots + H_nE_n}{H} \tag{5}$$

where H_i is the thickness of the i'th soil layer and H is the total thickness of all layers. In this study individual layers are not considered directly, although spatial variability may lead to the appearance of layering, so it will be assumed that n samples will be taken at equispaced distances over the depth H along a vertical line centered on the footing. In this case, the estimated elastic modulus just becomes the arithmetic average

$$\hat{E}_s = \frac{1}{n}\sum_{i=1}^{n} E_i \tag{6}$$

No attempt is made in this study to account for measurement error. The goal here is to assess the settlement variability conditioned by actual observations of the elastic modulus at a few points.

Using the estimated elastic modulus, the settlement predicted by Janbu's method becomes

$$\delta_{pred} = \mu_0 \left[a + b \ln \left(\frac{H}{B} \right) \right] \cdot \frac{P}{\hat{E}_s} \qquad (7)$$

If a maximum allowable settlement of 40 mm is to be designed for, then by setting $\delta_{pred} = \delta_{max} = 0.04$ m, Eq. (7) can be solved for the required footing width B,

$$B = H \exp \left\{ -\frac{1}{b} \left(\frac{\hat{E}_s \delta_{max}}{P \mu_0} - a \right) \right\} \qquad (8)$$

Since the soil elastic modulus field $E_s(\underset{\sim}{x})$ is random, the estimate $\hat{E}_s$ will also be random, which means that B is random. This is to be interpreted as follows; consider a sequence of similar sites on each of which a footing is to be designed and placed to support the load P such that, for each, the settlement prediction is δ_{max}. Because the sites involve different realizations of the soil elastic modulus field, they will each have a different estimate $\hat{E}_s$ obtained by sampling. Thus, each site will have a different required footing width B.

The task now is to assess the distribution of the actual settlement experienced by each of these designed footings. If the prediction equation is accurate, then it is expected that approximately 50% of the footings will experience settlements in excess of δ_{max} while the remaining 50% will experience less settlement. But, how much more or less? That is, what is the variability of settlement in this case? Note that this is a conditional probability problem. Namely, the random field $E_s(\underset{\sim}{x})$ has been sampled at n points to obtain the design estimate $\hat{E}_s$. Given this estimate, B is obtained by Eq. (8). However, since the real field is spatially variable, $\hat{E}_s$ may or may not represent the actual elastic modulus as 'seen' by the completed footing so that the actual settlement experienced by the footing will inevitably differ from the design target.

3 PROBABILISTIC ASSESSMENT OF SETTLEMENT VARIABILITY

The settlement variability will be assessed by Monte Carlo simulation. Details of the finite element model and random field simulator can be found in Fenton and Griffiths (2002). The finite element model is 60 elements wide by 40 elements in depth, with nominal element sizes $\Delta x = \Delta y = 0.15$ m, giving a soil regime of size 9 m wide by 6 m in depth. The Monte Carlo simulation consists of the following steps;

1) generate a random field of elastic modulus local average cells using the Local Average Subdivision (LAS) method (Fenton and Vanmarcke, 1990) which are then mapped onto the finite elements themselves.
2) 'virtually' sample the random field at 4 elements directly below the footing centerline (at depths 0, $H/3$, $2H/3$, and H). Then compute the estimated design elastic modulus, $\hat{E}_s$, as the arithmetic average of these values.
3) compute the required footing width, B, using Eq. (8).
4) adjust both the (integer) number of elements, n_B, underlying the footing in the finite element (FE) model and element width, Δx, such that $B = n_B \Delta x$. Note that the FE model assumes that the footing is an integer number of elements wide. Since B, as computed by Eq. (8) is continuously varying, some adjustment of Δx will be necessary. The final value of Δx is constrained to lie between (3/4)0.15 and (4/3)0.15 to avoid excessive element aspect ratios (Δy is held fixed at 0.15 m to maintain $H = 6$ m). Note also that the random field is *not* regenerated for the adjusted element size, so that some accuracy is lost with respect to the local average statistics. However, the approximation is deemed acceptable, given all other sources of uncertainty. Finally, the actual value of B used is constrained so that the footing is not less than 4 elements wide, nor more than 48 elements wide. This constraint is actually a more serious limitation, leading to some possible bias in the results. This is discussed further below.
5) use the FE code to compute the simulated settlement, δ_{sim}, which is interpreted as the settlement that the footing would actually experience on this particular realization of the spatially varying elastic modulus field.
6) repeat from step (1) $n_{sim} = 2000$ times to yield 2000 realizations of δ_{sim}

