STABILITY ANALYSIS OF SLOPES IN VARIABLE SOILS BY FINITE ELEMENTS

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ABSTRACT: The paper will review the use of finite element (FE) methods as a powerful alternative to classical limit equilibrium method for tackling slope stability problems. The benefits of FE are particularly obvious when dealing with non-typical geometries and soils with variable properties. The paper will show several examples of finite element methods applied to slope stability problems ranging from 1-D "infinite" slopes, to recent work on 3-D slopes

1. INTRODUCTION

The finite element method offers a powerful alternative to classical limit equilibrium methods of slope stability that have remained essentially unchanged for decades. The method offers the following main advantages:

- No assumption needs to be made in advance about the shape or location of the failure surface. Failure occurs "naturally" through the zones within the soil mass in which the soil shear strength is unable to sustain the applied shear stresses.

- Since there is no concept of slices in the finite element approach there is no need for assumptions about slice side forces. The finite element method preserves global equilibrium until "failure" is reached.

- If realistic soil compressibility data is available, the finite element solutions will give information about deformations at working stress levels.

- The finite element method is able to monitor progressive failure up to and including overall shear failure.

It is certainly not the case that the finite element method of slope stability analysis is a new technique. The first paper to tackle the subject by Smith & Hobbs (1974) is over 35 years old and this was followed by an important paper on the topic by Zienkiewicz *et al.* (1975). The Zienkiewicz paper had a very significant influence on the author's finite element slope stability software developments over the years. Early publications date back to Griffiths (1980) and the first ever published source code for finite element slope stability appeared in the second edition of the text by Smith & Griffiths (1988, 2004). Readers are also referred to Griffiths & Lane (1999) for a thorough review of how the methodology works.

This paper will focus on demonstrating the use of the finite element method as applied to several slope examples that would not necessarily be amenable to traditional limit equilibrium "slip circle" approaches.

2. LONG SLOPES

2.1 How long is "infinite"?

It has been noted previously (e.g. Duncan & Wright 2005) that the infinite slope assumptions can be expected to lead to conservative estimates of the factor of safety. This is primarily due to support provided at the ends of a finite slope that is not accounted for in the infinite slope model.

In this section finite element slope stability analyses have been performed on "long slopes" with uphill and downhill conditions, to assess the range of validity and conservatism of the "infinite slope" assumptions. The main question to be addressed is; How long must a slope be for it to be considered "infinite"?

A typical finite element mesh of 8-noded quadrilateral elements is shown in Figure 1.

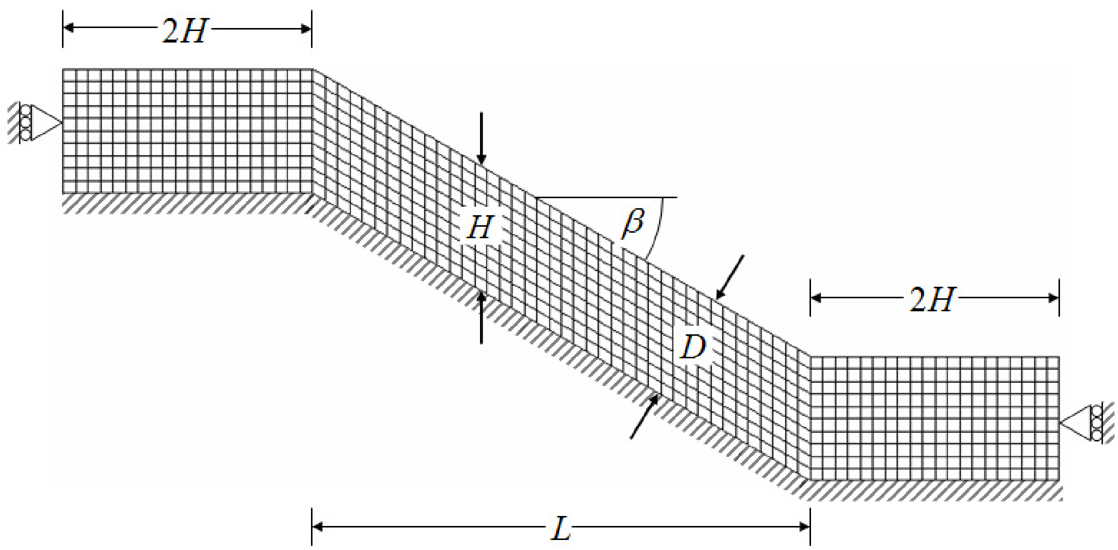


Figure 1: Mesh of 8-node quadrilateral element for "long slope" analysis.

For simplicity, we have considered an undrained clay slope for which the infinite slope equation would give the following factor of safety

$$FS = \frac{c_u}{\gamma_{sat} H \cos\beta \sin\beta} \quad (1)$$

In this example, the properties shown in the caption of Figure 2 were held constant while L/H was gradually increased. As shown, the computed factor of safety converged on the infinite slope solution of $FS = 1.15$ from equation (1) for L/H greater than about 16. It may be noted that the infinite slope solution is always conservative. For example, with $L/H = 2$ the computed factor of safety was $FS = 2.86$, more than twice the infinite slope value. A typical failure mechanism for a steeper slope is shown in Figure 3. The figure indicates that as the slope gets longer, the infinite slope mechanism starts to dominate and the "toe" failure at the downhill end becomes less important.

