A Probabilistic Approach to Karstic Foundation Variability Karst and Sinkholes:

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When a heavily loaded industrial building was expanded, a thick raft saved the day.

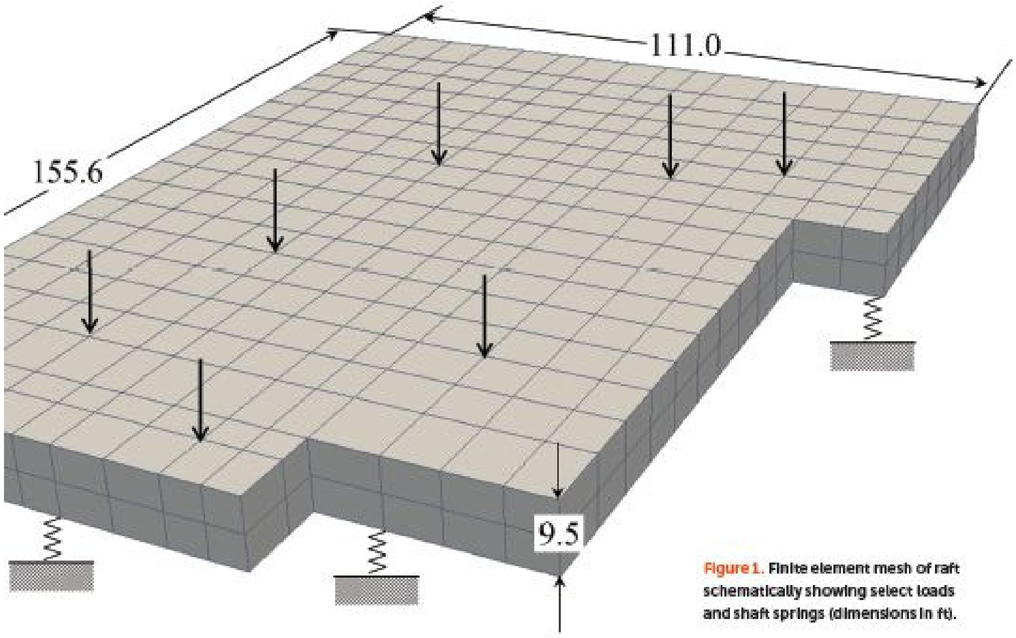


Figure 1. Finite element mesh of raft schematically showing select loads and shaft springs (dimensions in ft).

Karst is a landscape formed in soluble rocks caused by movement of water that has become slightly acidic. Limestone, dolomite, and gypsum are vulnerable to these influences and may be characterized by sinkholes, caves, and underground drainage systems. Such conditions clearly present particular challenges and uncertainties for foundation engineers. This article describes how probabilistic tools were used to assess the influence of randomly distributed underground voids and caverns on the performance of drilled shaft foundations in karst to support a factory expansion project.

What to Do with the Voids?

A concrete raft on 231 drilled shafts was proposed for support of the expansion of a heavily loaded cement manufacturing plant located in an alluvial setting on the banks of the Ohio River. The plant, which had been in operation since the 1960s, was constructed on a driven H-pile foundation. The underlying bedrock is Mississippian-aged limestone, which is a relatively pure, jointed, and fractured stratum with an average compressive strength of 26 ksi. Overlying the limestone bedrock are poorly consolidated to unconsolidated silt, clay, sand, and gravel. The subsurface investigation prepared by a local geotechnical consultant for the proposed expansion included 52 borings. Upon reaching auger refusal in two of the borings, 10 ft of rock coring yielded an RQD value of 87 percent in both borings.

Although most of the existing plant was founded on driven H-Piles, drilled shafts were proposed for the expansion, with each shaft socketed two feet into what had been considered competent and unweathered limestone bedrock. However, during installation of the drilled shafts, significant slurry and concrete loss suggested the presence of numerous voids in the bedrock. As part of a more detailed subsurface investigation, a second local firm cored 20 borings in response to the slurry and concrete loss. Subsequently, the Owner asked AMEC to provide additional engineering consultancy and to take the task to completion. Seventy-six additional rock cores were obtained, primarily by drilling pilot holes at the proposed shaft locations. Thirty-five percent of these exploratory borings encountered randomly distributed bedrock voids. Clearly, the performance of any shaft intersecting a void would be reduced, especially if the location of the void coincided with the shaft tip.

To account for the lack of detailed knowledge about the locations of the voids, a probabilistic, Monte-Carlo, 3-D, finite element simulation was proposed to determine the probability of excessive shaft settlements and moments. The results of the study would facilitate risk management of the foundation system and provide guidance as to whether remedial measures, such as grout filling, additional structural elements, or raft thickening, might be economically justified.