The sequence of realizations for δ_{sim} can then be statistically analyzed to determine its conditional probability density function (conditioned on $\hat{E}_s$).

The elastic modulus field is assumed to be lognormally distributed with parameters

$$\sigma^2_{\ln E_s} = \ln\left(1 + V^2\right), \quad \mu_{\ln E_s} = \ln(\mu_{E_s}) - \tfrac{1}{2}\sigma^2_{\ln E_s} \quad (9)$$

where $V = \sigma_{E_s}/\mu_{E_s}$ is the coefficient of variation. Since $E_s(\underset{\sim}{x})$ is lognormally distributed, its logarithm is normally distributed and $E_s(\underset{\sim}{x})$ can be obtained from a Gaussian random field through the transformation

$$E_s(\underset{\sim}{x}) = \exp\{\mu_{\ln E_s} + \sigma_{\ln E_s} G(\underset{\sim}{x})\} \qquad (10)$$

where $G(\underset{\sim}{x})$ is a zero mean, unit variance, Gaussian random field, realizations of which are generated by the LAS method.

The underlying Gaussian field is assumed to have a Markovian correlation structure, having correlation function

$$\rho(\tau) = \exp\left\{-\frac{2|\tau|}{\theta_{\ln E}}\right\} \qquad (11)$$

where τ is the distance between any two points in the field and $\theta_{\ln E}$ is the scale of fluctuation, loosely defined as the separation distance beyond which two points of $\ln E_s(\underset{\sim}{x})$ become largely uncorrelated. The random field has been assumed isotropic in this preliminary study, leaving the more site specific anisotropic considerations for later work.

The simulation is performed for various statistics of the elastic modulus field. In particular, the mean elastic modulus, μ_{E_s}, is held fixed at 30 MPa, while the coefficient of variation, V, is varied from 0.1 to 1.0, and the scale of fluctuation, $\theta_{\ln E_s}$, is varied from 0.1 to 15.

4 PREDICTION OF SETTLEMENT MEAN AND VARIANCE

It is hypothesized that if Janbu's relationship is sufficiently accurate for design purposes, it can also be used to predicted the actual (simulated) settlement, δ_{sim}, reasonably accurately. That is, it is supposed that Eq. (4),

$$\delta = \mu_0 \left[a + b \ln\left(\frac{H}{B}\right)\right] \cdot \frac{P}{E_s^*}$$

will predict δ_{sim} for each realization if a suitable value of E_s^* can be found. Fenton and Griffiths (2002), found that settlement is very well predicted by setting E_s^* equal to the geometric average of the elastic modulus field over the region directly under the footing. This is what will be used here.

One difficulty is that the value of B in Eq. (4) is also derived from a sample of the random elastic modulus field. This means that δ is a function of both E_s^* and $\hat{E}_s$ and that E_s^* is a local geometric average over a rectangle of *random* size $B \times H$. If Eq. (8) is substituted into Eq. (4), then δ can be expressed as

$$\delta = \frac{\hat{E}_s}{E_s^*}\delta_{max} \qquad (12)$$

Since E_s^* is a geometric average, over a random area of size $B \times H$, of a lognormally distributed random field, then E_s^* is *conditionally* lognormally distributed with parameters,

