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Dynamic active earth pressure analysis

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SUMMARY

The paper presents a review of work done in the area of dynamic active earth pressure analysis. This is followed by some finite element analyses of the earth pressure problem in which the soil is treated as an elastic perfectly plastic solid, and the accelerations are applied pseudo-statically. Good agreement is obtained with exisiting analytical methods. The effect of horizontal accelerations on the line of action of the resultant force due to earth pressures has also been observed. The tendency of this parameter to move from the one-third point to the mid-height of the wall as the acceleration increased was in line with observations made by other researchers.

1. REVIEW OF DYNAMIC ACTIVE EARTH PRESSURE ANALYSIS

1.1 Dynamic Lateral Earth Pressure

Nazarian and Hadjan [6] conducted a review of the dynamic lateral earth pressures on retaining walls and divided all previous work into three categories,

1. fully plastic solutions,
2. elasto-plastic solutions,
3. elastic wave solutions.

The review presented in this paper will essentially concentrate on the first and second categories.

Seed and Whitman [11] suggested that possible failures of retaining walls, when subjected to earthquake loading, were due to the increase in the lateral earth pressure behind the wall and liquefaction of the backfill material. Probably the most common method used to estimate this increase in lateral pressure and to design retaining walls against earthquake loading is the Mononobe-Okabe Active Earth Pressure Theory (Mononobe and Matsuo [4], Okabe [7]). The Mononobe-Okabe solution is based on the theory proposed by Coulomb for the active earth pressure on retaining walls due to a dry cohensionless backfill and modified to take into account vertical and horizontal accelerations. The inertia forces in the horizontal and vertical direction are described by,

k_h=(horizontal component of earthquake acceleration)/g

k_v=(vertical component of earthquake acceleration)/g

$g = 9.81 m/s^2$

It can be shown that the dynamic active pressure with earthquake effects can then be given by,

$$P_{AE} = \frac{1}{2}\gamma H^2(1 - k_v)K_{AE} \tag{1}$$

γ =unit weight of the backfill, H is the height of the wall and K_{AE} respresents the active coefficient of earth pressure (Figure (1)), with earthquake effects,

$$K_{AE} = \frac{\cos^2(\phi - \beta - \theta)}{\cos\theta \cos^2\beta \cos(\delta + \beta + \theta)\left[1 + \left\{\frac{\sin(\delta+\phi)\sin(\phi-i-\theta)}{\cos(\delta+\beta+\theta)\cos(\beta-i)}\right\}^{1/2}\right]^2} \tag{2}$$

$$\theta = \tan^{-1}\left[\frac{k_h}{(1 - k_v)}\right] \tag{3}$$

Seed and Whitman [11] have shown that the effect of the soil/wall friction angle δ is very small, however small errors in the soil friction angle ϕ can lead to large errors in the prediction of K_{AE}.

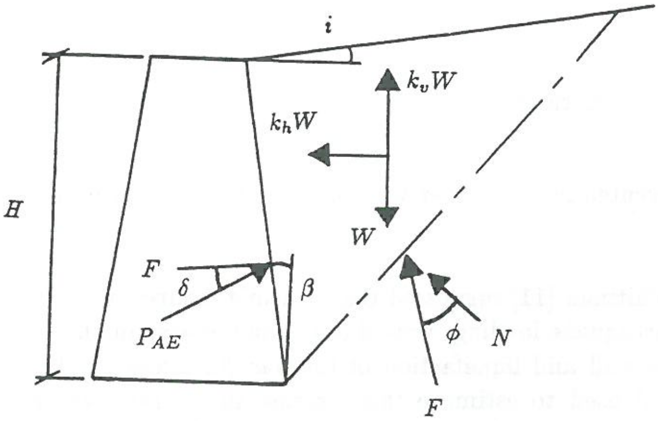

Figure 1 - Derivation of the Mononobe-Okabe equation.

The Mononobe-Okabe method however has several disadvantages,

1. the inertia of the wall is neglected and the dynamic amplification of the backfill is not considered,
2. the method is based on rigid body motions,
3. reversal of the ground motion is not considered.

