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Computation of the ultimate pressure of a laterally loaded circular pile in frictional soil

P.A.Lane & D.V.Griffiths
Manchester University, UK

ABSTRACT: The paper presents numerical solutions for the ultimate pressure to cause failure of a laterally loaded circular pile in frictional soil. The analysis considers a slice of pile at sufficient depth below ground level such that approximately plane strain conditions apply. The analysis is performed using a 'non-axisymmetric' approach involving Fourier expansions in the tangential directions and this allows a rather simple transformation from rough to smooth conditions at the pile/soil interface. The pile is displaced incrementally into the elasto-plastic soil, and the reactions back-figured from the converged stresses. The effects of tension behind the pile are fully accounted for, and the influence of volumetric changes in the soil during shearing demonstrated. The ultimate pressure is presented in the form of dimensionless 'bearing capacity factors', and comparisons with closed-form solutions made where available.

1 INTRODUCTION

The failure of a laterally loaded pile is governed by the soil resistance and the pile strength [1,2]. As an initial step to the numerical analysis of the full-depth pile, the behaviour of the pile at depth may be considered as a 'slice' of pile and soil under plane strain conditions.

The limiting lateral pressure on the pile disc has been assesed as between 8.3 and 11.4 Cu for a purely cohesive soil [2]. A characteristic solution by Randolph and Houlsby [3] refined this further. Their results of $(4\sqrt{2} + 2\pi)$Cu and $(6 + \pi)$Cu for perfectly rough and perfectly smooth discs respectively, are used here for comparison.

Several features are considered as a basis for future work on a full-depth model, to include pile/soil detaching and soil with both cohesion and internal friction. As no suitable analytical solution was available for this problem a parallel analysis using a conventional plane strain, finite element mesh, technique was conducted wherever possible and good agreement obtained.

2 ANALYSIS METHOD AND TECHNIQUES

The behaviour of a rigid pile at depth was modelled as a disc displaced laterally into a slice of elasto-visco-plastic material, under plane strain conditions. A rigid boundary was placed at 10 pile diameters distant, sufficient to minimize boundary effects for most cases. The pile/soil interface was considered as perfectly rough or smooth by the use of decoupled freedoms in the mesh [4].

A Mohr-Coulomb yield criterion was applied to the soil and the effects of reducing tension in the soil behind the pile were examined by the imposition of a 'no tension' criterion.

$$\sigma_3 < \text{TVAL} \qquad (1)$$

where TVAL may vary from zero (no tension) to some permitted limit.

Within the pile and soil slice tangential stresses, strains and displacements were modelled using Fourier expansions [5]. This technique for modelling axisymmetric structures subjected to non-axisymmetric loading has been shown to be an efficient and flexible method with potential savings in storage and computational costs compared to the conventional plane strain analysis [6].

For the cases considered here NHAR harmonics were used with NANG angular sampling points between 0° and 180°. The relationship between NANG and NHAR to obtain the 'best' solution varies for different soil properties. For example, a purely cohesive soil, allowing tension to develop, needs only odd harmonics (1,3,5 etc) and where the number of harmonics (NHAR) is given by:

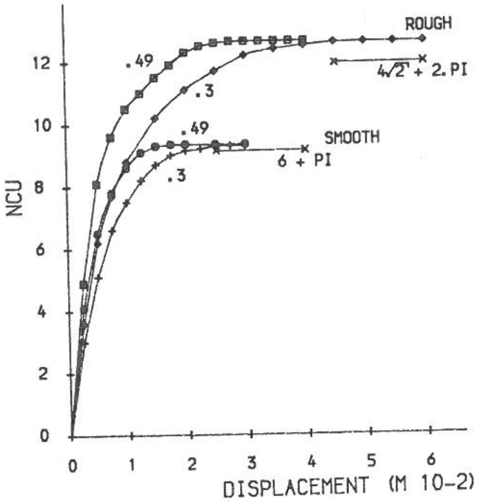

Figure 2.1 Load/displacement response for a rigid pile in a purely cohesive soil permitting tension,Cu=100,E=10E5.

where

$$NCu = \frac{P}{Cu.D} \qquad (5)$$

and P is the load per unit length of pile (kN/m) with diameter D (m).

$$NHAR = \frac{NANG - 1}{2} \qquad (2)$$

When no tension is permitted:

$$NHAR = \frac{NANG + 1}{2} \qquad (3)$$

with all harmonics (odd, even and zero) necessary for a good solution [7].

