Reliability-based exit gradient design of water retaining structures

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Abstract

The traditional approach for estimating the exit gradient due to seepage downstream of water retaining structures is to proceed deterministically, perhaps using flow-net techniques, and to incorporate large safety factors of at least 5 or 6. In comparison to the safety factor usually adopted in other areas of geotechnical design these values are exceptionally high (e.g. slope stability factors of safety are of the order of 1.3). The reason for this conservative approach is twofold. Firstly, the consequence of piping and erosion brought about by i_e approaching i_c is very severe, leading to complete and rapid failure of civil engineering structures with little advance warning. Secondly, the high safety factors reflect the designer's uncertainty in local variations of soil properties at the exit points and elsewhere within the flow domain.

This paper presents an alternative to the safety factor approach by expressing exit gradient predictions in the context of Reliability-Based Design. Random field theory and finite element techniques are combined with Monte-Carlo simulations to study the statistics of exit gradient predictions as a function of soil permeability variance and spatial correlation in a steady seepage 2-d boundary value problem. The approach enables conclusions to be drawn about the probability of critical conditions being approached and

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hence failure at a given site. Such a reliability-based approach represents a more rational methodology for tackling predictions of exit gradients for design purposes.

Introduction

This work presented in this paper brings together Finite Element Analysis and Random Field Theory in the study of a simple boundary value problem of steady seepage. The aim of the investigation is to observe the influence of soil variability on the exit gradient i_e at the downstream side of a water retaining structure in two-dimensions. Smith and Freeze (1979, Pts. 1 and 2) were among the first to study the problem of confined flow through a stochastic medium using finite differences, in which examples of flow between parallel plates and beneath a single sheet-pile were presented. Other workers, including the present authors (see e.g. Griffiths and Fenton 1993) have further considered the probabilistic seepage problem using finite element methods for a range of boundary value problems.

In addition to defining the soil mass as having a randomly distributed permeability k (defined here in the classical geotechnical sense as having units of *length/time*), the current study also includes the ability to vary the spatial correlation of the field as this has been shown to have a significant influence. The importance of the spatial correlation was highlighted in a recent conference on probabilistic methods in geotechnical engineering (Li and Lo 1993). For example Mostyn and Li (1993) emphasised the importance of taking account of the spatial correlation of soil properties in probabilistic analyses. It was pointed out that the "vast majority of existing models do not do this", and although their particular application was the analysis of slope stability in which the random soil properties in question were the shear strength parameters, the same arguments could be applied to soil permeability in a seepage problem. White (1993) also described how early probabilistic analyses typically represented soil property uncertainty by the use of a single 'perfectly correlated' random variable which was varied from one realization to the next.

The use of *random fields* (Vanmarcke 1984, Fenton and Vanmarcke 1990) was considered to be an important refinement, in that the soil property at each location within the soil mass was itself considered to be a random variable. An important feature of the random field approach is that it appropriately takes into account the positive correlation that is observed between soil properties measured at locations that are 'close' together.

In previous studies of seepage through random soil, the seepage quantity Q has tended to be the focus of the investigations, and although results relating to the exit gradient i_e were presented, no serious interpretation was attempted. This was partly due to the added complexity of interpreting the first derivative of the total head (or "potential"), itself a random variable, with respect to length at the exit points. In this paper the exit gradients are studied in more detail for a range of parametric variations of the input permeability statistics. For the purposes of this initial study, a simple boundary value problem has been considered – that of two-dimensional seepage beneath a single sheet pile wall penetrating to half the depth of a soil layer. This problem has been chosen because it is well understood and a number of theoretical solutions exist for computing flow rates and exit gradients in the deterministic (constant permeability) case (see e.g. Harr 1962, Verruijt 1970).

Brief Description of the Finite Element Model

In this paper a random field generator called the Local Average Subdivision Method (LAS) (Fenton 1990) is combined with the Finite Element Method which is naturally suited for modeling spatially varying soil properties.

