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Finite Element Assessment of the Method of Fragments for Problems of Confined Seepage

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INTRODUCTION

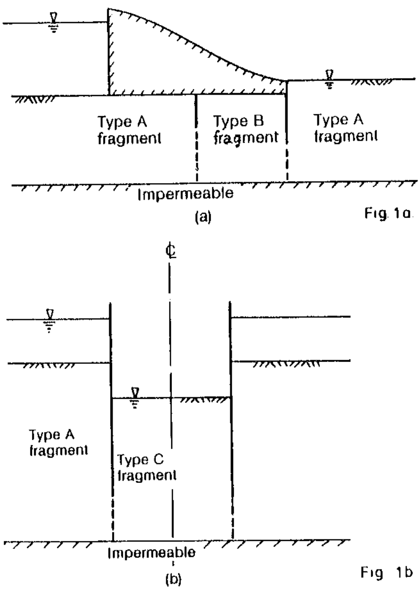
The Methods of Fragments (Pavlovsky 1933) has been around for many years for solving problems of confined seepage, but has not received the attention it deserves. The method claims to give quick and reliable estimates of flow rates, uplift pressures and exit gradients due to seeping water beneath retaining structures. This paper assesses these claims using finite element analysis and finds that for the majority of cases excellent agreement is obtained. The crucial assumption in the method of fragments is that certain equipotentials are vertical. This assumption is found to become less valid as cut-off walls are shortened relative to the depth of the permeable stratum. The assumption that head is lost linearly within fragments is also examined and found to be generally satisfactory. Finally, a novel application of the method of fragments to the problem of non-symmetric double-walled cofferdams is presented. It is found that this can be achieved by the introduction of a vertical impermeable surface between the walls. The 'best' location of this impermeable boundary is presented in chart form as a function of the relative lengths of the cut-off walls.

Many methods are available for tracing the route taken by water as it seeps through a porous soil under steady state conditions. Making the usual assumptions regarding soil/fluid incompressibility and validity of Darcy's law, all the methods must amount to a solution of Laplace's equation

$$k_H \frac{\partial^2 h}{\partial x^2} + k_V \frac{\partial^2 h}{\partial y^2} = 0 \qquad (1)$$

Undoubtedly, the best known and most widely taught method for civil engineers uses 'flow nets', Casagrande (1940). The drawing of flow nets is, however, something of an art, and is rarely straightforward except for the simplest boundary conditions. Small changes in the geometry of a problem, such as the length of a cut off wall or apron, require a completely new flow net and if several different geometries were to be considered for design purposes, much time could be wasted on the trial and error process, only to find that a particular configuration was unsuitable.

The method of fragments can be considered to be a semi-analytical approach which will usually give approximate solutions. The method was first proposed by Pavlovsky (1933) and was 'publicised' by Harr (1962). More recently, Griffiths (1984) has rationalised the method into chart form enabling convenient implementation for engineers. The method, and the inherent assumption contained with it, are summarised in the next section.



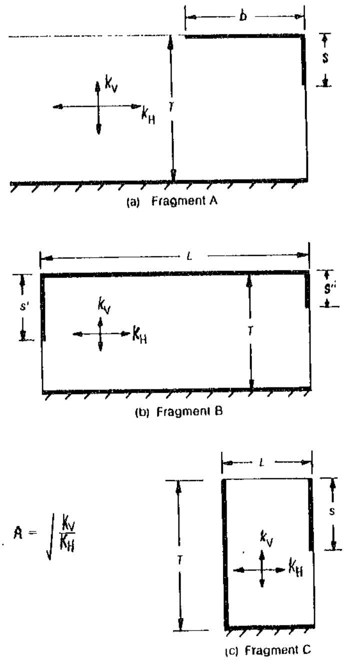


Fig.1 : Subdivision into fragments Fig.2 : Three fragment types

REVIEW OF THE METHOD OF FRAGMENTS

Although the method can also be applied to unconfined flow problems (Harr 1977) the present discussion is concerned with confined flow problems in which all boundary conditions are known.

The crucial assumption in the method is that the equipotential lines at certain locations within the flow regime can be considered vertical. These 'certain locations' are usually chosen to be places where cut-off walls are situated or sharp changes of geometry occur. Figure 1 shows some examples of how confined flow problems could be divided into fragments. The subdivision results in fragments which are rectangular in shape and although this means that the method can only be applied to problems with fairly simple boundaries, the rectangular shapes have the advantage that they can be easily analysed by mapping techniques or finite element approaches.

