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Finite element slope stability analysis – Why are engineers still drawing circles?

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ABSTRACT: Limit equilibrium methods of slope stability analysis can be wholly superseded by finite element analysis for any potential slope failure mechanism. Reluctance to use the finite element method for slope stability analysis in practice has been partly due to concerns that it is complex and computationally time consuming. A finite element (FE) program to model slope failure mechanisms has been developed which allows rapid analysis of a wide range of slope stability problems including the influence of water and submerged loading conditions. Mesh density has been balanced with accuracy and economy as essential criteria for the successful engineering application of FE programs. The program has been validated against traditional slip circle analyses and documented case histories.

1 INTRODUCTION

Slope stability analyses are usually conducted for one of the following reasons: assessment of an existing natural slope; design of a proposed embankment or cut; or back-analysis of a failed or failing slope. Any method must therefore be capable of analysis using in-situ conditions; forecast of potential behaviour using design conditions and back-analysis of previous conditions. Any such analysis should be as insensitive as possible to 'a priori' assumptions or constraints. Traditional method of slope stability analysis, based on the slip circle approach, are governed by the imposed circle (or modified arc) and the detail of the analysis method. Finite element analyses are less dependent on detail of method, requiring only general classification of drained/undrained behaviour; cohesive or predominantly granular material and possibly large-strain considerations in extreme cases. The pre-condition of an assumed failure mechanism is not required.

1.1 *The influence of water*

Given the crucial influence of water on the stablity of a slope and the possible variation of this effect over time it is also vital that any method can accommodate the full range of effects and in a realistic manner. Efforts have been made to incorporate such conditions into traditional slip circle methods but all are constrained by the imposition of the mechanism and the importance of its assumed location in the calculation. Comparison with the work of Bishop and Morgernstern (1960) and Lambe and Silva (1995) illustrates the importance of the correct modelling of pore pressure variation beyond that of a global r_u value.

1.2 *Submerged and drawdown conditions*

The ability of the FE method to allow an almost infinite range of properties to be accommodated dependent only on the mesh density and machine capability is especially important in allowing a fine variation in pore pressures to be included and the modelling of submerged and *rapid drawdown* conditions allows utilisation of the program for the most extreme and usually critical cases. The charts of Morgernstern (1963) are themselves based on traditional slip circle analysis and therefore suffer the same constraints. The FE program allows the full range of conditions to be tested without constraint and the most critical case pinpointed.

2 THE FE PROGRAM: FEEMB1LG

The program is an expanded version of 'FE-EMB1' developed by Griffiths (1996) for 2-dimensional slope stability analysis by finite elements using 8-node quadrilateral elements of elastic-perfectly plastic soil with a Mohr-Coulomb failure criterion. The primary development has been the inclusion of free-surface and/or external reservoir loading. The soil's self-weight is modelled by a gravity 'turn-on' procedure (Smith and Griffiths, 1988) with nodal loads added in the first increment.

2.1 *Factor of Safety and 'Failure'*

The Factor of Safety (FoS) for the slope is defined by division of the original shear strength parameters where:

$$c_f^{'} = c^{'}/FoS \tag{1}$$

$$\phi_f^{'} = \arctan(\frac{\tan\phi^{'}}{FoS}) \tag{2}$$

Failure of the slope can be defined in different ways (Abramson et al, 1996). The program uses the failure of the visco-plastic algorithm to converge within an iteration limit (usually 250), with a nodal displacement criterion on successive iterations. This is considered to be a physically real criterion. The FoS 'at failure' lies between the FoS at which the iteration limit is reached and the immediately previous value. By comparison, the FoS generated by traditional methods represents the ratio between the driving and restoring forces.

Piezometric surfaces, pore water pressure conditions and submerged effects can be included by definition within the data file. As well as the numerical results, displaced mesh and displacement vector plots are produced to assist in the determination of the failure mechanism. Figure 1. shows such a plot.

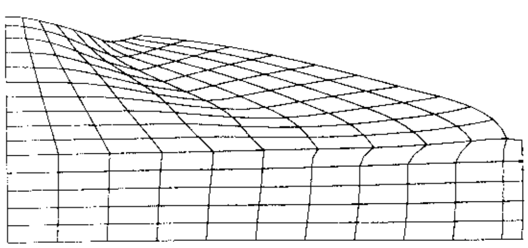

Figure 1. Displaced mesh plot from FEEMB1LG

3 BENCHMARKING WITH BISHOP'S SLOPES

3.1 *Bishop and Morgernstern's slopes*

The classic papers of Bishop and Morgernstern were taken as benchmarks for the finite element approach. The example calculations for the slope in Figure 15 of their joint 1960 paper where reproduced giving FoS of 1.65 (for $r_u = 0.5$) and 3.0 for $r_u = 0$, compared to their Stability Coefficients results of 1.65 and 3.06. Similarly the results for Figure 2 of their paper were recalculated giving excellent agreement as shown in Figure 2 here.