2.2 Influence of slope angle

A curiosity of the infinite slope equation (1) is that for constant H, γ_{sat} and c_u, the factor of safety starts to *increase* as the slope steepens in the range $\beta > 45°$. This result seems counter intuitive since our experience of finite slopes is that the factor of safety always falls as a slope gets steeper. An explanation of this effect for infinite slopes comes from the fact that as the slope gets steeper, the length of the potential failure surface available to resist sliding is increasing at a faster rate than the down-slope component of soil weight trying to cause sliding. Even a long slope analysis with $L/H = 2$ demonstrates this effect as shown in Figure 4.

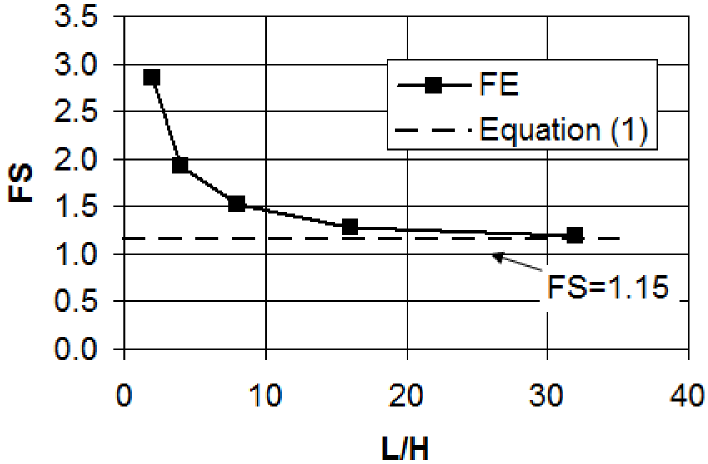


Figure 2: Influence of length ratio L/H on the computed factor of safety FS for slope with $H = 2.5$ m, $\beta = 30°$, $c_u = 25$ kN/m^2 and $\gamma_{sat} = 20$ kN/m^3.

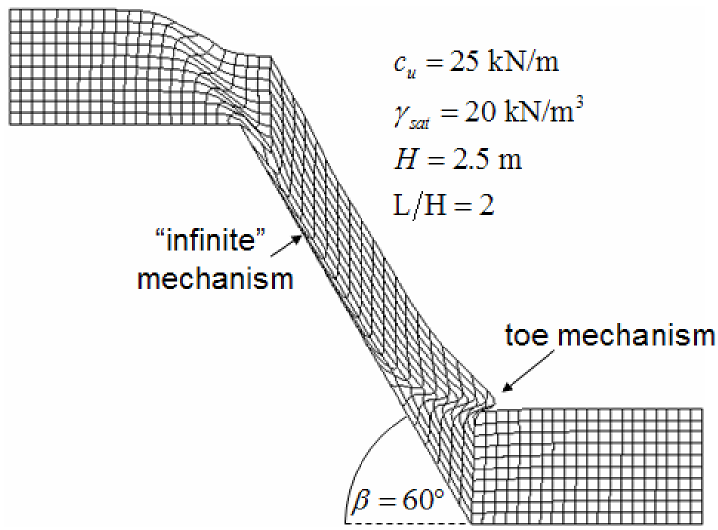


Figure 3: Deformed mesh at failure corresponding to a slope with $L/H = 2$ and $\beta = 60°$ indicating a toe mechanism with $FS = 1.58$.

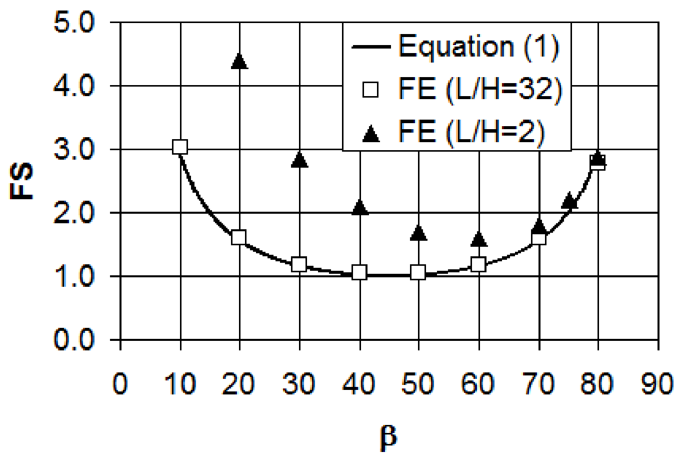


Figure 4: Influence of slope angle β on the computed factor of safety FS for an undrained clay slope with $H = 2.5$ m, $c_u = 25$ kN/m^2 and $\gamma_{sat} = 20$ kN/m^3 for two different length ratios.

3. STRATIFIED SLOPES

The next example to be considered here is the James Bay Dike slope shown in Figure 5.