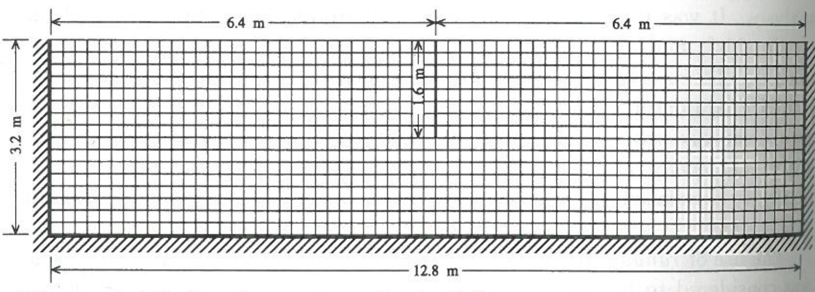

Figure 1: Finite element mesh used for seepage analyses.

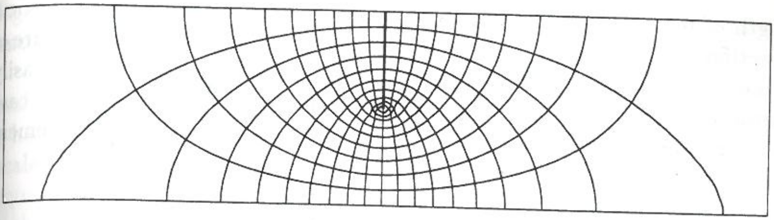

Figure 2: Flow net for deterministic analysis.

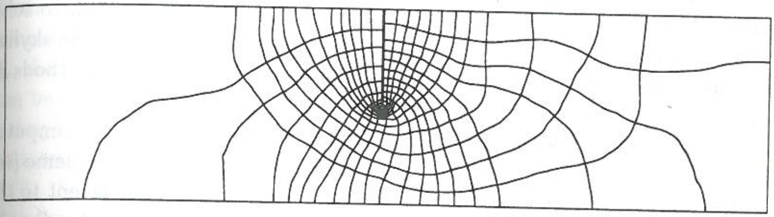

Figure 3: Typical flow net with random permeability for the case θ_k =2m. and CV_k =1.

The finite element mesh and dimensions used in the present study is shown in Figure 1. Figure 2 shows the classical smooth flow net corresponding to a constant permeability field, and Figure 3 shows a typical case in which the permeability is randomly distributed in space. In the particular case shown by Figure 3, it appears that the permeability to the right of the wall is generally lower than that to the left.

The finite element program used for the solutions of Laplace's equation presented in this paper is published in full in the text by Smith and Griffiths (1988). As shown in Figure 1, all analyses used a uniform mesh of 4-node elements with 64 elements in the x-direction (32 on each side of the wall) and 16 elements in the y-direction. All elements are square with a side length of 0.2 m.. The wall length corresponds to 8 elements in the vertical direction and penetrates to half the soil layer depth. A time-saving feature of square (or rectangular) elements is that their conductivity matrices are easily computed explicitly without the need for numerical integration. In this case assuming the permeability of the i^{th} element is k_i, the symmetrical element conductivity matrix is given by:

$$\mathbf{k_i} = \frac{k_i}{6} \begin{bmatrix} 4 & -1 & -2 & -1 \\ & 4 & -1 & -2 \\ & & 4 & -1 \\ & & & 4 \end{bmatrix} \tag{1}$$

During assembly of the global conductivity matrix, a 'skyline' storage strategy was used together with a Cholesky factorization approach to solve the simultaneous equations (see e.g. Griffiths and Smith 1991). The skyline approach runs faster than conventional (constant band-width) methods as well as giving substantial savings on memory requirements.

The exit gradient against the downstream side of the wall was computed using a four-point backward difference numerical differentiation scheme (see e.g. Griffiths and Smith 1991). The gradient was computed adjacent to the wall since this location will record the highest values on average. In all analyses, the head difference across the wall, H, was set equal to unity because this is just a linear scaling factor on the computed exit gradient.