Raft and Shaft Geometry and Properties

The proposed design consisted of an approximately rectangular, reinforced concrete raft, 9.5-ft thick, with plan dimensions as shown in Figure 1. The raft was to be supported by 49, 5-ft diameter shafts. To help evaluate the impact of voids on foundation performance, a 3-D finite element mesh of the raft was developed using 544 elastic, 20-node, hexahedral elements and 3,277 nodes, ensuring that the nodes coincided with the shaft locations. This element choice was made because it performs well in bending and is easier to mesh in a problem such as this compared to a tetrahedral element. The shafts were modeled as 1-D springs and assigned one of two possible stiffness values. A shaft founded on competent rock was assigned twice the stiffness of a shaft founded in a void. The rationale for this 2:1 ratio was based on the results from an O-Cell load-test performed on a shaft in competent rock that indicated similar stiffness contributions from the tip and the sides. Table 1provides a summary of the properties of the mesh.

Young's modulus of raft	453,600.0 k/ft²
Poisson's ratio of raft	0.17
Probability of shaft tip in a void	35%
Shaft stiffness (tip in competent rock)	89,000 k/ft
Shaft stiffness (tip in void)	44,500 k/ft
Total vertical load on raft	47,870 k

(Table 1. Data for raft/shaft foundations)

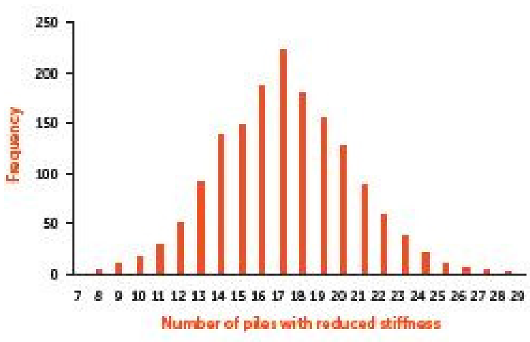
Vertical forces were applied at numerous locations on the slab according to the locations of the various structures to be supported. In addition, two moments were included to model wind action on one of the taller buildings. Some of the loads and springs are shown schematically on Figure 1.

Deterministic Analyses

Initially, two deterministic analyses were performed in which all shafts were assigned: (a) an intact stiffness of 89,000 k/ft, assuming no voids, and (b) a reduced stiffness of 44,500 k/ft, assuming all shafts were founded in voids. With all springs set to 89,000 k/ft, a maximum vertical displacement of 0.0275 ft (0.33 in) was obtained, which is in close agreement with the value of 0.32 obtained from an independent analysis using the structural engineering software package STAAD. The maximum in-plane moments back-figured from the stresses across the raft sections were Mx = 575 k-ft/ft and My = 1,301 k-ft/ft. The corresponding displacement with all springs set to a reduced shaft stiffness of 44,500 k/ft was 0.0455 ft (0.55 in) with essentially the same maximum moments.

Probabilistic Analyses

In the probabilistic analyses, each of the 49 shafts was assigned a random value from a standard normal distribution. If this random value lay within bounds that were calibrated to have a probability of 35 percent, the shaft was assigned a reduced stiffness; otherwise, the shaft was assigned its full stiffness. The analysis was repeated 1,600 times using Monte-Carlo simulations. (A total of 1,600 analyses was determined to be a reasonable number to provide reproducible results.) With the assumption of independence between shafts (i.e., one shaft encountering a void would not change the likelihood of one of its neighbors also encountering a void), the analysis represents a classical Bernoulli process. It has a mean of 49 x 0.35 = 17 (i.e., number of shafts x probability that shaft was assigned a reduced stiffness) and a standard deviation of √49x 0.35x (1x 0.35) = 3.34 , as shown inFigure 2.



(Figure 2. Histogram of number of shafts with reduced stiffness under rat following Monte-Carlo simulations)

The point of this histogram is that, in a probabilistic analysis, each individual simulation does not necessarily include 17 shafts with reduced stiffness. Although the average number of shafts with reduced stiffness over the entire 1,600 simulations is 17, individual simulations will typically have a different number. For example from Figure 2, some simulations generate as few as 8 or as many as 26 shafts with reduced stiffness, albeit with a low probability.

From each simulation, the maximum vertical displacement and the maximum moments in the slab were recorded, and using the 1,600 Monte-Carlo simulations, histograms of these quantities were plotted and their means and standard deviations computed. Figure 3 shows a histogram of the maximum vertical displacement which, in the great majority of simulations, occurred in the same place as in the deterministic analyses. A summary of results for the maximum vertical displacement following the 1,600 Monte-

Carlo simulations is shown in Table 2. The coefficient of variation of the maximum displacement — the ratio of the standard deviation of the maximum displacement to the mean of the maximum displacement — is **v**δ(max) = **σ**δ(max)/**μ**δ(max) = 0.07, or 7 percent. This value is quite low, presumably due to, in part, the stiff, 9.5-ft–thick, reinforced-concrete raft.