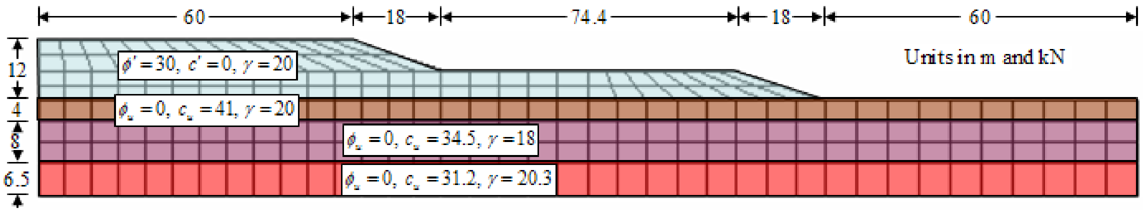


Figure 5: FE geometry and soil properties assigned to the James Bay dike.

The geometry has a terraced cross-section with four different soil types consisting of cohesionless soil in the embankment and undrained clays in the foundation. Even though the slope was never actually built, this configuration has attracted considerable interest (see e.g., El Ramly *et al.* 2002, Duncan *et al.* 2003) because published limit equilibrium solutions that assumed circular failure mechanisms (e.g. Bishop's method), led to unconservative estimates of the factor of safety. Although limit equilibrium procedures are available for estimating the factor of safety associated with non-circular surfaces (e.g. Spencer's method), it is still hard to guarantee that the critical surface corresponding to the minimum factor of safety has been found.

The benefits of the FE slope stability approach are even more striking in an example such as this. By using a strength reduction factor (SRF) to failure as described in Griffiths & Lane (1999), the factor of safety can be accurately estimated, and the corresponding failure mechanism observed. The sudden displacement increase shown in Figure 6 indicates that $FS \approx 1.27$ and the deformed mesh at failure given in Figure 7 clearly shows the non-circular critical failure mechanism.

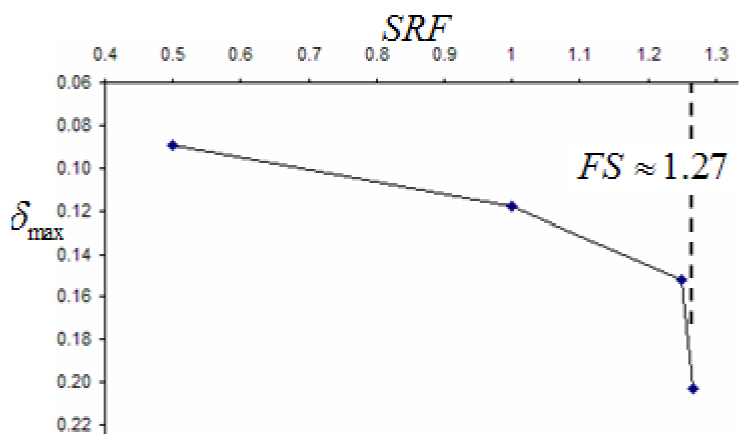


Figure 6**:** FE solution of James Bay Dike by strength reduction indicating $FS \approx 1.27$

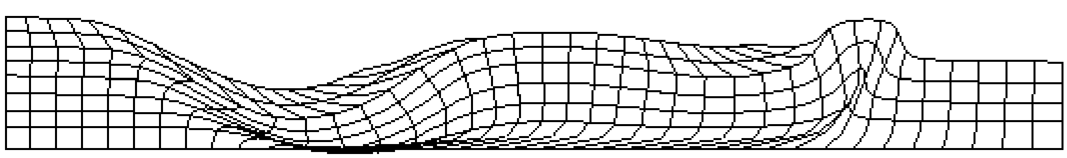


Figure 7: Deformed mesh at failure demonstrating non-circular critical mechanism

4. 3-D SLOPES

The vast majority of slope stability analyses are performed in 2-D under the assumption of plane strain conditions. Even when 2-D conditions are not appropriate, 3-D analysis is rarely performed. There are a number of reasons for this. The vast majority of work on this subject has shown that the 2-D factor of safety is conservative (e.g. lower than the “true” 3-D factor of safety), and existing methods of 3-D slope stability analysis are often complex, and not well established in practice. A further disadvantage of some 3-D methods, is that being based on extrapolations of 2-D “methods of slices” to 3-D “methods of columns”, they are complex, and not readily modified to account for realistic boundary conditions in the third dimension. The advantages of FE slope stability methods become even more attractive in 3-D. The paper demonstrates some 3-D slope stability analyses by finite elements and shows that great care must be taken in subscribing to the received wisdom that “2-D is always conservative”.

4.1 When is plane strain a reasonable approximation?

The first issue addressed here for a homogeneous slope, is to consider the question “how long does a slope need to be in the third dimension for a 2-D analysis to be justified?”

Figure 8 shows a simple mesh that might be used for a 3-D analysis of an undrained slope. The “rough-smooth” boundary condition implies a symmetric analysis about the plane $z = L/2$, thus only half of the actual depth L of the slope is analyzed. The bottom $(y = D)$ and far-side $(z = 0)$ of the slope are fully fixed, while the back $(x = 0)$ and front-side $(z = L/2)$ of the slope are constrained by vertical rollers.