$$\mathrm{E}\left[\ln E_s^* \mid B\right] = \mu_{\ln E_s} \qquad (13a)$$

$$\mathrm{Var}\left[\ln E_s^* \mid B\right] = \gamma(B,H)\sigma^2_{\ln E_s} \qquad (13b)$$

where $\gamma(B,H)$ is the so-called variance function (Vanmarcke, 1984) which gives the reduction in the variance due to local arithmetic averaging (the geometric average of a lognormally distributed random field corresponds to an arithmetic average of the underlying normally distributed field). The variance function is defined as the average correlation coefficient between every pair of points in the field,

$$\gamma(B,H) = \frac{\int_0^B\int_0^B\int_0^H\int_0^H \rho(x_1-x_2, y_1-y_2)dy_1 dy_2 dx_1 dx_2}{(HB)^2}$$

where, for the isotropic correlation function under consideration here, $\rho(x,y) = \rho(\sqrt{x^2+y^2}) = \rho(\tau)$, see Eq. (11). The variance function is determined numerically using Gaussian quadrature. The unconditional distribution parameters of $\ln E_s^*$ are obtained by taking expectations of Eqs. (13) with respect to B;

$$\mu_{\ln E_s^*} = \mu_{\ln E_s} \qquad (14a)$$

$$\sigma^2_{\ln E_s^*} = \mathrm{E}\left[\gamma(B,H)\right]\sigma^2_{\ln E_s} \qquad (14b)$$

where a first-order approximation to $\mathrm{E}\left[\gamma(B,H)\right]$ is

$$\mathrm{E}\left[\gamma(B,H)\right] \simeq \gamma(\mu_B, H) \qquad (15)$$

Although a second order approximation to $\mathrm{E}\left[\gamma(B,H)\right]$ was considered, it was found to be only slightly different than the first order approximation. It is recognized that the unconditional marginal distribution of E_s^* is probably no longer lognormal but histograms of E_s^* indicate that this is still a reasonable approximation.

The other random quantity appearing in the right-hand-side of Eq. (12) is $\hat{E}_s$, which is an arithmetic average of a set of n observations;

$$\hat{E}_s = \frac{1}{n}\sum_{i=1}^{n} E_i$$

where E_i is the ith observed elastic modulus. It is assumed that elastic modulus samples are of approximately the same physical size as a finite element (for example a CPT cone measurement involves a 'local average' bulb of deformed soil in the vicinity of the cone which might be in the order of 0.15 m in diameter). The first two moments of $\hat{E}_s$ are then

$$\mu_{\hat{E}_s} = \mu_{E_s} \tag{16a}$$

$$\sigma^2_{\hat{E}_s} = \left(\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\right)\sigma^2_{E_s} \simeq \gamma(\Delta x, H)\sigma^2_{E_s} \tag{16b}$$

where ρ_{ij} is the correlation coefficient between the ith and jth samples. The last approximation assumes that local averaging of $E_s(\underset{\sim}{x})$ results in approximately the same level of variance reduction as does local averaging of $\ln E_s(\underset{\sim}{x})$. This is not a bad approximation given all other sources of uncertainty.

If we can further assume that $\hat{E}_s$ is at least approximately lognormally distributed, with parameters given by the transformations of Eq. (9), then δ in Eq. (12) will also be lognormally distributed with parameters

$$\mu_{\ln\delta} = \mu_{\ln\hat{E}_s} - \mu_{\ln E_s^*} + \ln(\delta_{max}) \tag{17a}$$

$$\sigma^2_{\ln\delta} = \sigma^2_{\ln\hat{E}_s} + \sigma^2_{\ln E_s^*} - 2\mathrm{Cov}\left[\ln\hat{E}_s,\ \ln E_s^*\right] \tag{17b}$$