The Fourier amplitudes from each expansion are conveniently summed using the repeated Trapezium rule e.g.

$$a_i = \frac{2}{\pi} \int_0^{\pi} f(\theta).\cos i\theta.d\theta \qquad (4)$$

The problem was first solved for a purely cohesive soil, permitting tension, and excellent agreement obtained with the closed-form solution [3,8], see Fig 2.1.

3 'NO TENSION' CRITERION

The introduction of a criterion for permissable tension in the soil is necessary to account for the gapping that occurs behind a laterally loaded pile. It was considered here to examine the reduction in stiffness and ultimate pressure that results. The elastic case shown in Fig 3.1 was first solved with full tension permitted, giving exact agreement with the

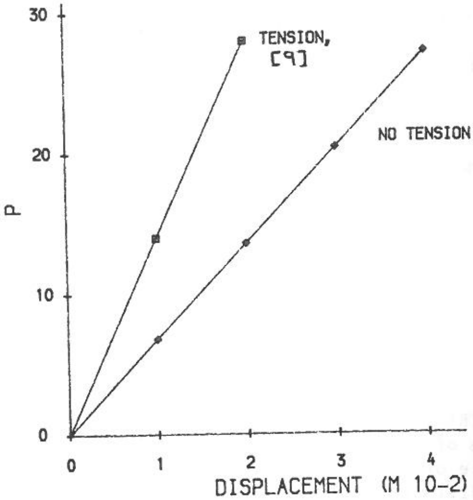

Figure 3.1 Elastic response of a pile in soil with and without tension.

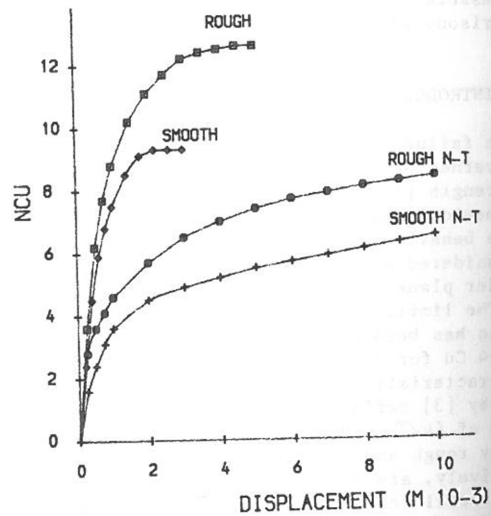

Figure 3.2 Effect of permissable soil tension on the load/displacement response of a pile in cohesive soil.

closed-form solution of Baguelin et al [9]. The no tension case has a lower stiffness.

When a similar plastic analysis was performed on a cohesive soil the reduced stiffness and ultimate pressure was also seen, although the maximum is not well defined.

When plastic displacement vectors are plotted for the soil with and without ten-

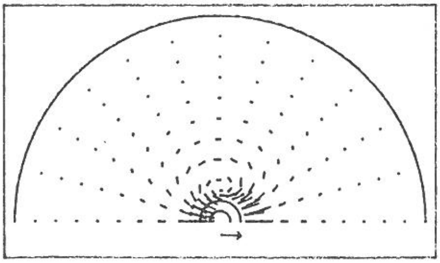

Figure 3.3 Plastic displacement vectors for cohesive soil with tension (NU=0.3)

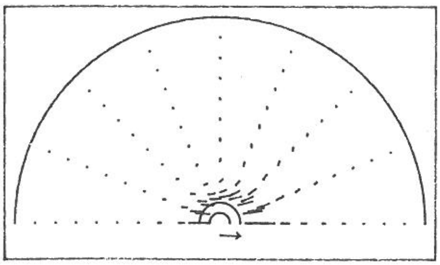

Figure 3.4 Plastic displacement vectors for cohesive soil without tension (0.3)