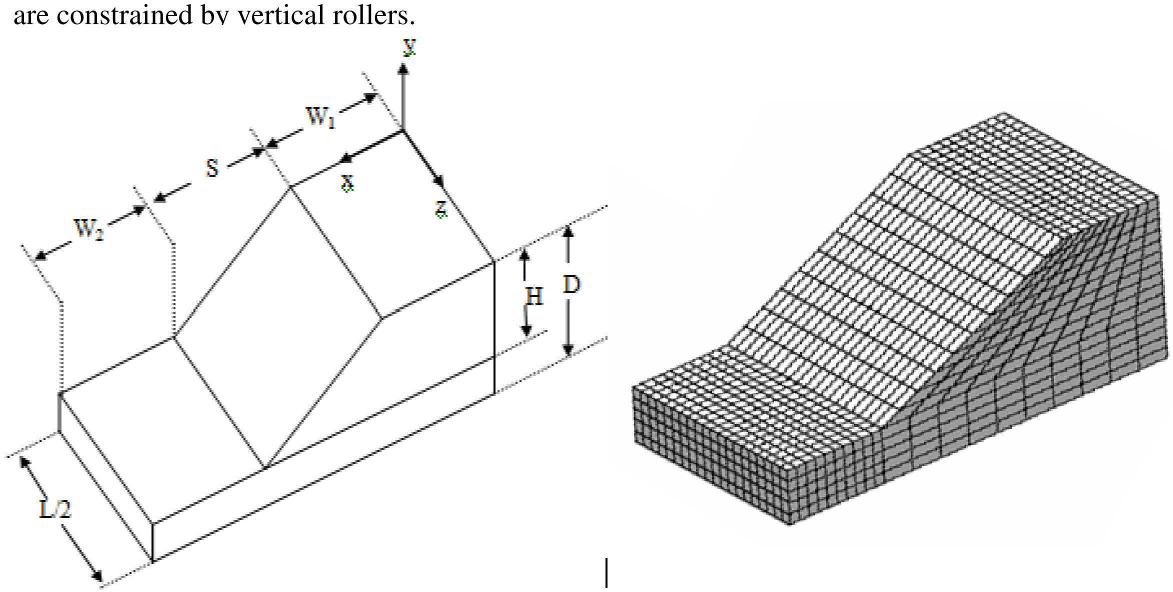


Figure 8. 3-D layout and typical mesh $(L/H = 2)$. FE analyses used 20-node hexahedral elements.

The results from a series of FE analyses with different depth ratios (L/H) while keeping all other parameters constant are shown in Figure 9. It can be seen that the factor of safety in 3-D was always higher than in 2-D but tended to the plane strain solution for depth ratios of the order of $L/H > 10$. It is shown that results of the same analysis with a coarser mesh gave slightly higher values of FS.

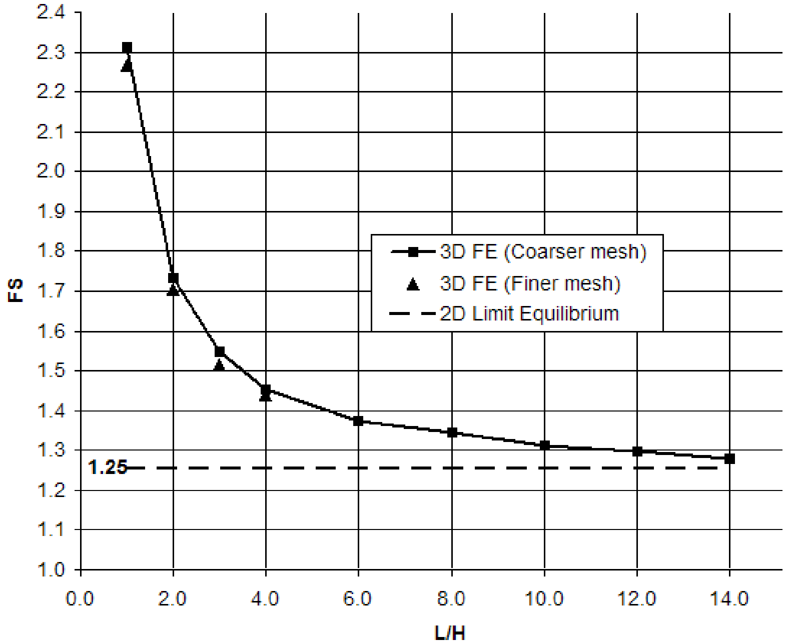


Figure 9: Comparison of 3-D FE and 2-D limit analyses for a $\phi_u = 0^o$ slope with $c_u / (\gamma \mathrm{H})$=0.20 .