Three types of fragments have been identified in the present work and are shown in Figure 2. These enable a wide range of problems to be tackled although other fragment types could be developed if required.

THE FINITE ELEMENT MODEL

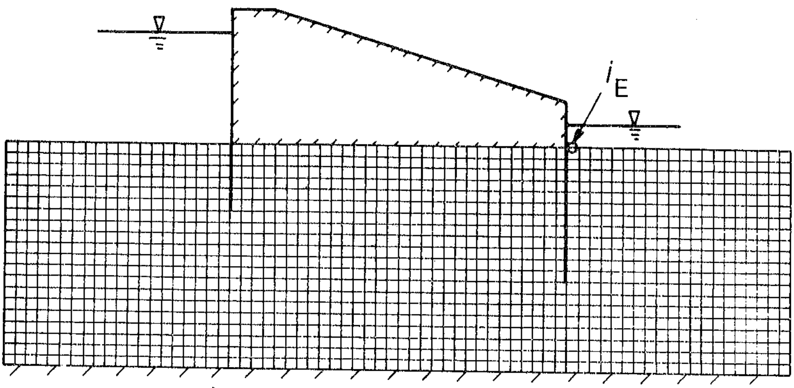


Figure 3 : Typical mesh with two cut-off walls

Four-noded quadrilateral elements were used throughout and a typical mesh is shown in Figure 3 for a dam with two cut off walls. Due to the symmetry of the governing Laplace's equation, and to avoid controversy over aspect ratios, it was decided to use square elements throughout. Boundary conditions involved fixing the potential head at the up- and downstream sides and defining impermeable boundaries ($\partial h/\partial n=0$). The impermeable cut-off walls were assumed to have zero thickness but, in order to model impermeable conditions, each node on the walls had two freedoms, one to the left and one to the right.

For details of the formulation and listings of the programs, the reader is referred to Smith (1982), but in all cases the problem was reduced to the solution of linear simultaneous equations, thus

$$\underline{q} = \underline{k}\,\underline{h} \qquad (2)$$

where $\underline{h}$ is a vector of nodal values of head, $\underline{q}$ is a vector of net nodal inflow/outflow and $\underline{k}$ is the system 'stiffness' matrix.