Richards and Elms [10] proposed a method based on limit displacements; in their method the wall inertia is taken into consideration and shown to be an important factor when designing for earthquake loading. Nadim and Whitman [5] used finite elements to model the seismic response of retaining walls, they observed that as the frequency of excitation approached the fundamental frequency of the backfill, the amplification of the backfill will contribute significantly to the walls displacements. Non-linear seismic response of retaining walls has been conducted by Siller and Bielak [12] using finite elements, they used sinusoidal displacements at two different frequencies as the input motion and looked at both rotating and non-rotating walls. They found that the displacements and forces for the non-rotating wall were larger than those of the rotating wall. Dynamic centrifuge testing of retaining walls subjected to earthquake loading has been conducted by Ortiz, Scott and Lee [8]. They found reasonable agreement between their results and the Mononobe-Okabe solution, they also found that the earth pressure distribution down the wall is non-linear and that the dynamic behaviour of the wall occurs mainly in the first mode of vibration. Dynamic centifuge testing of retaining walls have also been conducted by Steedman and Zeng [14]. They found good agreement with the Mononobe-Okabe solution, but showed that amplification of the horizontal acceleration up through the soil layer has an important effect when the wall is subjected to a horizontal base acceleration.

1.2 Point of Application of the Resultant Active Pressure

Prakash and Basavanna [9] showed that the point of application of the resultant active earth pressure increases from $H/3$ to $H/1.9$ above the base of the wall for a horizontal earthquake of $0.3g$. Bakeer and Bhatia [1] found the average position of the resultant earth pressure to be $0.42H$ above the base of the wall and that the earth pressure distribution was non-linear.

2. DESCRIPTION OF THE PROBLEM

The problem being considered is shown in Figure (2).

To compare the results to the Mononobe-Okabe solution (Equations 1,2) the wall is assumed to be 'weightless' and $5m$ in height. The backfill is assumed to be dry and

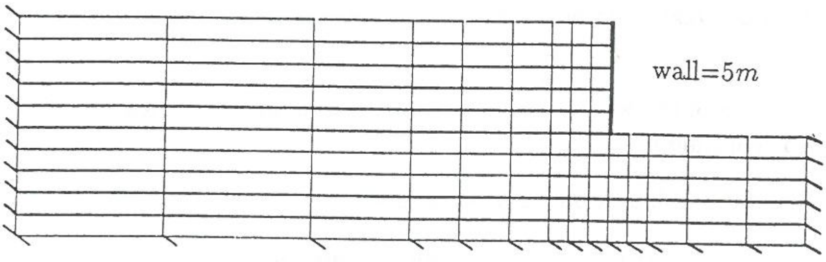

Figure 2 - Problem being considered

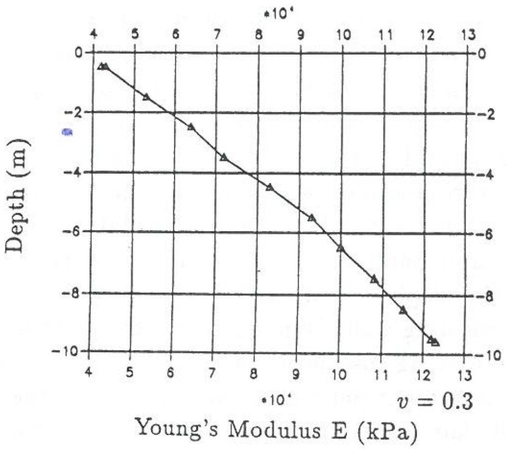

Figure 3 - Assumed variation in Young's Modulus with depth

cohesionless and modelled using the elastic perfectly plastic Mohr-Coulomb failure criteria, with a soil friction angle of 30^o and a dilation angle $\psi = 0$. Figure (3) shows the assumed variation of Young's modulus E with depth (Griffiths and Prevost [2]). In the pseudo-static analyses presented here, the Initial Stress Method, (see eg. Smith and Griffiths [13]), with a tension-cut off and drift correction is applied. The horizontal stresses are assumed to remain constant across the base of the wall and are computed as a function of K_o, ($K_o \approx 1 - \sin\phi$). The excitation is in the form of a suddenly applied constant horizontal acceleration, which would cause no dynamic amplification of the backfill, in accordance with the Mononobe-Okabe solution.

3. PSEUDO-STATIC ANALYSIS

3.1 Description of the Algorithm

The algorithm used in the pseudo-static analysis applied the 'accelerations' as incremental gravity loads in the horizontal direction. Displacement increments were then applied to the wall in order to induce active failure of the wall.

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3.2 Results

Figure (4) shows a typical result of the pseudo-static analysis, for a friction angle of $\phi = 30^o$ and $k_h = 0.1$.