4.2 Is plane strain conservative?

The assumption that 2-D analyses lead to conservative factors of safety needs some qualification. Firstly, a conservative result will only be obtained if the most pessimistic section in the 3-D problem is selected for 2-D analysis (see e.g., Duncan 1996). In a slope that contains layering and strength variability in the third dimension, this "most pessimistic" 2-D section may not be intuitively obvious. Secondly, the corollary of a conservative 2-D slope stability analysis is that back analysis of a failed slope will lead to an unconservative overestimation of the soil shear strength (e.g. Arellano & Stark 2000). Although some investigators (e.g. Hutchinson & Sarma 1985, Hungr 1987) have asserted that the factor of safety in 3-D is always greater than in 2-D, it cannot be ruled out that an unusual combination of soil properties and geometry could lead to a 3-D mechanism that is more critical. Bromhead & Martin (2004) argued that some landslide configurations with highly variable cross-sections could lead to failure modes in which the 3-D mechanism was the most critical. Other investigators have indicated more critical 3-D factors of safety (e.g., Chen & Chameau (1982) and Seed *et al*. (1990)) although this remains a controversial topic.

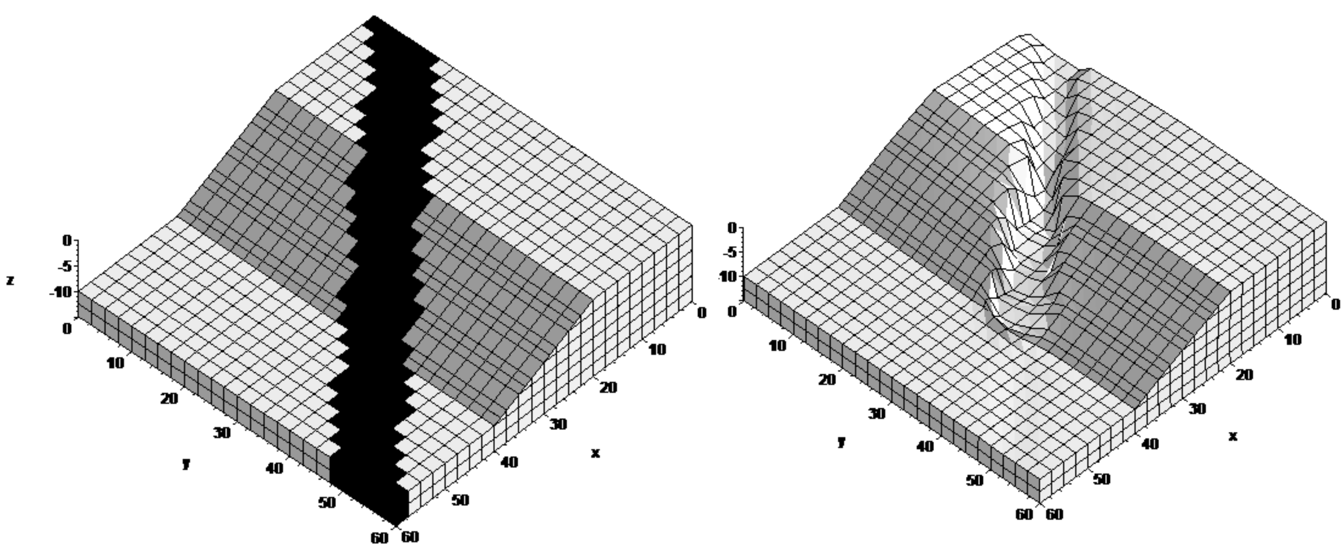


Figure 10: Three-dimensional slope at failure including an oblique layer of weak soil.

Finite element slope stability analysis offers us the opportunity to perform objective comparisons in which 2-D and 3-D factors of safety are compared for variable soil conditions. This point is highlighted in the 3-D example shown in Figure 10 which represents a 2:1 slope of height 10 m, foundation depth 5 m and a length in the out-of-plane direction of 60 m with smooth boundary conditions. An oblique zone of weak soil (shaded black) with undrained strength $c_u = 20\text{ kN/m}^2$ has been introduced into the slope with the surrounding soil four times stronger at $c_u = 80\text{ kN/m}^2$. The 3-D factor of safety is found to be approximately 1.5 and the mechanism clearly follows the weak zone as also shown in Figure 10.