Brief Description of the of the Random Field Model

Field measurements of permeability have indicated an approximately log-normal distribution (see e.g. Hoeksema and Kitanidis 1985, and Sudicky 1986). The same distribution has therefore been adopted for the simulations generated in this paper.

Essentially, the permeability field is obtained through the transformation

$$k_i = \exp\{\mu_{\ln k} + \sigma_{\ln k}\, g_i\} \tag{2}$$

in which k_i is the permeability assigned to the i^{th} element, g_i is the local average of a standard Gaussian random field, g, over the domain of the i^{th} element, and $\mu_{\ln k}$ and $\sigma_{\ln k}$ are the mean and standard deviation of the logarithm of k (obtained from the 'point' mean and standard deviation μ_k and σ_k).

The LAS technique (Fenton 1990, Fenton and Vanmarcke 1990) generates realizations of the local averages g_i which are derived from the random field g having zero mean, unit variance, and a spatial correlation controlled by the scale of fluctuation, θ_k. As the scale of fluctuation goes to infinity, g_i becomes equal to g_j for all elements i and j – that is the field of permeabilities tends to become uniform on each realization. At the other extreme, as the scale of fluctuation goes to zero, g_i and g_j become independent for all $i \neq j$ – the soil permeability changes rapidly from point to point.

In the two dimensional analyses presented in this paper, the scales of fluctuation in the vertical and horizontal directions are taken to be equal (isotropic) for simplicity. It should be noted that for a layered soil mass the horizontal scale of fluctuation, θ_h is generally larger than the vertical scale, θ_v, due to the natural stratification of many soil deposits. This anisotropy can be transformed to a problem with isotropic scales of fluctuation through a simple shrinking of the horizontal dimensions by the ratio of the vertical to horizontal scales of fluctuation, that is by scaling the horizontal coordinate by θ_v/θ_h. Such a scaling is strictly only valid for so-called ellipsoidal correlation structures, but is a reasonable approximation in any case. Thus the assumption of isotropy employed herein is not a serious limitation. However, the actual spatial correlation structure of soil deposits is not usually well known, especially in the horizontal direction, hence in this paper a parametric approach has been employed to study the influence of θ_k (see e.g. DeGroot and Baecher 1993, Marsily 1985, Asaoka and Grivas 1982).

The input to the random field model therefore comprises of the three parameters $(\mu_k, \sigma_k, \theta_k)$. Based on these underlying statistics, each of the 1024 elements in the mesh is assigned a permeability from a realization of the permeability random field. A series of realizations are generated, each with the same underlying statistics, but each having quite different spatial distributions of permeability. The analysis of sequential realizations and the accumulation of results comprises a Monte-Carlo process. In the current study 2000 realizations were performed for each parametric combination enabling statistical information to be computed on the output quantity of interest–in this case the exit gradient i_e.

The 2-d model used herein implies that the out-of-plane scale of fluctuation is infinite – soil properties are constant in this direction – which is equivalent to specifying that the streamlines remain in the plane of the analysis. This is clearly a deficiency of the present model, however it is believed that useful information regarding the variability of exit gradients can still be obtained from the 2-d model. Three-dimensional analyses form part of a continuing study of this problem (Griffiths and Fenton 1995).

Summary of Results from the Seepage Analyses

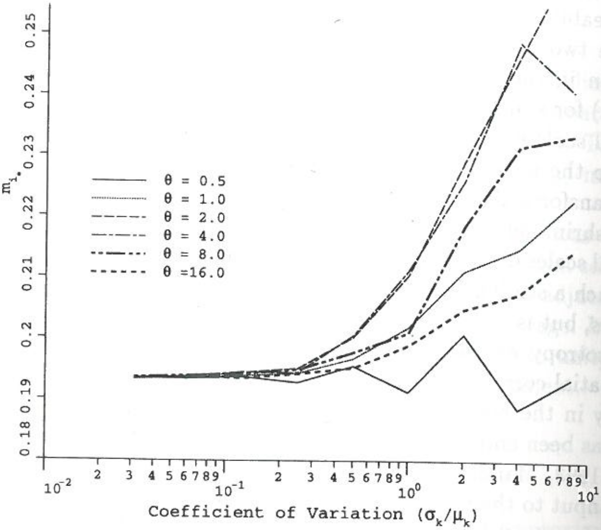

Figure 4: Exit gradient mean (m_{i_e}) vs. Coefficient of variation of permeability (CV_k)

The deterministic analysis of this seepage problem, in which the permeability everywhere is taken equal to μ_k, indicated an exit gradient of around $i_{det} = 0.193$ which agrees closely with the analytical solution for this problem (see e.g. Lancellotta 1993) .