VERTICAL EQUIPOTENTIAL ASSUMPTION

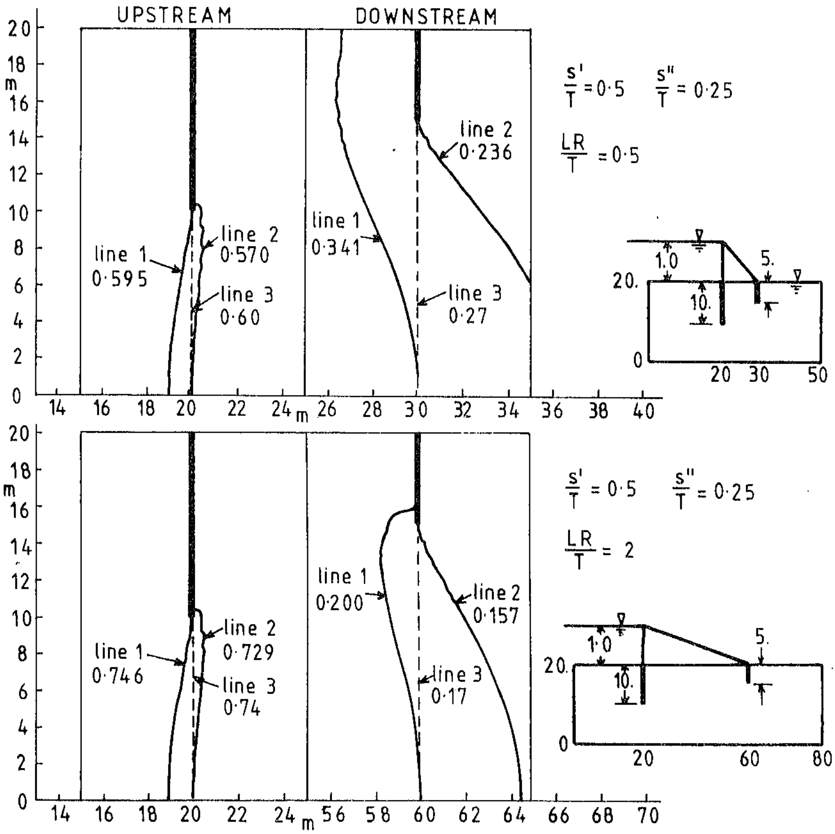


Figure 4 : Assessment of the vertical equipotential assumption

The results of some finite element runs on the mesh of Figure 3 are given in Figure 4. For such a boundary value problem, the method of fragments would place equipotentials directly beneath the cut-off walls, so it is in this vicinity that attention has been focussed. Each of the results in Figure 4 shows a cut-off wall together with three equipotentials - line 1 the upstream equipotential, line 2 the downstream equipotential and line 3 the equipotential by the method of fragments from charts (Griffiths 1984). The two equipotentials from the finite element analysis (line 1 and line 2) were chosen to be those passing through the tip of the wall and through a point on the impermeable surface directly below the wall. Both the up-and downstream walls are considered. The error introduced by assuming a vertical equipotential beneath the walls should be considered relative to the total head loss of 1 unit across the full problem.

Although the equipotentials were not always vertical, the range of values and the error introduced by the fragments assumption was quite small. In the majority of cases the equipotentials drawn through the tip of the wall and the base (lines 1 and 2) lay very close to each other. The downstream wall of the shorter dam (LR/T=0.5) behaved rather differently in that line 1 did not meet the wall itself but intersected the base of the dam. The error was still small, however, as indicated by the head values associated with lines 1 and 2 which had a spread of only 10% of the total head (0.34 - 0.24). This trend would tie in with the results of Figure 5, which shows the fragments assumptions regarding linear loss of head to be less satisfactory for short dams with short cut-off walls.

FLOW RATES, UPLIFT PRESSURES AND EXIT GRADIENTS

The accuracy of the method of fragment for predicting flow rates and exit gradients has been considered in some detail in an earlier publication (Griffiths 1984). Regarding uplift pressures, however, an additional assumption has to be made that head is lost linearly along the uppermost streamline within a Type-B fragment. Figure 5 shows comparisons between the head along line AB beneath a dam by fragments and that computed using a mesh of the type shown in Figure 3. The results indicate that the method of fragments accounts quite well for the uplift pressures, although the assumption of linearity is an approximation. The 'worst' cases are shown in Figures 5(a) and 5(b) where the dam is short relative to the depth of the permeable layer. In these cases, the uplift pressure by the method of fragments was slightly unconservative, but still within 10% of the more rigorously obtained finite element values. In spite of this, all the solutions presented by the method of fragments gave adequate predictions for engineering purposes. It may be noted that when applying the method to obtain uplift pressures due to seepage through anisotropic soils, the 'transformed' width RL, rather than the original width L, should be used.