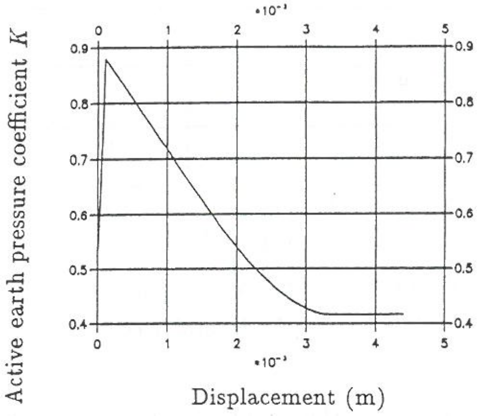

Figure 4 - Typical result of pseudo-static analysis, for $\phi = 30^o$ and $k_h = 0.1$

The lateral stresses initially increase sharply with the applied acceleration (hence the sharp increase in K) and then as the wall is displaced they gradually fall until active failure of the wall is achieved and the value of K_{AE} becomes constant. Figure (5) shows the results of the pseudo-static analysis as compared to the Mononobe-Okabe solution (Equations 1,2).

In general the results are in good agreement with the Mononobe-Okabe solution, although at higher acceleration values the numerical results indicate slightly higher values of K_{AE}.

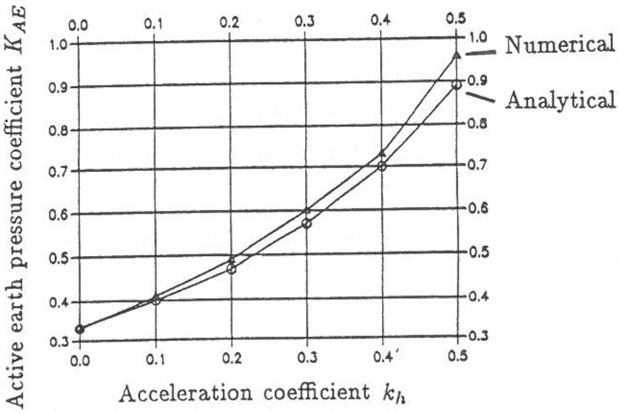

Figure 5 - Active earth pressure coefficient K_{AE} with increasing horizontal acceleration

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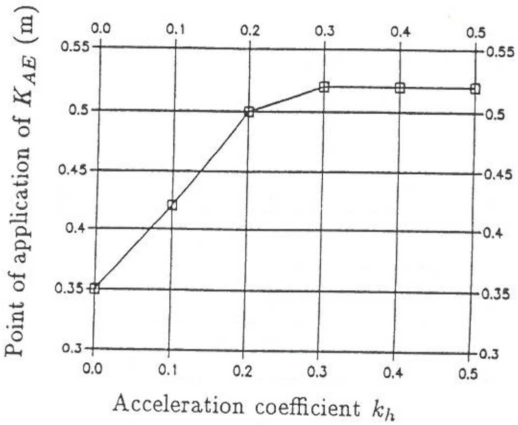

Figure 6 - Point of application of K_{AE} with increasing horizontal acceleration

Figure (6) shows the increase in the point of application of the resultant active force with increasing acceleration. The cause of this increase is clearly shown by looking at the horizontal stress distribution behind the wall at failure for the different levels of horizontal acceleration (Figure (7)).

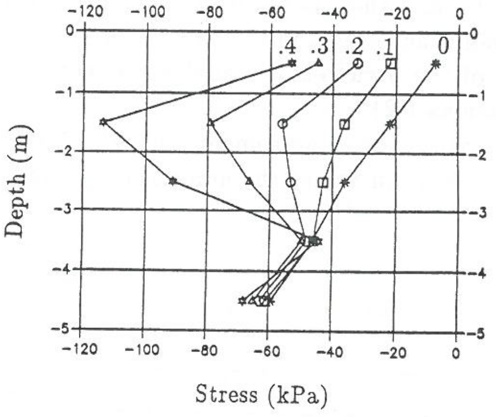

Figure 7 - Stress distribution behind the wall at failure for values of k_h

As the acceleration level increases there is a significant increase in the lateral pressure above $1/3H$ of the wall.

4. CONCLUDING REMARKS

A review of dynamic active earth pressure analyses has been presented, in which the problem has been considered from analytical, experimental and numerical viewpoints. In this paper the active problem has been considered using a pseudo-static

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finite element analysis in which horizontal forces were applied by forming the product of the (consistent) mass matrix and the required acceleration vector. Good agreement was obtained with existing analytical solutions in terms of the magnitude and the line of action of the resultant force during horizontal loading.

Current research by the authors seeks to extend this work in order to observe the behaviour of retaining structures during genuine dynamic loading events. This involves the coupling of time integration algorithms with realistic nonlinear constitutive laws for soil. Such a model is called ALTERNAT (Molenkamp [3]) which incorporates many of the features observed in soil behaviour during cyclic loading. The importance of modelling the volumetric behaviour of soil during cyclic loading is strongly apparent in the undrained environment. If the volume change tendency during shear is contractive, the soil may liquefy and the lateral thrust on any retaining structure would increase dramatically.

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