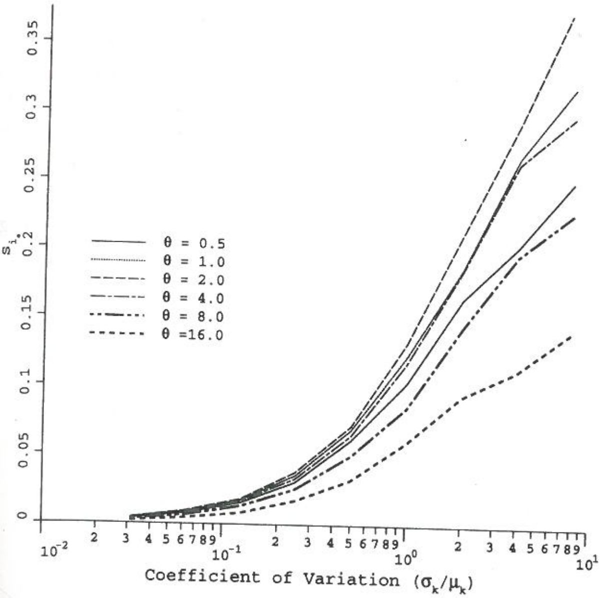

Figure 5: Exit gradient standard deviation (s_{i_e}) vs. Coefficient of variation of permeability (CV_k)

Given that the critical exit gradient i_c (i.e. the value that would initiate piping) for a typical soil is approximately equal to unity, this deterministic value implies a factor of safety of around 5 – a conservative value not untypical of those used in design of water retaining structures (see e.g. Holtz and Kovacs 1984).

When incorporating a random field analysis, a number of parametric runs were performed using the program `rflow2d` developed by the authors. The point mean permeability was fixed at $\mu_k = 1 \times 10^{-5}$ m/s while the point standard deviation and spatial correlation of permeability were varied in the ranges: $0.03125 < \sigma_k/\mu_k < 32.0$ and $0.5 < \theta < 16.0$ m. For each of these parametric combinations the Monte-Carlo process led to *estimated* values of the mean and standard deviation of the exit gradient given by m_{i_e} and s_{i_e} respectively.

Graphs of m_{i_e} and s_{i_e} against CV_k $(= \sigma_k/\mu_k)$ for a range of θ_k-values have been plotted in Figures 4 and 5 respectively. Figure 4 shows that as the coefficient of variation of the input permeability tends to zero, the mean exit gradient tends, as expected, to the deterministic value of 0.193. For small scales of fluctuation the mean exit gradient remains constant or even falls slightly as the coefficient of variation is increased, however the amount by which the mean exit gradient increases is dependent on θ_k and appears to reach a maximum when $\theta_k \approx 2$. This is shown more clearly in Figure 6 where the same results have been plotted the other way round with θ_k along the abscissa.