The covariance term can be expressed as

$$\mathrm{Cov}\left[\ln\hat{E}_s,\ \ln E_s^*\right] = \sigma_{\ln\hat{E}_s}\sigma_{\ln E_s^*}\rho_{ave} \tag{18}$$

where ρ_{ave} is the average correlation between every point in the domain defining E_s^* and every point in the domains defining $\hat{E}_s$. This can be expressed in integral form and solved numerically, but a simpler empirical approximation is suggested by observing that there will exist some 'average' distance between the samples and the soil block under the footing, τ_{ave}, such that $\rho_{ave} = \rho(\tau_{ave})$. For the particular problem under consideration with H = 6 m, the best value of τ_{ave} was found by trial and error to be

$$\tau_{ave} = 0.1\mu_B \tag{19}$$

Finally, two of the results suggested above depend on the mean footing width, μ_B. This can be obtained approximately as follows. First of all, taking the logarithm of Eq. (8) gives us

$$\ln B = \ln H - \frac{1}{b}\left(\frac{\delta_{max}\hat{E}_s}{\mu_0 P} - a\right) \tag{20}$$

which has first two moments

$$\mu_{\ln B} = \ln H - \frac{1}{b}\left(\frac{\delta_{max}\mu_{\hat{E}_s}}{\mu_0 P} - a\right) \tag{21a}$$

$$\sigma^2_{\ln B} = \left(\frac{\delta_{max}}{b\mu_0 P}\right)^2\sigma^2_{\hat{E}_s} \tag{21b}$$

and since B is non-negative, it can be assumed to be at least approximately lognormally distributed (histogram plots of B indicate that this is a reasonable assumption) so that

$$\mu_B \simeq \exp\left\{\mu_{\ln B} + \tfrac{1}{2}\sigma^2_{\ln B}\right\}$$

With these results, the parameters of the assumed lognormally distributed settlement can be estimated using Eqs. (17) given the three parameters of the elastic modulus field, μ_{E_s}, σ_{E_s}, and $\theta_{\ln E_s}$.

5 COMPARISON OF PREDICTED AND SIMULATED SETTLEMENT

Before discussing the results, it is worth pointing out some of the difficulties with the comparison. First of all, as the coefficient of variation $V = \sigma_{E_s}/\mu_{E_s}$ increases, it becomes increasingly likely that the sample observations leading to $\hat{E}_s$ will be either very small or very large. If $\hat{E}_s$ is very small, then the resulting footing width, as predicted by Eq. (8), may be wider than the finite element model (although, as discussed above, the footing width is arbitrarily restricted to being between 4 and 48 elements wide). It is recognized, however, that it is unlikely that a footing width in excess of 9 m would be the most economical solution. In fact, it is very likely that the designer would search for an alternative solution, such as a pile foundation, when faced with such a soft soil.

What this means is that it is difficult to evaluate the *unconditional* reliability of any single design solution since design solutions are rarely used in isolation – each is only one amongst a suite of solutions available to the designer and each has its own range of applicability (or, rather, economy). This implies that the reliability of a single design solution must be evaluated *conditionally*, that is for the range of soil properties which make the solution economically optimal.

This conditional reliability problem is quite complex and beyond the scope of this study. Research by the authors in this area is ongoing. Here the study is restricted to the unconditional reliability problem with the recognition that some of the simulation results at higher coefficients of variation are biased by restricting the 'design' footing widths. In the worst case, where V = 1.0, the fraction of footing widths found to be too wide, out of the 2000 realizations, ranged from 0% (for $\theta_{\ln E_s}$ = 0.1) to 12% (for $\theta_{\ln E_s}$ = 15).

The log-settlement mean, as predicted by Eq. (17a), is shown in Figure 3 along with the sample mean obtained from the simulation results for the minimum (V = 0.1) and maximum (V = 1.0) coefficients of variation considered. For small V, the agreement is excellent. For larger V, the maximum relative error is only about 7%, occurring at the smallest scale of fluctuation. Although the relative errors are minor, the small scale behaviour is not properly predicted by the analytical results and subsequent approximations built into Eq. (17a). It is believed that the major source of the discrepancies at small scales is due to the approximation of the second moment of $\hat{E}_s$ using the variance function $\gamma(\Delta x, H)$.