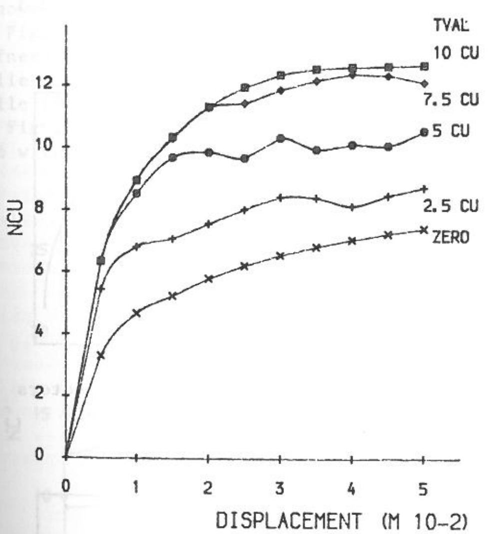

Figure 3.5 Effect of permissable tension on the load/displacement response of a rough pile in cohesive soil (NU=0.3).

sion, the effect of the criterion is seen reducing soil movement behind the pile and changing the overall mechanism, Figs 3.3 and 3.4, (some movement still occurs behind the pile to maintain mesh continuity). The change in the nature of the displace-

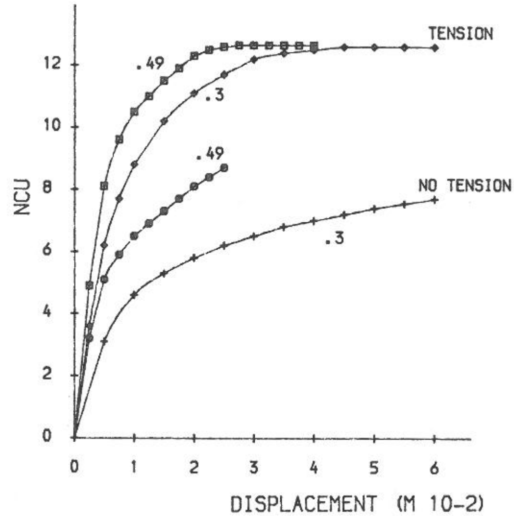

Figure 3.6 Effect of Poisson's Ratio on the load/displacement response of a rough pile in cohesive soil with and without tension.

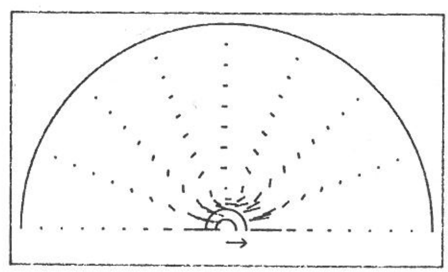

Figure 3.7 Displacements for a rough pile in cohesive soil with no tension (NU=0.49)

ments illustrates the change in the load/ displacement response in Fig3.2.

The variation in stiffness and ultimate load with permissable tension is shown in Fig 3.5. In practice TVAL ≯0.5 Cu approximates to no tension and TVAL >10 Cu to full tension.

With a higher Poisson's Ratio (NU) the stiffness of a 'no tension' case increases but it is still lower than with full tension Fig 3.6. The effect of Poisson's Ratio on the plastic displacements can be seen from Fig 3.7 (NU = 0.49) compared to Fig 3.4 (NU = 0.3).

The reduced soil disturbance for a smooth pile can be seen from comparing Fig 3.8 to Fig 3.7. The smaller disturbance pattern of the smooth pile accounts for the reduced stiffness and ultimate pressure compared to the rough (Fig 3.2).

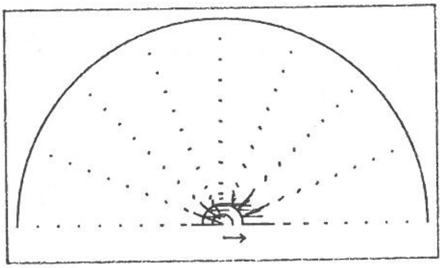

Figure 3.8 Plastic displacement vectors for a smooth pile in cohesive soil with no permissable tension (NU=0.49).

4 FRICTIONAL SOIL

A weightless material with cohesion and interparticle friction was also considered As the model was so confined dilation was reduced to zero with a non-associated flow rule (PSI=0) in the following examples to prevent locking of the mesh.

The effect of the internal angle of friction (PHI) is shown in Fig 4.1, where

$$Nc = \frac{P}{c.D} \qquad (6)$$

There is an increase in strength with PHI although the ultimate pressure is not well defined for high PHI. The displacement required to reach 'ultimate' pressure (Δult) also increases with PHI (Fig 4.1).