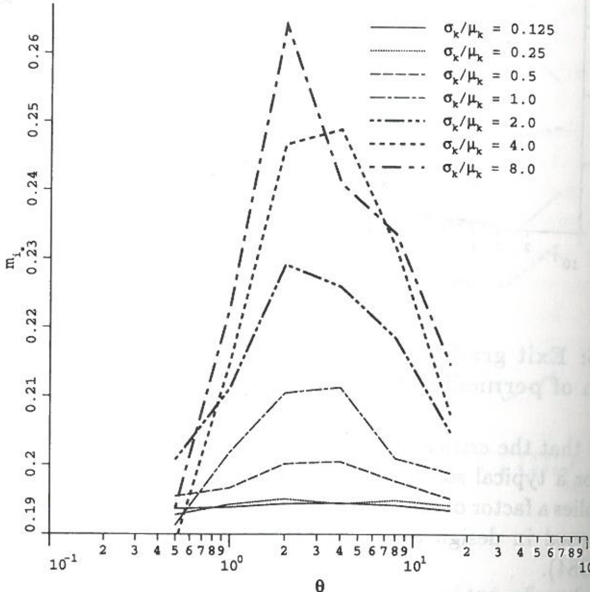

Figure 6: Exit gradient mean (m_{i_e}) vs. Scale of fluctuation (θ_k)

The return to deterministic values as θ_k increases is to be expected if one thinks of the limiting case where $\theta_k = \infty$. In this case each realization would have a constant permeability, thus the deterministic exit gradient would be obtained. The ragged nature of the results in Figure 4 as the coefficient of variation is increased suggests the need for a greater number of realizations than the 2000 conducted in this study. Reproducibilty studies to address this question have been performed and will be published separately.

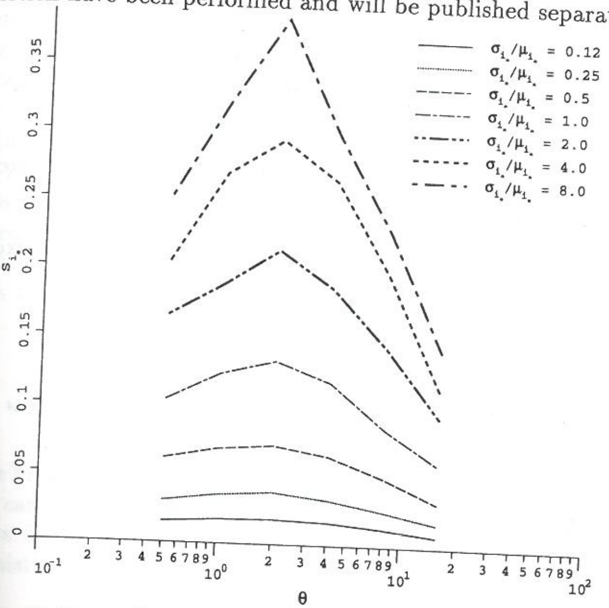

Figure 7: Exit gradient standard deviation (s_{i_e}) vs. Scale of fluctuation (θ_k)

Figure 5 shows that as the coefficient of variation of the input permeability increases, so the standard deviation of the exit gradient also increases. However the standard deviation increases more substantially for some values of θ_k than others. Again the plot against θ_k shown in Figure 7 shows this effect more clearly, the peak in the standard deviation results again occurring around $\theta_k \approx 2.0$. It would appear that there is a 'worst-case' value of θ_k which corresponds to the maximum mean *and* standard deviation of the exit gradient..

Reliability-Based Design Interpretation

A Factor of Safety applied to a deterministic prediction is intended to eliminate any serious possibility of failure but without any objective attempt to quantify the risk. Reliability-Based Design attempts to quantify risk by seeking answers to the following questions:

1. "What is the *probability* that the actual exit gradient will exceed the deterministic prediction?".
2. "What is the *probability* that the actual exit gradient will exceed the critical value, resulting in failure?".

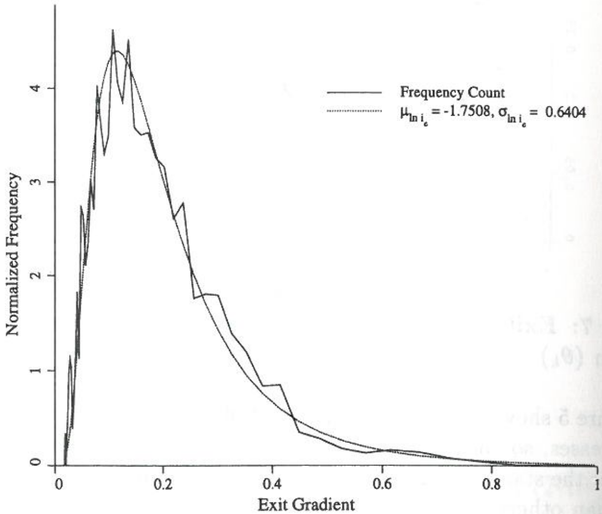

Figure 8: Histogram of exit gradient values following 2000 realizations for the case θ_k =2m. and CV_k =1.

