It's All the RAGE

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By D. V. Griffiths, Ph.D., P.E., F.ASCE and Gordon A. Fenton, Ph.D., P.Eng., M.ASCE

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Risk assessment in geotechnical engineering, or RAGE, is an exciting and rapidly growing area of interest and study for both geotechnical practitioners and academics. Evidence of this growth is attested by increased sessions on the topic at G-I symposia, new practitioner-oriented journals, recent textbooks, and regularly scheduled ASCE Continuing Education short courses.

Soils and rocks in their natural state are among the most variable of engineering materials. Geotechnical engineers often must "make do" with materials at a particular site. In a perfect world with no economic constraints, numerous boreholes would be drilled and multiple samples returned to the laboratory for measurement of soil properties such as permeability, compressibility, and shear strength. Engineering designs following such a thorough site characterization would lead to confident performance predictions. In reality, rather limited site investigation data are available and the traditional approach for dealing with uncertainty in geotechnical design has been through the use of characteristic values of the soil properties coupled with a generous factor of safety.

If the multitude of data for one of the soil properties from the “perfect world” site investigation were plotted as a histogram, a broad range of values would be observed in the form of a bell-shaped curve. The most likely values of the property would be somewhere in the middle, but a significant number of samples would display higher and lower values. This variability, inherent in soils and rocks, suggests that geotechnical systems are highly amenable to a statistical interpretation.

This is quite a different philosophy to the traditional approach: in the probabilistic approach, input soil properties are characterized in terms of their means, variances, and covariances, leading to estimates of the probability of failure (p_f) or reliability index (β) of a design. Specific examples might involve estimation of the probability of failure of a slope, the probability of excessive differential settlement of a foundation, or the probability of excessive leakage from a reservoir.

Risk is defined as the probability of design failure weighted by the consequences of design failure (e.g., fatalities, cost, and unacceptable performance). Design of geotechnical systems will typically include a target acceptable risk, defined as the risk that the stakeholders consider acceptable under given conditions. The acceptable risk built into a design will likely be much lower for a major earth dam in a populated area than for an embankment retaining an irrigation pond in a remote rural location. Regardless of the type of project, however, risk assessment is unavoidably quantitative in nature and an engineer performing a risk assessment must ultimately develop numerical estimates of p_f.

Methods of Probabilistic Analysis

While there are several tools available for probabilistic analysis in geotechnical engineering, event trees, the first order reliability method, and the random order finite element method of probabilistic analysis, are representative of tools with increasing levels of complexity and mathematical sophistication.

Level I: Event Trees. Event trees are typically used for probabilistic analysis in practice, and are performed prior to deciding whether more detailed mathematical or numerical modeling is warranted. Agencies such as the Bureau of Reclamation who deal regularly with critical geotechnical structures, such as earth dams, use event trees to estimate the probability of different modes of design failure.

Event trees consist of nodes and branches that must be constructed carefully and adhere to certain rules to be useful in calculations. From a starting node, two or more branches leave. At the end of each branch there is another node from which more branches may leave and go to separate nodes. The idea is repeated from the newer nodes as often as required to completely depict all possibilities. A probability is associated with each branch and, for all branches except those leaving the starting node, the probabilities are conditional; that is, they are probabilities of events occurring given that other events (earlier branches) have already occurred.

Event trees can become quite complicated for complex problems. Figure 1 presents a simple example for an embankment potentially vulnerable in the event of an earthquake or a flood. All the numbers on the figure represent probabilities, which in practice are developed by probabilistic models and/or an expert panel of engineers based on experience and similar case histories.

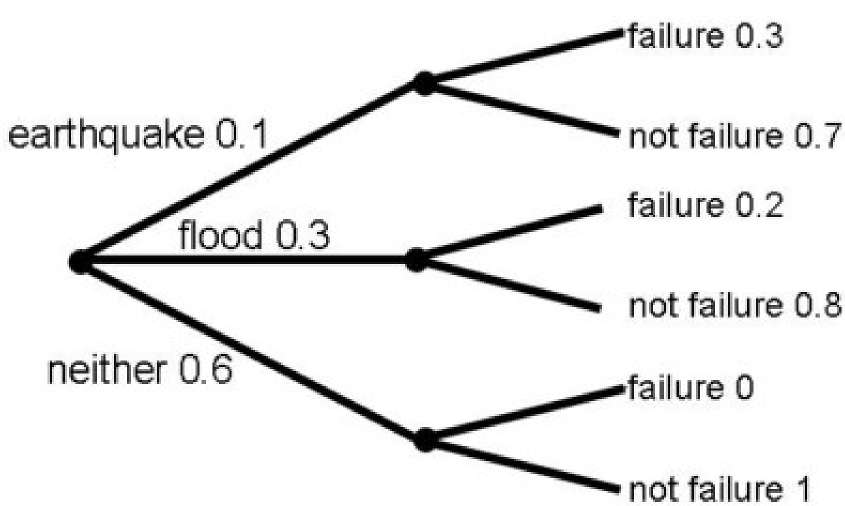

Figure 1. A simple event tree showing conditional probabilities that might lead to failure of an embankment.