Mean	$\mu_{\delta(\max)} = 0.0321$ ft (0.385 in)
Standard deviation	$\sigma_{\delta(\max)} = 0.0024$ ft (0.029 in)
Minimum	$\delta_{(\min)} = 0.0273$ ft (0.327 in)
Maximum	$\delta_{(\max)} = 0.0423$ ft (0.507 in)

(Table 2. Statistical output for minimum and maximum vertical displacement in raft)

Mean	$\mu_{M_{x(\max)}} = 626$ k-ft/ft	$\mu_{M_{y(\max)}} = 1304$ k-ft/ft
Standard deviation	$\sigma_{M_{x(\max)}} = 72$ k-ft/ft	$\sigma_{M_{y(\max)}} = 46$ k-ft/ft
Minimum	$M_{x(\max)\max} = 978$ k-ft/ft	$M_{y(\max)\max} = 1422$ k-ft/ft
Maximum	$M_{x(\max)\min} = 509$ k-ft/ft	$M_{y(\max)\min} = 1224$ k-ft/ft

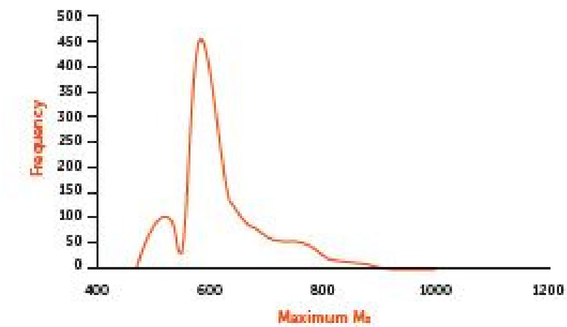
(Table 3. Statistical output for the maximum and minimum Mx and My moments in raft)

Figures 4 and 5 show histograms of the maximum moments in the raft in the x- and y- directions, and Table 3 summarizes the results. Note that the coefficients of variation of the maximum moments are given as shown in the formula below, or 12 percent and 4 percent, respectively.

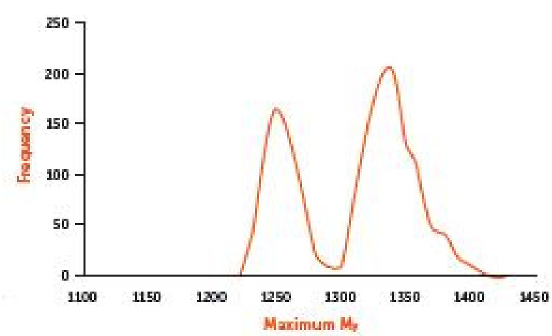
$$v_{M_{x(\max)}} = \sigma_{M_{x(\max)}} / \mu_{M_{x(\max)}} = 0.12 \text{ and } v_{M_{y(\max)}} = \sigma_{M_{y(\max)}} / \mu_{M_{y(\max)}} = 0.04$$

While the variability of **M**x(max) is three times higher than**M**y(max), it can still be concluded that the level of variability of the maximum moment in both directions is quite low, with a worst-case coefficient of variation of about 12 percent.

The histogram of My(max) given in Figure 5 displays an unusual bi-modal distribution that might warrant further investigation. The bi-modal behavior is thought to be due to a nearby critical underlying shaft that causes the moment to switch between the two values, depending on whether its stiffness is high or low.



(Figure 4. Histogram of maximum Mx moment (in k-ft/ft) in raft following Monte-Carlo simulations)



(Figure 5. Histogram of maximum My moment (in k-ft/ft) in raft following Monte-Carlo Simulations)

Risk Assessment in Geotechnical Engineering

The growth of risk assessment methodologies and their use in geotechnical engineering is likely to continue. Risk assessments can lead to better, safer, and more efficient designs. For this project, despite the high void content in the karstic subsurface, the thick raft employed by the designers led to rather low coefficients of variation of maximum raft displacements and moments of about 7 percent and 12 percent, respectively.

Subsequent analyses could be performed using these values to estimate the probability of threshold displacements or moments being exceeded. However, the low variations observed for these quantities suggest that these additional analyses might be of marginal value because relatively small safety margins would lead to extremely low probabilities of unacceptable performance. Ultimately, based on these probabilistic analyses, the designers concluded that poor performance of the raft foundation was unlikely and that the project could continue without extensive mitigation measures.

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Categorized as: Karst

Categories