The Monte-Carlo scheme described in this paper enables probabilistic statements to be made. For example, if out of 2000 realizations, 100 gave an exit gradient $i_e \geq 1$, it could be concluded that the probability of piping or erosion was of the order of 100/2000, or 5%. In general though, a histogram can be plotted and the probabilities computed via standard tables of the area beneath a normal curve.

A typical histogram of exit gradient values corresponding to $\theta_k = 2$m. and $CV_k = 1$ is shown in Figure 8. The ragged line comes from the frequency count (unit-area normalized) obtained over the realizations and the smooth dotted line is based on a lognormal fit to that data. The good agreement suggests that the actual distribution of exit gradients is indeed lognormal. The mean and standard deviation of the underlying *normal* distribution of $\ln i_e$ is also printed on the figure. It should be noted that the relationships between the statistics of $\ln i_e$ and i_e are given by:

$$\mu_{i_e} = \exp\left\{\mu_{\ln i_e} + \frac{1}{2}\sigma^2_{\ln i_e}\right\} \tag{3}$$

$$\sigma^2_{i_e} = \mu^2_{i_e}\left\{\exp(\sigma^2_{\ln i_e}) - 1\right\} \tag{4}$$

Since Figure 8 shows a fitted lognormal probability density function, probabilities can be deduced directly. For example, in the particular case shown in Figure 8, the probability that the actual exit gradient will exceed the deterministic value of 0.193 is approximated by:

$$P\left[i_e > 0.193\right] = 1 - \Phi\left(\frac{\ln 0.193 + 1.7508}{0.6404}\right) \tag{5}$$

where $\Phi(.)$ is the cumulative normal distribution function.

In this case $\Phi(0.17) = 0.568$, thus:

$$P\left[i_e > 0.193\right] = 0.43 \tag{6}$$

and there is a 43% probability that the deterministic prediction of $i_{det} = 0.193$ is unconservative.

A similar calculation has been performed for all the parametric variations considered in this study. In each case the following probability was calculated:

$$P\left[i_e > \alpha i_{det}\right] \tag{7}$$

where α is a simple scaling factor on the deterministic exit gradient. When $\alpha = 1$ (as in equation 5), the result is just the probability that the actual exit gradient will exceed the deterministic value. Larger values of α are interesting for design purposes where a prediction of the probability of failure is required. In the current example, the deterministic exit gradient is approximately equal to 0.2, so it would be of interest to know the probability of the actual exit gradient exceeding the critical hydraulic gradient $i_c \approx 1$. For this comparison therefore α would be set equal to 5.

Although a full range of probability values have been computed in this study, the one corresponding to $\theta_k = 2$ is presented here in Figure 9. This value was chosen because the variability of the exit gradient appeared to be a maximum in this range as shown in Figure 7.

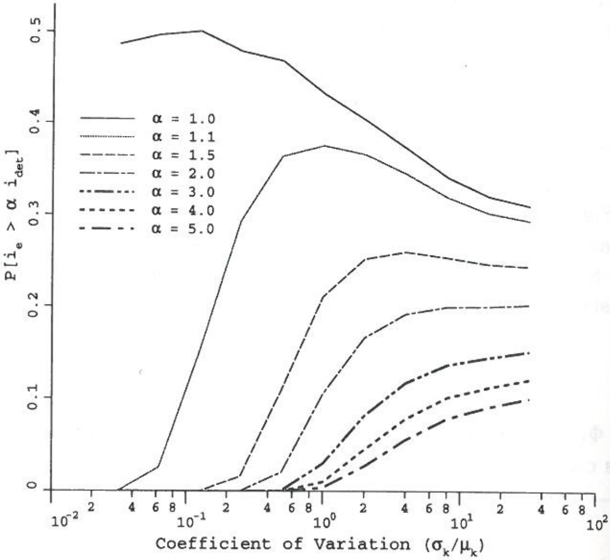

Figure 9: Probability that i_e exceeds αi_{det} vs. CV_k for the case θ_k =2m.