The probability of a specific type of failure is found by multiplying together the probabilities along the branches that lead to that failure. From Figure 1, the probability of failure due to an earthquake (p_{feq}) would be given by:

$$p_{feq} = 0.1 \text{ x } 0.3 = 0.03$$

The total p_f, regardless of cause, would be obtained by adding together the branch products due to earthquake and flood as:

$$p_f = 0.1 \text{ x } 0.3 + 0.3 \text{ x } 0.2 = 0.09$$

Level II: First Order Reliability Methods (FORM). This method has gained significant attention in recent years as a relatively simple way of obtaining probabilities of failure for geotechnical systems involving random input variables. The method is also easily run using familiar software such as Excel.

The starting point for a FORM analysis is a performance function for the system under investigation. A performance function separates safe from failure combinations of input variables and is the locus of $FS = 1$. Usually, the function is arranged such that if it is negative, failure conditions are implied; if it is positive, safe conditions are implied. A performance function may be based on a familiar equation from classical geotechnical analysis or, if no convenient function exists, it may be generated numerically using curve fitting.

The performance function for a bearing capacity analysis in which a strip footing is subjected to an allowable bearing pressure (q_{all}) might be written as:

$$g = FS - 1$$

where $FS = q_{ult}/q_{all}$ and q_{ult} is obtained from Terzaghi's bearing capacity equation. Let us assume that the width of the footing (B), the soil unit weight ($\gamma¢$), surface surcharge (q) and groundwater conditions are confidently known (deterministic), but that the shear strength parameters (*c ¢, tanf¢*) are uncertain and to be treated as random input variables (stochastic), characterized by their means and standard deviations ($\mu_{c¢},\sigma_{c¢}$) and ($\mu_{tanf¢}$, $\sigma_{tanf¢}$).

A typical bivariate probability density function with generic random variables *x* and *y* might look like the “hill” shown in Figure 2a. Figure 2b shows a plan view of the probability density function in normalized space (μ = 0, σ = 1) together with contours of the reliability index *ß,* which measures standard deviations units away from the mean. For example, the contour marked *ß* = 1.5, represents the locus of random variables 1.5 standard deviations away from their mean values. Also shown on Figure 2b is the performance function labeled *FS* = 1.

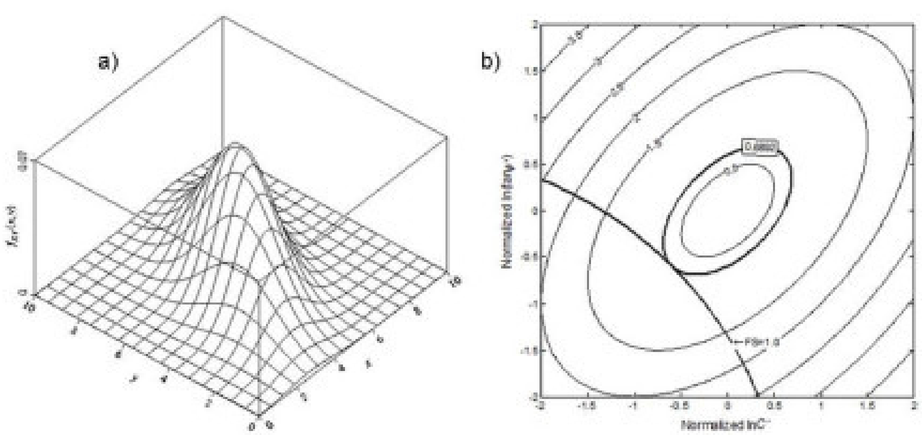

Figure 2. a) Probability density function (pdf) involving two random variables. b) Plan view of a normalized pdf together with a performance function marked FS=1 and the minimum reliability index contour marked b=0.6892.

FORM is essentially an optimization method that iteratively finds the most likely values of the random variables that would result in failure. In Figure 2b, this is given by the contour *ß* = 0.6982 that just touches the performance function. The reliability index is easily converted to a probability of failure through standard cumulative distribution tables. In this case, *ß* = 0.6982 corresponds to p_f = 0.243.

Random Finite Element Method (RFEM). This method was developed by the authors in the early 1990s and involves a combination of finite element and random field methodologies with Monte-Carlo simulations. The method is more computationally intensive than FORM but properly accounts for spatial variability and correlation, which recognizes that at any given site, soil properties are more likely to have similar properties if they are located close together rather than far apart. In particular, in addition to the means and standard deviations of input parameters (as required by FORM), RFEM also requires input of the spatial correlation length, defined as the distance over which properties tend to be positively correlated. Anisotropic spatial correlation lengths can also be considered where the horizontal spatial correlation length may be longer than in the vertical direction.

An advantage of RFEM, which becomes especially clear in the study of the collapse of soil masses, is its ability to realistically allow the failure mechanism to "seek out" the most critical and weakest path through the soil mass. This can lead to quite convoluted failure mechanisms that are significantly different to the classical mechanisms that occur in homogeneous soils. More importantly, the "seeking out" phenomenon, not easily accounted for by methods such as FORM, generally gives lower factors of safety and higher p_f values than would be predicted by traditional, but "incorrect," mechanisms.