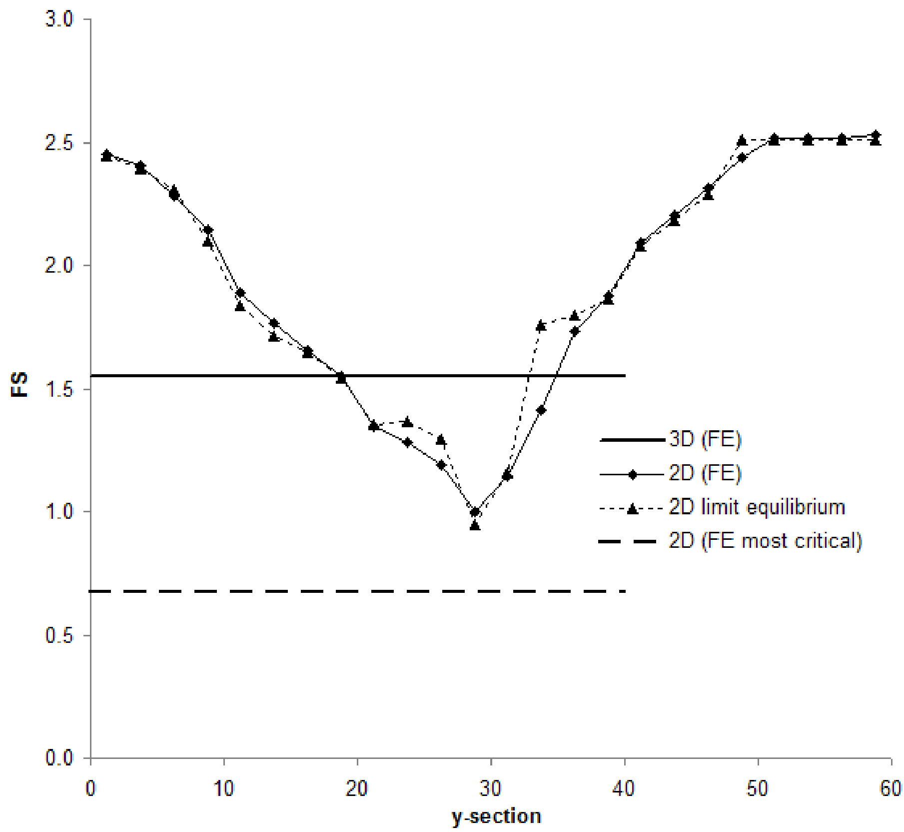


Figure 11: Factors of safety from 3d analysis and various 2d sections.

When 2-D stability analyses are then performed on successive slices in the $x-z$ plane moving from $y = 0\text{ m}$ to $y = 60\text{ m}$, the result shown in Figure 11 is obtained. As a check, the 2-D analyses were performed both by finite elements and by a standard limit equilibrium program. It can be seen that towards the boundaries of the 3-D slope ($y < 21\text{ m}$ and $y > 34\text{ m}$) where the majority of soil in the sections is strong, the 2-D results led to higher and therefore unconservative estimates of the factor of safety. On the other hand, at sections towards the middle of the slope ($21\text{ m} < y < 34\text{ m}$) where there is a greater volume of weak soil, the 2-D results led to lower, and therefore conservative estimates of the factors of safety. The 2-D factor of safety closely approached unity at $y = 29\text{ m}$. An even more critical 2-D plane however, is the one that runs right down the middle of the weak soil. This 2-D plane has a 2.5:1 slope which is flatter than the $x-z$ planes considered previously, however it is homogeneous and consists entirely of the weaker soil. A 2-D slope stability analysis on this plane gives an even lower factor of safety of about 0.7. This result, also shown on Figure 11, is less than half of the factor of safety given by the 3D analysis and would be considered excessively conservative, even by geotechnical design standards.

Even in the rather simple problem considered here, the results have shown a quite complex relationship between 2-D and 3-D factors of safety. The results confirm that 2-D analysis will deliver conservative results if a pessimistic plane in the 3-D problem is selected, however this may lie well below the "true" 3-D factor of safety. It has also been shown however, that selection of the "wrong" 2-D plane could lead to an unconservative result.

5. RISK ASSESSMENT OF SLOPES

Risk assessment and probabilistic analysis in geotechnical engineering is a rapidly growing area of interest and activity for practitioners and academics. It fair to say that slope stability analysis has received greater attention from probabilistic tools than any other application of conventional geotechnical engineering (see e.g. Li and Lumb 1987, Mostyn and Lee 1993, Griffiths and Fenton 2000, Duncan 2000, El Ramly *et al.* 2002, Huang *et al.* 2010).

Soils and rocks are the most variable of all engineering materials, so when an engineer chooses "characteristic values" of the soil shear strength for a limit analysis (say), it is very likely that some parts of slope consist of soil that is stronger than the characteristic values, and other parts are weaker. How do the stronger and weaker soils interact and which of them have the upper hand in determining the factor of safety?

5.1 Checkerboard slope stability analysis.

In this section we take a simple 2-D slope and assign the slope two different properties arranged in a checkerboard pattern (Zhou & Griffiths 2009) as shown in Figure 12.