It should be noted that irrespective of the θ_k or CV_k, the probability that the actual exit gradient exceeds the deterministic value is always less than 50%. This is a reassuring result from a design standpoint. In fact the probabilities which approach 50% correspond to a very low CV_k and are somewhat misleading in that the computed exit gradients have a very low variance and are approaching the deterministic value. The 50% merely refers to an equal likelihood of the actual exit gradient lying on either side of an essentially normal distribution with a very small variance. For small CV_k, this is shown clearly by the sudden reduction to zero of the probability that i_e exceeds i_{det} scaled up by a small factor (say 10% as indicated by $\alpha = 1.1$).

As α is increased further, the probability consistently falls, although each curve exhibits a maximum probability corresponding to a different value of CV_k. This interesting observation implies that there is a 'worst-case' combination of θ_k and CV_k that gives the greatest likelihood of i_e exceeding i_{det}.

In consideration of failure conditions, the value of $P[i_e \geq 1]$, as indicated by the curve corresponding to $\alpha = 5$, is small but not insignificant, with probabilities approaching 10% for the highest CV_k cases considered. In view of this result, it is not surprising that for highly variable soils a Factor of Safety against piping of up to 10 has been suggested by some commentators (see e.g. Harr 1987) .

Concluding Remarks

The paper has presented results which form part of a broad study conducted by the authors into the influence of random soil properties on geotechnical design. In this paper, random field methodology has been combined with the finite element method to study the exit gradient due to steady seepage beneath a single sheet-pile wall embedded in a layer of random soil. The influence of spatial correlation of soil properties has been fully incorporated through a scale of fluctuation parameter θ_k.

The spatial correlation and the coefficient of variation (CV_k) of the input permeability were varied over a wide range of values. For each parametric combination, 2000 realizations of a Monte-Carlo process were performed. A backward difference numerical differentiation formula was used to calculate the exit gradient just to the right of the sheet pile wall.

Generally speaking the computed variance of the exit gradient was considerably higher than other quantities of interest in the flow problem such as the flow rate. This is hardly surprising when one considers that the exit gradient involves differentiation of the computed total head with respect to distance at the exit point. An interesting result was that the computed exit gradient was found to reach a maximum for a particular value of the scale of fluctuation given by $\theta_k \approx 2$m. The significance of this value in relation to the dimensions of the particular boundary value problem used in this study is still under consideration.

When the results were interpreted in the context of Reliability-Based Design, conclusions could be reached about the probability of exit gradient values exceeding the deterministic value, or even reaching levels at which stability and piping could occur. For the particular case of $\theta_k = 2$m and for certain values of CV_k, it was found that the probability of the actual exit gradient being at least 10% higher than the traditional deterministic value could be as high as 40%. The probability of an unconservative deterministic prediction was generally found to exhibit a maximum point corresponding to a particular combination of θ_k and CV_k. From a design point of view this could be considered a 'worst-case' scenario corresponding to maximum uncertainty in the prediction of exit gradients.

With regard to the possibility of piping, erosion and eventual failure of the system, a relationship was established between the traditional Factor of Safety and the "Probability of Failure". For the particular case mentioned above, and assuming that the critical exit gradient is of the order $i_c \approx 1$, a Factor of Safety of 5 could still imply a probability of failure as high as 10% if the soil permeability variance is also high. This result suggests that Factors of Safety as high as 10 might be needed for critical structures founded in highly variable soil.

Further studies are continuing on the exit gradient problem with the aim of distilling results into a form that will help designers of water retaining structures and lend objectivity to the assessment of risk.

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