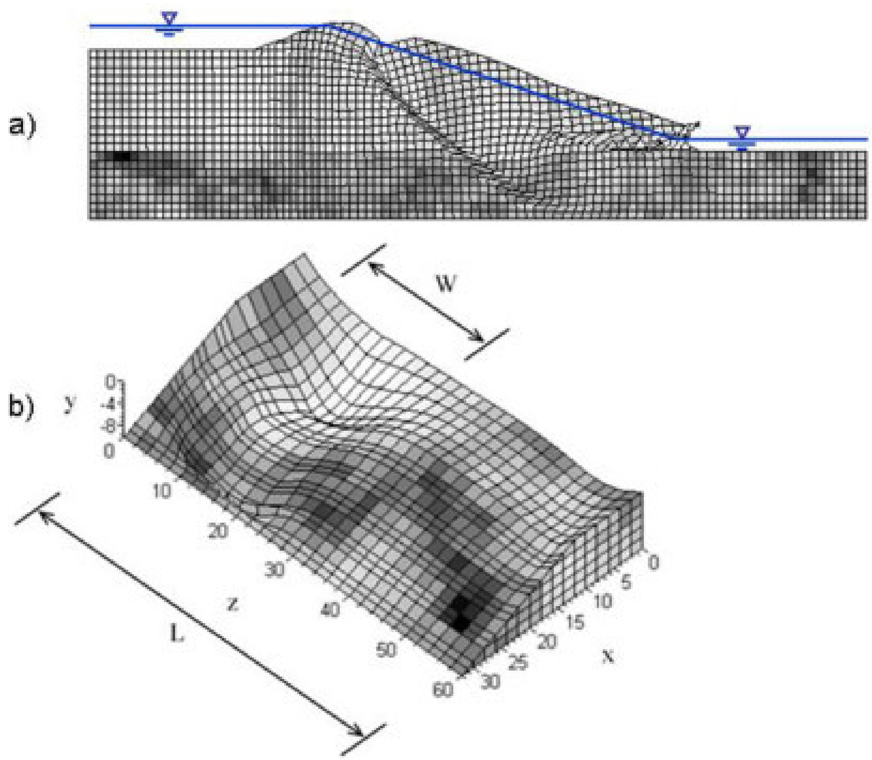

Figure 3. Typical RFEM simulations of slopes showing failure mechanisms "seeking out" paths through the weakest soils. a) 2D simulation of a tailings dam showing the development of two different failure mechanisms. b) 3D simulation of a long dam or levee showing a localized failure mechanism due to a zone of weaker soil.

Figures 3a and 3b show, respectively, typical failure mechanisms that might be displayed in two-dimensional (2D) and three-dimensional (3D) slopes modeled by RFEM. The 2D case represents a tailings dam with different random materials in the embankment and the foundation. Two different mechanisms have formed simultaneously through the weaker soil formations, indicating a tendency for rotational and horizontal sliding mechanisms. The 3D case is of a long dam or levee in which spatial correlation effects have led to a concentration of weaker soils at a particular location resulting in a localized failure zone. The p_f predicted by a RFEM is simply the number of simulations that fail divided by the total number of Monte-Carlo simulations performed.

The Road to RAGE

Although probabilistic concepts have been utilized by geotechnical engineering for many years, they have tended to be confined to "high tech" projects such as offshore and earthquake engineering where a statistical treatment of loading (e.g., the 100-year event) was an essential consideration. Nowadays, engineers are increasingly required to explicitly consider risk and reliability in more conventional investigations such as slopes and foundations. Detailed probabilistic analysis of two different earth slopes might conclude that the slope with the higher factor of safety also has a higher probability of failure than the slope with the lower factor of safety! Only a probabilistic method could reveal such a counter-intuitive outcome.

The increased use of reliability-based design in geotechnical engineering is also an incentive for a greater awareness of probabilistic methods. These methods feed directly into the choice of load and resistance factors needed to achieve a target reliability level.

Risk-based methodologies are here to stay because they offer a more scientific and informative approach to assessing the reliability of geotechnical designs. Geotechnical engineers should become familiar with these concepts and include some of them in their routine “toolbox” for geotechnical analysis.

V. Griffiths, Ph.D., P.E., F.ASCE, is professor of Civil Engineering at the Colorado School of Mines in Golden, CO. Professor Griffiths’ research interests include numerical and probabilistic methods in geotechnical engineering. He offers regular ASCE Continuing Education short courses for geotechnical engineers on these topics. He can be reached at: d.v.griffiths@mines.edu

Gordon A. Fenton, Ph.D., P.Eng., M.ASCE, is professor in the Departments of Civil Engineering and Engineering Mathematics at Dalhousie University in Halifax, Nova Scotia, Canada. Dr. Fenton’s research interests include the development of reliability-based geotechnical design provisions and probabilistic modeling of geotechnical problems. He joins Dr. Griffiths in the offering of short courses on these topics. He can be reached at: Gordon.Fenton@Dal.Ca

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