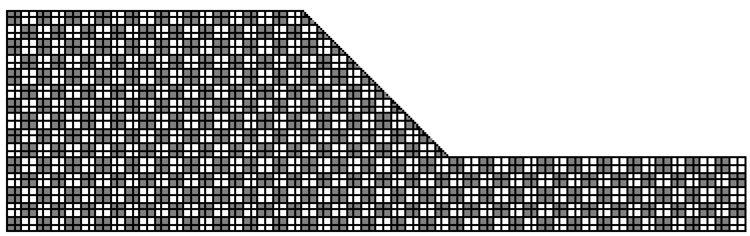


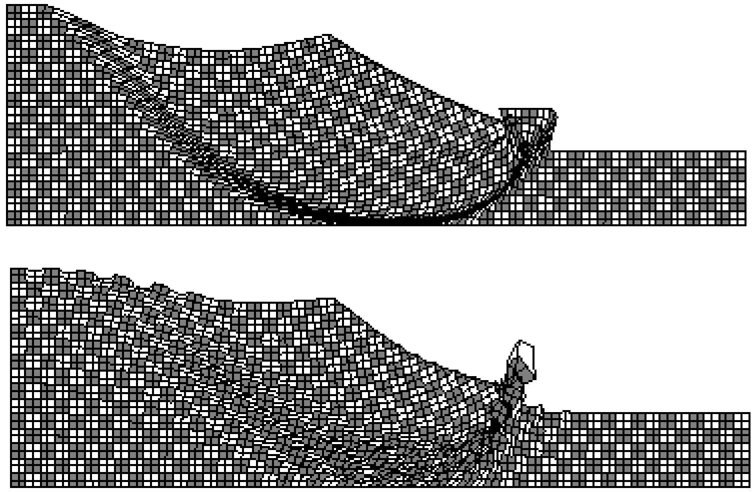
Figure 12: Slope stability analysis with checkerboard strength pattern. The darker zones are stronger.

The $45°$ undrained clay slope has a height of $H = 10$ m and a foundation depth ratio of $D = 1.5$. The mean strength of $c_u = 50$ kPa was held constant, while the stronger soil was made stronger and the weaker soil was made weaker. The results of the factor of safety analysis by strength reduction are shown in Table 1. Clearly the weaker soil "wins"!

Table 1: Influence of variable soil in a checkerboard pattern

$C_{u(\text{strong})}$ (kPa)	$C_{u(\text{weak})}$ (kPa)	$C_{u(\text{strong})} / C_{u(\text{weak})}$	FS
50	50	1.00	1.39
60	40	1.50	1.30
70	30	2.33	1.17
80	20	4.00	1.03
90	10	9.00	0.88

Failure mechanisms in the homogeneous and the most variable cases are shown in Figure 13. In the most variable case, it can be seen that multiple mechanisms are attracted to the "diagonals" of weak soil and show a more dramatic outcrop on the downhill side.



a)

b)

Figure 13: Failure mechanisms in "checkerboard" analysis. a) Homogeneous slope, b) Slope with strength ratio $C_{u(\text{strong})} \big/ \; C_{u(\text{weak})} = 9$

5.2 Probabilistic slope stability using RFEM.

The goal of a probabilistic slope stability analysis is to estimate the probability of slope failure as opposed to the ubiquitous factor of safety used in conventional analysis. Several relatively simple tools exist for performing this calculation that include the First Order Second Moment (FOSM) methods and the First Order Reliability Methods (FORM).

A legitimate criticism of these first order methods is that they are unable to properly account for spatial correlation length in the random material. This parameter recognizes that two sites could have the same mean and standard deviation of strength parameters, but quite different spatial correlation lengths. The spatial correlation length is the distance in length units, over which soil properties tends to be correlated.

To overcome these deficiencies, the author and Gordon A. Fenton of Dalhousie University, have developed an advanced probabilistic analysis tool called the Random Finite Element Method (RFEM) that combines random field theory with elasto-plastic finite element analysis. Input to RFEM is provided in the form of the mean, standard deviation and spatial correlation length of the soil strength parameters. Spatial correlation length may be expressed in dimensionless form as Θ_C in which the spatial correlation length is normalized with respect to the slope height. Following generation of a locally averaged random field the properties are assigned to the mesh and gravity loads are applied. The slope either fails or not, and the process is repeated. Following a sufficient number of Monte-Carlo simulations, the probability of failure is simply the proportion of the total number of simulations that failed. The interested reader is directed to publications by Griffiths and Fenton (2000, 2004) and the textbook by Fenton and Griffiths (2008) for more detail. The method is becoming recognized as the state-of-the-art in probabilistic geotechnical analysis and is being used by several research groups worldwide. The RFEM codes developed by Griffiths and Fenton have now been applied to numerous areas of geotechnical engineering and are freely available in source code from the authors' web site at www.mines.edu/~vgriffit/rfem.

A typical failure mechanism from an RFEM slope analysis is shown in Figure 14 with a spatial correlation length equal to half the slope height.

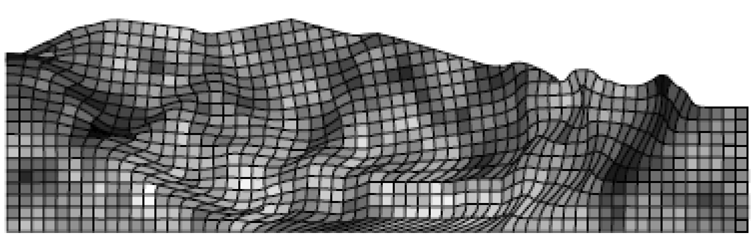


Figure 14: Typical failure mechanisms from an RFEM analysis. A complex pattern of partially formed failure mechanisms is displayed

The computed probability of failure as a function of spatial correlation length (V_C) is given in Figure 15 for different input coefficients of variation of undrained strength. It can be seen that a rising correlation length may either increase or decrease the slope failure probability depending on the input coefficient of variation. First order methods can be shown to represent special cases of the RFEM results where the spatial correlation length is set to infinity. In summary, it is not necessarily the case that first order approaches will be conservative.