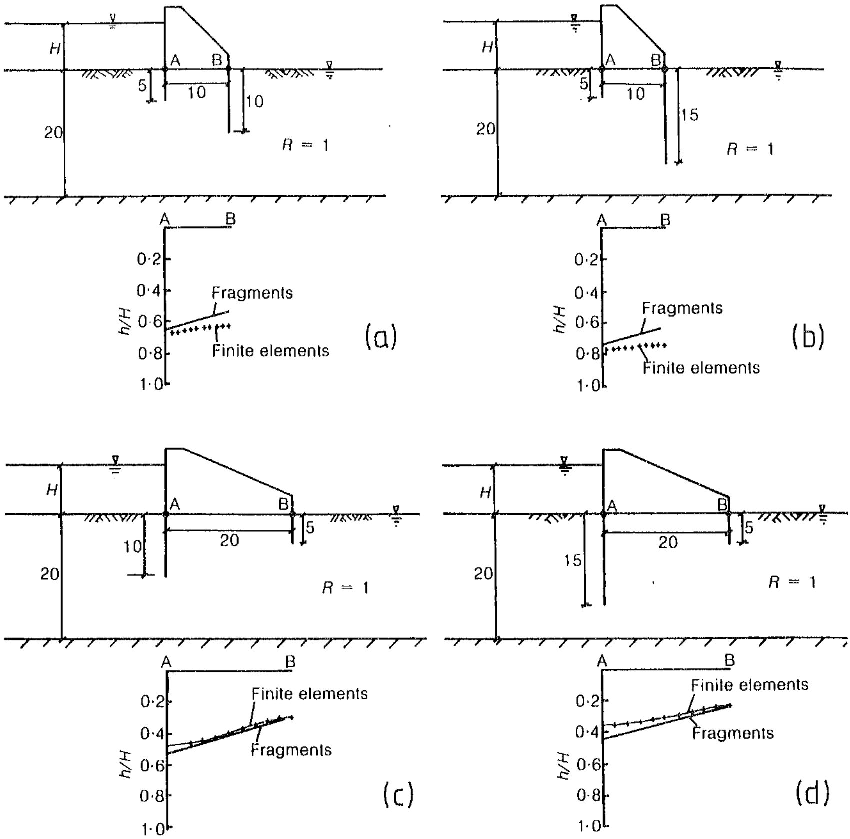


Figure 5 : Uplift pressures by fragments and finite elements

FLOW THROUGH UNSYMMETRICAL COFFERDAMS

An application of the method of fragments to a new problem is now considered. Figure 6 shows an unsymmetrical cofferdam with walls of unequal length. The symmetrical case (Figure 1) is easily dealt with by assuming that an impermeable surface lies at the centreline. This problem is then solved by superposing two identical type -C fragments. It is suggested that a similar approach can be used in the unsymmetrical case by assuming the existence of a vertical impermeable surface. If this assumption is acceptable the problem then amounts to finding the 'best' location of the impermeable surface.

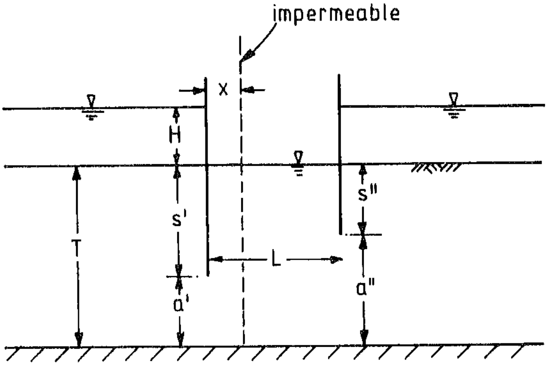


Figure 6 : Unsymmetrical double-wall cofferdam

It seems intuitively likely that the 'best' position depends on the relative size of the gaps beneath the walls thus

$$\frac{x}{L} = f\left(\frac{a'}{a' + a''}\right) \qquad (3)$$

Assuming at all times that $a' \leqslant a''$, finite element runs have been performed on a number of configurations. Boundary conditions of equation (3) that must be satisfied are given by

$$\frac{x}{L} = 0 \text{ when } \frac{a'}{a' + a''} = 0$$

and (4)

$$\frac{x}{L} = 0.5 \text{ when } \frac{a'}{a' + a''} = 0.5$$

The finite element runs involved a full analysis of the unsymmetrical problem which yielded a total flow rate and two exit gradients corresponding to each side. This was followed by a series of runs on the same problem, but with an impermeable wall moved gradually from left ($x/L = 0$) to right ($x/L = 1$). By this process the best position of the impermeable wall in order to simulate the full problem was obtained. A series of results for the case $LR/T = 0.5$ are given in Figure 7. These results emphasised that the solution of a potential problem of this type tends to maximise the kinetic energy of the system. The full solution for the total flow rate in all cases agreed closely with the maximum flow rate achieved by moving the impermeable wall from left to right. This maximum flow rate corresponded to the optimum value of x/L. It was found that this optimum position also gave the best values of the exit gradients on both sides.