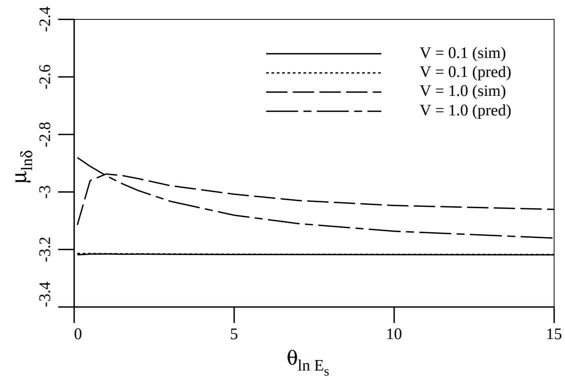

Figure 3. Comparison of predicted and simulated mean settlement.

The log-settlement variance, as predicted by Eq. (17b), is shown in Figure 4 along with the sample variance obtained from the simulation results for three different coefficients of variation, V. Again, the agreement improves for increasing scales of fluctuation but overall, the predicted variance is reasonably good and shows the same basic behaviour as seen in the simulated results.

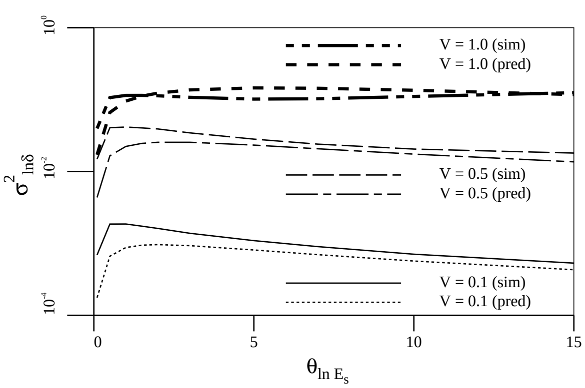

Figure 4. Comparison of predicted and simulated variance of settlement.

Figure 5 compares simulated versus predicted probability that the settlement exceeds some multiple of δ_{max} over all values of V and $\theta_{\ln E_s}$. The agreement is reasonable, tending to be somewhat conservative (predicted 'failure' probability exceeding simulated probability).

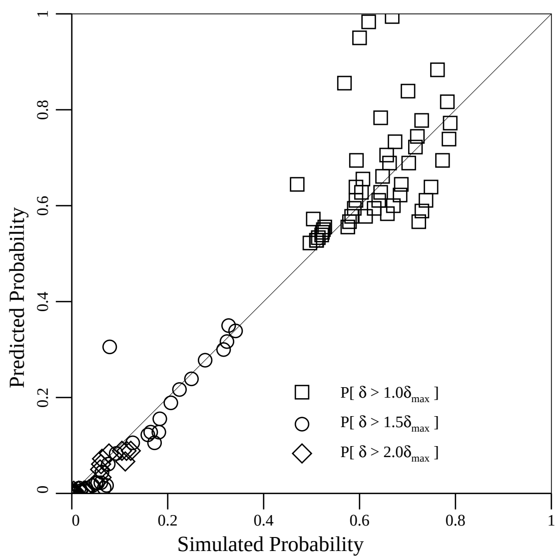

Figure 5. Comparison of predicted and simulated settlement probabilities.

6 Summary

The results of Figure 5 indicate that the Janbu settlement prediction given by Eq. (1) has a reliability, when used in design, that is reasonably well (and perhaps somewhat conservatively) estimated by Eqs. (17) so long as the basic statistics, μ_{E_s}, σ_{E_s}, and $\theta_{\ln E_s}$, of the elastic modulus field are known or estimated. Of these parameters, the most difficult to estimate is the scale of fluctuation, $\theta_{\ln E_s}$, since its estimator requires a large sample. However, Figure 4 indicates that there is a 'worst case', in the sense of maximum variance, which occurs at about $\theta_{\ln E_s} \simeq 1$. Thus, if the scale is unknown, it should be conservative to use $\theta_{\ln E_s} \simeq 1$.