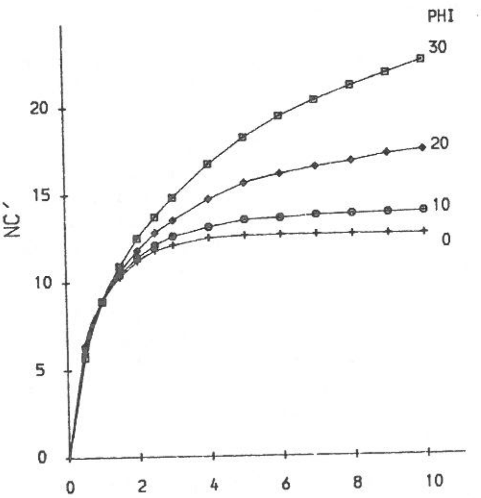

Figure 4.1 Variation in load/displacement response with PHI for a rough pile in soil with tension (c=10, NU=0.3).

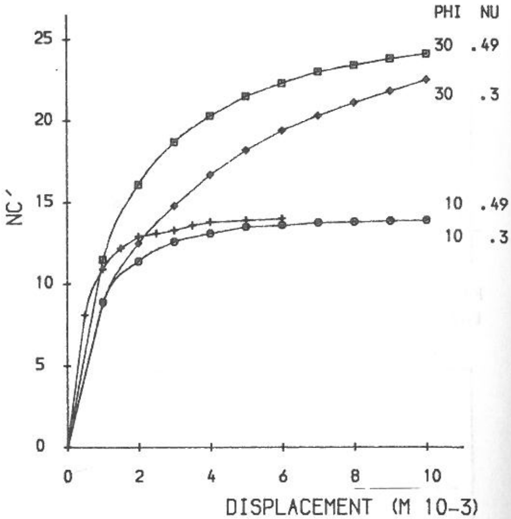

Figure 4.2 Effect of Poisson's Ratio on the load/displacement response of a rough pile in frictional soil with tension.

With a higher Poisson's Ratio the increased initial stiffness, reduced Δult and similar ultimate pressure seen in Fig 2.1 is again demonstrated, Fig 4.2.

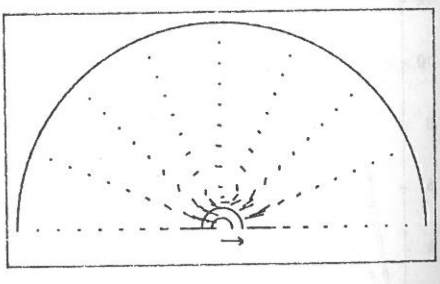

Figure 4.3 Plastic displacement vectors for a rough pile in frictional soil with tension (PHI = 10°)

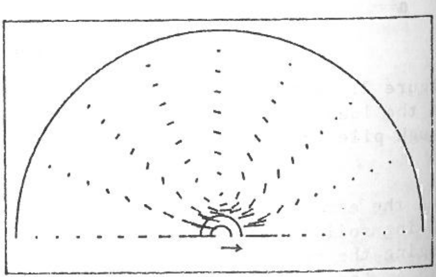

Figure 4.4 Plastic displacement vectors for a rough pile (PHI =30°).

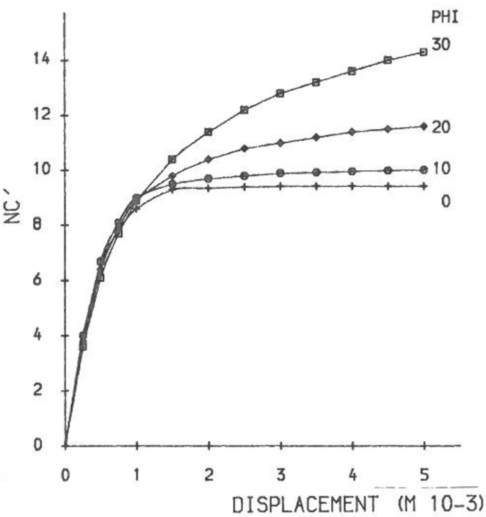

Figure 4.5 Effect of PHI on the load/ displacement response for a smooth pile in frictional soil with tension (NU=0.49)

The change in the mechanism of the plastic displacements of the soil with PHI is shown in Figs 4.3 and 4.4.