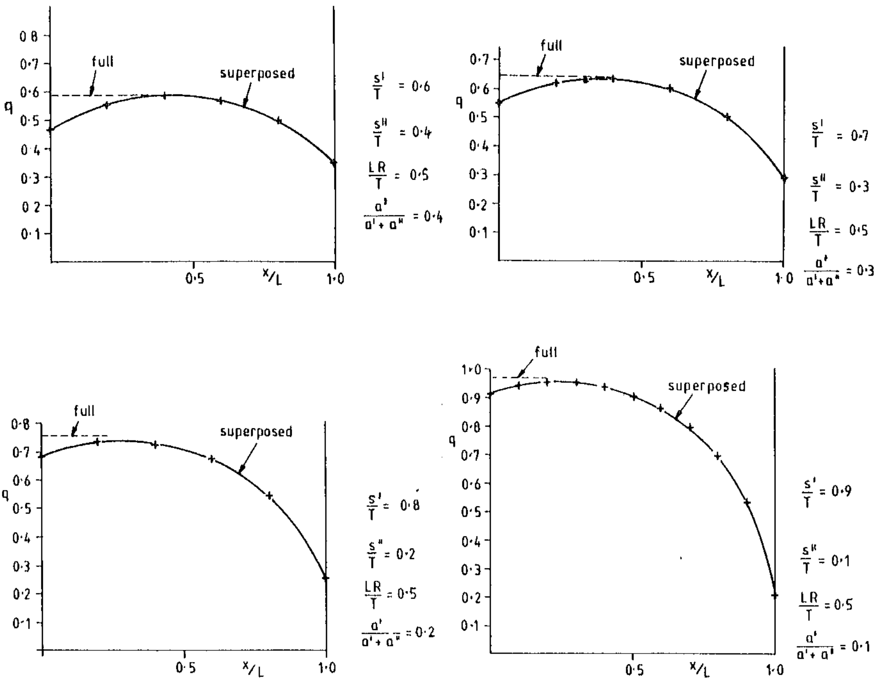


Figure 7 : Effect of impermeable surface on flow rate

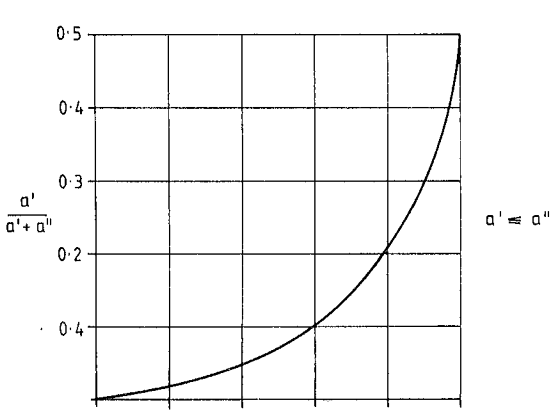


Figure 8 : Suggested location of impermeable surface

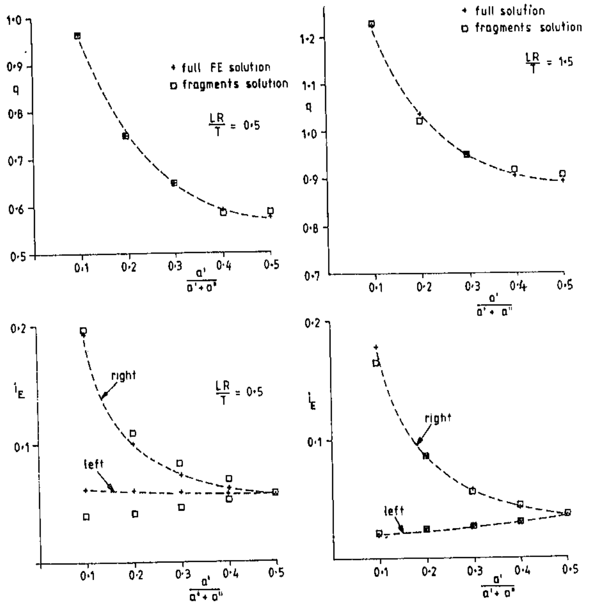


Figure 9 - Comparison of fragments and finite element solution

A series of other configurations involving different values of LR/T have resulted in the empirical curve of Figure 8. This curve estimates the 'best' position of the impermeable surface (x/L) depending on the relative lengths of the cut-off walls (a'/(a' + a")). Figure 9 shows a comparison of flow rates and exit gradients as computed using a full finite element analysis with those that would be obtained using the method of fragments in conjunction with Figure 8. It is seen that quite acceptable agreement is obtained. For the two different LR/T values considered, generally good agreement for both flow rates and exit gradients was observed. For the narrower dam, however, the method of fragments underestimated the exit gradient at the left side, but still gave excellent agreement with the more important and higher gradients to the right.

CONCLUSIONS

The finite element method has been applied to problems of steady state seepage in order to assess the assumptions inherent in the method of fragments. A detailed examination was made of the vertical equipotential assumption beneath cut-off walls. By examining the computed equipotentials in the vicinity of the walls it was found that the assumption was excellent provided the wall was not too short relative to the total depth of the permeable layer. The assumption of linear loss of head within a fragment was also examined with reference to the uplift pressures on water retaining structures. It was found that the assumption was quite acceptable although slightly unconservative for dams that were narrow relative to the depth of the permeable stratum. In all cases, however, the results were certainly of acceptable accuracy for engineering purposes and at least as good as those that could be obtained using a well drawn flow net

Finally, a new approach is suggested for the solution of unsymmetrical double walled cofferdams by the method of fragments. The method involves the assumption of a vertical impermeable surface at some point between the cut-off walls. A chart is presented for finding the 'best' position of the surface, and the accuracy of the ensuing estimates of flow rates and exit gradients is confirmed.

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