3.2 *'The Bishop slope'*

Lambe and Silva (1995) re-analysised 'the Bishop slope' of his 1955 paper for a critique of the Ordinary Method of Slices (OMS). Analysis by FEEMB1LG gave results that lay between the FoS values reported by them for OMS and their suggested correction method.

Table 1. Comparison of results for 'the Bishop slope'

r_u	Lambe and Silva	FEEMB1LG
0	2.6 - 2.5	2.5
0.6	0.7 - 1.0	0.83

But they commented '..engineers can average $\bar{B}$ to produce a constant r_u value. Our experience has never shown a section with a constant r_u.... A constant r_u simplifies analysis but does not make good sense and could provide misleading results.'

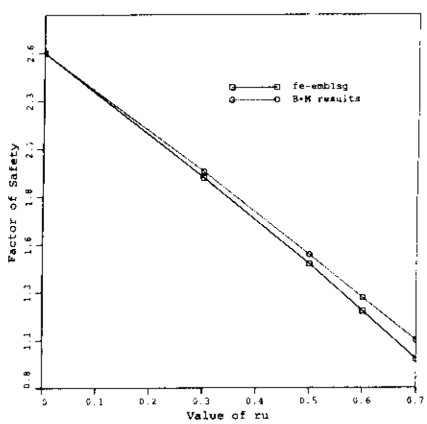

Figure 2. Comparison of Linear Relationship of r_u and FoS with Bishop and Morgernstern result.

3.3 *The Lodalen slope*

Bishop and Morgernstern (1960) attempted to account for this by averaging r_u values over the predicted slip circle. They used the 'Lodalen slope' (Sevaldson, 1956) as an example of the suggested method. Using their averaged value of r_u of 0.28 in FEEMB1LG gave a FoS of 1.07 compared to their result of 1.08. Utilising the original paper of Sevaldson to insert the observed water table levels and the measured r_u range of 0 to 0.49 in the program gave a FoS of 1.0, i.e. the actual slope failure reported at Lodalen. For maximum flexibility FEEMB1LG allows individual element specification of $\bar{B}$ and independent specification of water table levels.

4 SLOPE STABILITY UNDER SUBMERGED CONDITION

The effect of submergence on the slope is included in FEEMB1LG by the addition of nodal loads on the slope face. These are calculated automatically for a given water level but allows for the possibility of other loading sources through the use of a K_0 value. The imposition of water to the slope face is handled separately from the piezometric water levels. This allows total flexibility in the specification of conditions and, especially, the modelling of *rapid drawdown*. Taking the Example 6.14 of Smith and Griffiths (1988), the effect of varying the submerged water level was analysed and the results shown in Figure 3 for two sets of shear strength parameters.

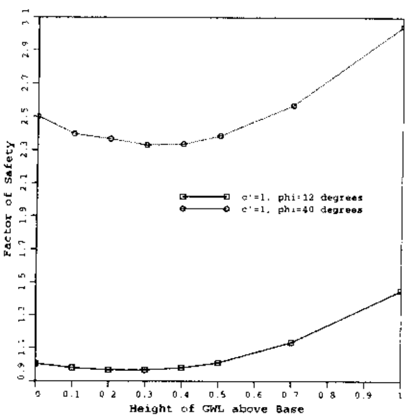

Figure 3. Submerged slope stability for variable water level.

The $\phi' = 40°$ result shows a minimum FoS of about 2.3 with 0.3m of water above the toe. The case with $\phi' = 12°$ also gives a minimum at 0.3m where the slope becomes unstable whilst it is stable either with no water or fully submerged.

These examples were run in a traditional slope stability package using slip circle searching techniques. Agreement was excellent for this simple case as the mechanism is predominately circular. However, the traditional method requires manual intervention in the searching strategy. The finite element approach generates the critical circle automatically and requires only a crude mesh (5x5 elements) as shown in Figure 4.

5 RAPID DRAWNDOWN

The most critical condition for most submerged slopes is the *rapid drawdown* case. The internal pore water pressures from the submerged condition cannot dissipate at the same speed as the external water level is reduced in a fine-grained material. Morgernstern's 1963 paper presented stability charts based on parametric studies using slip circle analysis automated on then available computers. He assumed $\bar{B}$ was unity and that no dissipation occurred during drawdown.

In FEEMB1LG the piezometric surface is specified as per the original water level but the face loads are based on the specified water surface level which in this case is below that of the piezometric values.

Morgernstern's charts are non-dimensional for various values of:

$$\frac{c'}{\gamma H} \tag{3}$$

and interpolation can be used for other values. Figures 5 and 6 illustrate the comparison of results between Morgernstern and FEEMB1LG for a range of cases. Excellent agreement was obtained although the finite element results are slightly lower especially over the higher drawdown ratios.