Figure 4.5 illustrates the reduced stiffness and ultimate pressure for a smooth pile with varying PHI compared to a rough pile (Fig 4.2).

Figure 4.6 summarizes the variation in Nc with PHI and NU at the given Δ .

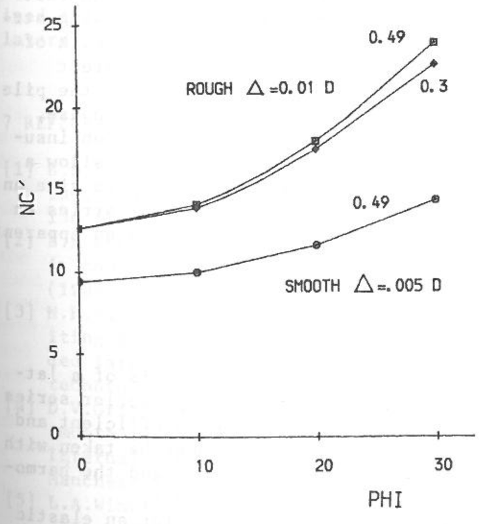

Figure 4.6 Variation in Nc with PHI and NU in frictional soil with tension.

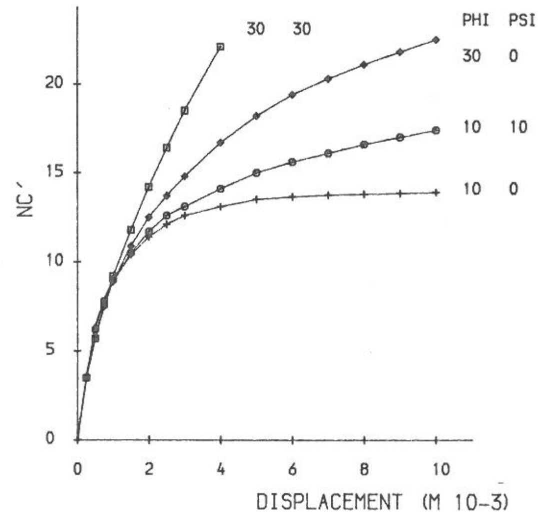

Figure 4.7 Effect of associated and non-associated flow rules on the response of a rough pile in frictional soil (NU=0.3)

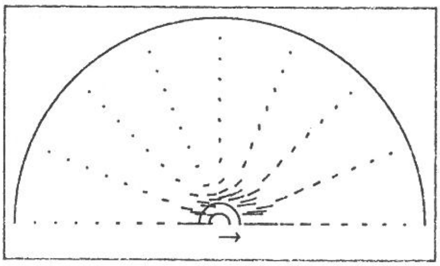

Figure 4.8 Plastic displacement vectors for associated flow for a rough pile in frictional soil (PHI=PSI=30°).

To demonstrate the problems expected with dilation in this confined model an associated flow rule was used, see Figs 4.7 and 4.8. Compared with Fig 4.4 the mechanism of plastic flow is no longer predominately rotational, with the soil in front moving towards the boundary increasing the apparent strength (Fig 4.7).

5 FRICTIONAL SOIL WITH NO TENSION

When the Mohr-Coulomb and No Tension yield criteria were implemented together for a frictional soil the pattern of results was rather different to those obtanied with the purely cohesive soil (Section 2). For low angles of friction the reduced stiffness and ultimate pressure and increased Δult were observed. For higher PHI an apparent increase occured in the 'plastic'

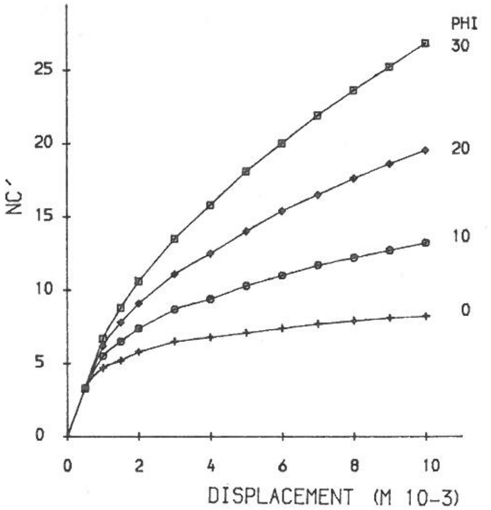

Figure 5.1 Variation in the response with PHI for a rough pile in frictional soil with no tension (NU = 0.3).

tiffness compared to soil with tension. Fig 5.1 shows the response for various PHI and the approximately linear stiffness at large displacement for high PHI.