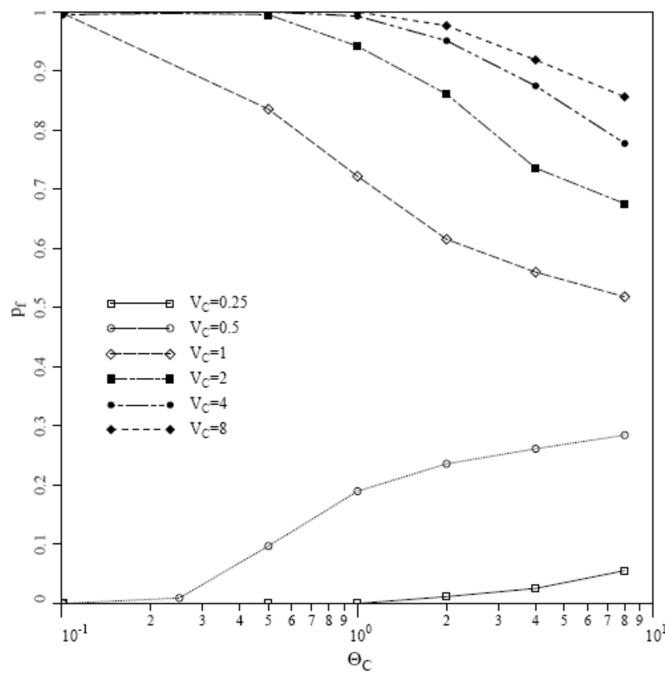


Figure 15: Influence of the (dimensionless) spatial correlation length and input coefficient of variation on the probability of failure of an undrained slope (see Griffiths and Fenton 2004 for further details).

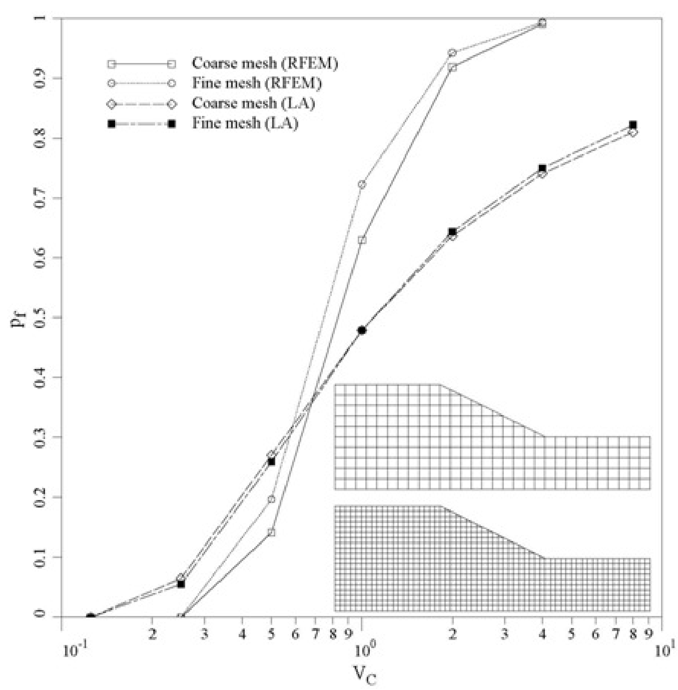


Figure 16: Influence of mesh density on computed probability of failure.

5.3 *Influence of mesh refinement.*

All finite element analyses are vulnerable to inadequate mesh refinement and the RFEM is no exception. It should be emphasized however, that RFEM takes full account of local averaging, so element size is properly accounted for and the input statistics adjusted accordingly. The "discretization error" therefore in RFEM is no worse than would be observed in a traditional deterministic slope stability analysis.

Figure 16 shows the sensitivity of the computed probability of failure by RFEM in two different meshes of different refinement. As might be expected, the more refined mesh indicates higher probabilities of failure, but the difference is quite small indicating that the coarser mesh would give acceptable levels of accuracy for most practical purposes.

6. CONCLUDING REMARKS

The paper has demonstrated the use of finite element methods for slope stability analysis. Examples were chosen to include 1-D infinite slopes, 2-D stratified slopes and 3-D slopes with variable properties. Observations were made on the length of a "long" slope and the depth of a 3-D slope in the out-of-plane direction needed to justify "infinite" slope and plane strain conditions respectively.

An investigation of the popular assumption that 2-D slope analysis is conservative compared to 3-D was found to rest entirely on the suitable selection of a "pessimistic" 2-D slice. A poorly selected 2-D slice could lead to unconservative predictions of the 3-D factor of safety.

Finally, the paper mentioned some probabilistic slope stability methods. These approaches target the *probability of failure* of a slope as opposed to the classical slope *factor of safety*. An important new method developed by the author and co-workers called the Random Finite Element Method (RFEM) was described. The importance of properly modeling spatial correlation length was highlighted, and it was shown that traditional "first order" probabilistic tools could lead to unconservative estimates of slope reliability.

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