For a particular site, the reliability assessment of the footing design proceeds as follows;

1) sample the site at a number of locations and produce an estimate of the mean elastic modulus $\hat{E}_s$. In current practice this estimate seems to be an arithmetic average of the observed values. Although the results of Fenton and Griffiths (2002) suggest that a geometric average would be more representative, the approach taken by current practice was adopted in this study.
2) compute the required footing width, B, by Eq. (8). This constitutes the design phase.
3) using the same data set collected in item (1), estimate $\mu_{\ln E_s}$ and $\sigma^2_{\ln E_s}$ by computing the sample mean and sample variance of the log-data. Assume that $\theta_{\ln E_s} \simeq 1$ unless a more sophisticated analysis is carried out.
4) using Gaussian quadrature, or some software package which numerically integrates a function, evaluate the variance reduction functions $\gamma(B, H)$ and $\gamma(\Delta x, H)$. Note that the latter assumes that the data in step (1) were collected along a single vertical line below the footing.
5) estimate $\mu_{\ln E_s^*}$ and $\sigma^2_{\ln E_s^*}$ using Eqs. (14) and (15).
6) estimate $\mu_{\ln \hat{E}_s}$ and $\sigma^2_{\ln \hat{E}_s}$ using Eqs. (16) in the transformations of Eq. (9).
7) compute τ_{ave} using Eq. (19) and then $\rho_{ave} = \rho(\tau_{ave})$ using Eq. (11). Compute the covariance of Eq. (18).
8) compute the mean and variance of log-settlement using Eqs. (17).

Assuming that the settlement is lognormally distributed, probabilities relating to the actual settlement of the designed footing can now be computed as

$$\mathrm{P}\left[\delta > \delta_{max}\right] = 1 - \Phi\left(\frac{\ln(\delta_{max}) - \mu_{\ln\delta}}{\sigma_{\ln\delta}}\right)$$

where Φ is the standard normal cumulative distribution function.

It is noted that this study involves a number of approximations and limitations, the most significant of which are deemed to be;

1) limiting the footing widths to some maximum upper value leads to some bias of the simulation results.
2) Janbu's influence factor, μ_1, is approximated as a straight line. In fact the curve flattens out for small values of H/B, or large values of B. This approximation error could easily be contributing to the frequency of predicting excessively large footing widths for low $\hat{E}_s$.
3) both E_s^* and $\hat{E}_s$ are assumed to be lognormally distributed, which is probably a reasonable assumption but which may lead to some discrepancies in extreme cases (such as for small scales of fluctuation). In addition, the variance of $\hat{E}_s$ is obtained using the variance function $\gamma(\Delta x, H)$. That is, a continuous local average over the height H in log-space is used to approximate the variance

reduction of the average of a discrete set of observations in real-space. The variance reduction is expected to be in the right ball-park, but to not be particularly accurate.

4) the covariance between $\ln E_s^*$ and $\ln \hat{E}_s$ is approximated by using an average correlation coefficient which is merely fitted by trial and error to the simulation results. Although the value obtained seems reasonable, it needs further verification.

Perhaps one of the main results of the paper, other than an approximate assessment of the reliability of a design methodology, is the recognition of the fact that the reliability assessment of design methodologies must be done conditionally. The task for the future is to determine how to specify the appropriate conditional soil property distributions as a function of design economies. Once this specification has been made, simulation can again be called upon to find the conditional reliabilities.

In addition, the results of this paper do not particularly address sampling issues. For example, in the discussion above outlining how the reliability assessment would proceed, it was assumed that the same data used to estimate $\hat{E}_s$ would provide a reasonable estimate of both $\mu_{\hat{E}_s}$ and $\mu_{\ln E_s^*}$ (the latter using the logarithm of the data). Clearly, this introduces additional bias and uncertainty into the assessment that is not accounted for above and needs further study.

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