Fig 5.2 shows the result of this higher plastic stiffness with no tension. At large displacements the load exceeds that for soil with tension and for higher Poisson's Ratio the effect is even more marked.

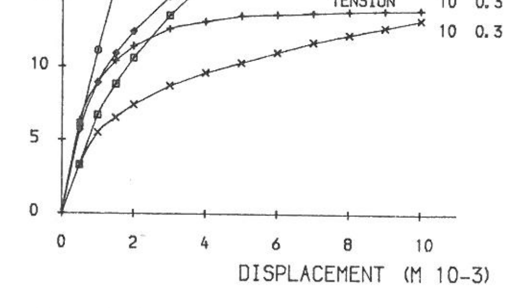

Figure 5.2 Comparison of load/displacement response for a rough pile in frictional soil with and without tension.

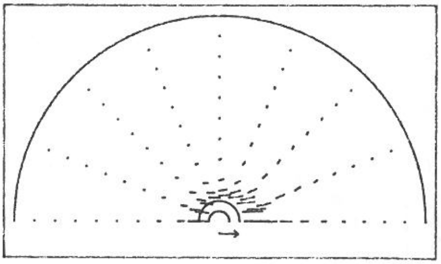

Figure 5.3 Plastic displacements for a rough pile in frictional soil with no tension (PHI = 30°, NU = 0.3).

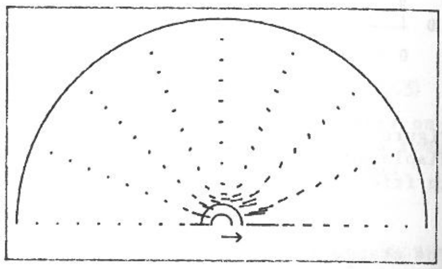

Figure 5.4 Plastic displacements for a rough pile in frictional soil with no tension (PHI = 30°, NU = 0.49).

An examination of the plastic displacement vectors shown in Figs 5.3 (NU = 0.3) and 5.4 (NU = 0.49) compared to Fig 4.4 demonstrates the relatively little soil movement occuring in the no tension case.

The no tension criterion is causing stress redistribution away from the back of the pile, this increases the apparent stiffness of the soil in front of the pile leading to its increased load response, Fig 5.2. At high angles of friction insufficient movement is occuring to allow a mechanism of failure to develop to give an ultimate pressure and the load carries on increasing with displacement in an apparent 'elastic' line.

6 CONCLUSIONS

The non-axisymmetric analysis of a laterally loaded rigid disc by Fourier series summation has so far proved efficient and flexible, although care must be taken with the selection of NANG, NHAR and the harmonics to achieve a good result.

The closed-form solution for an elastic material was reproduced. Good agreement was obtained with a closed-form solution for both rough and smooth piles in a purely cohesive soil. The ultimate pressure on a laterally loaded rigid disc in a cohesive plastic soil is not dependent on Poisson's Ratio for the soil.

When no soil tension is permitted there is a reduction in stiffness and ultimate pressure for a laterally loaded rigid disc in cohesive soil. The ultimate pressure of such a soil is not always well defined but 65% of the full-tension Nc value may be obtained at increased Δult. The effective range of permissable tension is 0.5Cu to 10Cu.

The ultimate pressure on a laterally loaded disc is dependent on the internal angle of friction of the soil with both friction and cohesion. In this confined model non-associated flow rules are used as dilation causes 'locking' of the mesh and an over-stiff, over-strong response at high angles of friction.

The reduction of permissable soil tension in a frictional soil reduces the initial stiffness of the load/displacement response. The ultimate pressure is not defined for high PHI in such a material due to the 'elastic' response at large displacements. The ultimate pressure for a 'no tension', frictional soil is not independent of the soil's Poisson's Ratio.

The features of various soil properties (c, PHI, NU), the pile/soil interface condition (rough or smooth) and the maximum permissable soil tension are all factors in the load/displacement behaviour and ultimate pressure of the disc. They have been considered in this 'at depth' model to illustrate their individual and combined effects on the response of a rigid laterally loaded pile.

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