Figure 4. Displaced mesh for Example 6.14

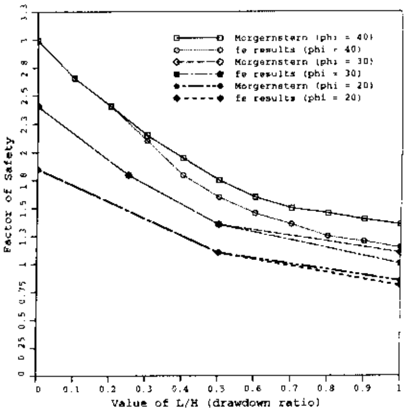

Figure 5. Comparison with Morgernstern results for a 2 : 1 slope and $c'/\gamma H = 0.05$

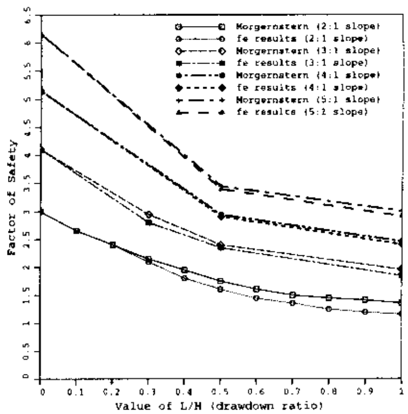

Figure 6. Comparison with Morgernstern results for $\phi' = 40^{\circ}$

The drawdown ratio (L/H) is the ratio between the slope height (H) and the depth below the crest to which the water level falls (L). All slopes are assumed to be initially fully submerged ($L = 0$).

6 CASE STUDY

An existing dam (WC) was being investigated for stability. The particular concern is for partially submerged conditions. A traditional slip circle analysis had been conducted and the Bishop and Morgernstern Stability Coefficient calculation was also performed. The dam had two layers of material - the embankment itself with $\phi' = 40^{\circ}$ and the

Table 2. Results comparison for WC Dam

Condition	Bishop etc	Slip Circle	FEEMB1LG
Dry slope	1.78		1.85
Partially submerged		2.36	1.7
Fully submerged		2.56	1.95
Empty with internal pwp		2.11	1.65

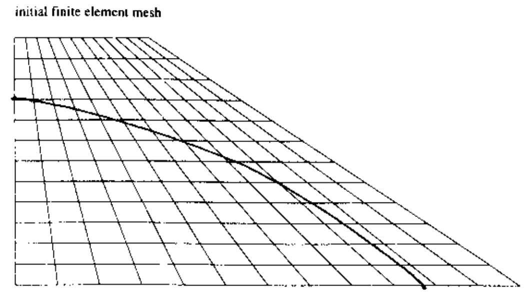

Figure 7. Initial 12×12 mesh for Lambe and Whitman Example 24.3

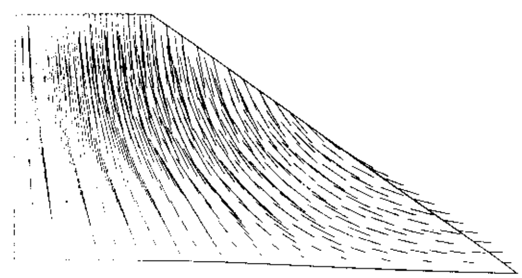

Figure 8. Displacements for Example 24.3

stronger foundation layer with $\phi' = 55^{\circ}$. For various conditions for the dam the FoS results were . The worst case condition was found to be that of emptying from a partially submerged condition. This corresponds well with the earlier results of the case of lowest FoS at partial submergence and the *rapid drawdown* effect. The traditional slip circle approach produced consistently higher estimates of FoS although the trend between conditions was the same. Further investigations are continuing as the difference between these results is important in deciding on the acceptability of its current condition.

7 MESH DENSITY

A study of mesh density was carried out to determine its' impact on the results. Lambe and Whitman's (1979) Example 24.3 was considered with mesh densities of 5×5, 10×10 and 12×12. The FoS for 250 iterations was taken and the result was found to vary by less than 2% between the meshes (from 1.23 to 1.21). Figures 7 and 8 show the initial mesh and phreatic surface for the Example 24.3 and the displacement plot for a 12×12 mesh.
The greatest difference between the meshes is the clearer illustration of the denser meshes in terms of identifying the failure mechanism. The numerical results are only marginally affected. The finite element result is in good agreement with the slip circle result, although slightly lower.

8 CONCLUSIONS

A finite element program has been shown to give consistent results over a wide range of the most critical slope stability problems. The finite element method has fewer constraints and initial assumptions than traditional slope stability analysis methods and automatically identifies the critical failure mechanism without the need for manual intervention. With commonly available computer power the finite element method is readily accessible to practicing engineers. Traditional slip circle methods suffer their own inherent problems which make them susceptible to misuse. The finite element method, with its greater potential, should should now be the basis of engineering analysis and design.

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