Influence of Spatial Variability on Rock Slope Reliability Using 1-D Random Fields

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Abstract

In this work, the theory of random fields is used to account for the influence of spatial variability on slope reliability. Within this framework the friction coefficient along a discontinuity is treated as a Gaussian random field which is fully described by its mean value, standard deviation and spatial correlation length. The random field is simulated using the Local Average Subdivision (LAS) method. As shown by the examples presented herein, the spatial correlation of shear strength along a failure plane can have an important influence on slope performance, as expressed by the failure probability. This is a significant observation since ignoring the influence of spatial correlation in design may lead to non-conservative estimations of slope reliability. The planar mode of failure is considered.

Keywords

Random fields • Rock slope • Local average subdivision • Spatial variability • Slope reliability

216.1 Introduction

In this work, the influence of spatial variability on rock slope reliability is studied using 1-d random fields simulated by the Local Average Subdivision Method (Fenton and Vanmarcke 1990). Specifically, the friction coefficient is treated as a random field along the discontinuity. Key element of the random field approach is that the influence of spatial variability on slope reliability is explicitly taken into account. The concept of random fields has already been applied to various geotechnical engineering problems (e.g. stability of soil slopes, spread and pile foundations, retaining walls) as part of a finite element approach, best known as the Random Finite Element Method (Fenton and Griffiths 2008). The present work offers an application of the theory of *random fields* in the area or rock slope stability assessment. The planar mode of failure is considered.

216.2 Computations Based on the Local Average Subdivision Method

Assuming that the rock block may slide along a planar discontinuity (plane AB in Fig. 216.1), the friction coefficient $\tan\varphi(x)$ is treated as random field. For the sake of simplicity, any possible external loading (water pressure, seismic forces, footing etc.) has been ignored. However, all equations given below may easily be transformed according to specific loading situations.

Following Coulomb's failure criterion, the safety factor of a rock slope against planar sliding is given by the formula

$$F = \frac{cL + \int_0^L t(x)\tan\varphi(x)dx}{W\sin\beta_d} \tag{216.1}$$

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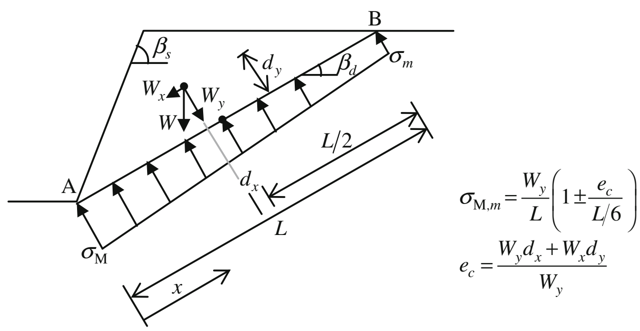

Fig. 216.1 Geometric elements of the problem

where, $t(x)$ is the normal reaction at the base of rock block (per unit length of slope), $\tan\varphi(x)$ is the friction coefficient along the discontinuity which is assumed to be a function of x (x is a distance along discontinuity on the cross-section plane measured from the lower end of discontinuity), c is the cohesion along the discontinuity which is assumed constant, L is the total length of discontinuity on the cross-section plane, β_d is the inclination angle of discontinuity considering the planar type of failure and *W* is the total weight of rock block. The friction coefficient $\tan\varphi(x)$ is treated as a random field with specified stochastic properties.

From Eq. 216.1 it is clear that in order to calculate the safety factor for a given configuration of the field $\tan\varphi(x)$ the normal force (or reaction) at every point along the discontinuity must be known. The rational assumption that the normal reaction varies linearly along the contact area can be made, especially in the present case where a rigid body lays on a rigid body. The stress distribution under the rock block could be assumed to follow a trapezoidal pattern with maximum and minimum stress value (σ_M and σ_m, respectively) as given by the following equation:

$$\sigma_{M,m} = \frac{W\cos\beta_d}{L}\left(1 \pm \frac{e_c}{L/6}\right) \tag{216.2}$$

where, e_c is the eccentricity of the self-weight of rock block, which, here, is the resultant force acting on the base (Fig. 216.1).

It can be noted that, Eq. 216.2 is commonly used in retaining wall and spread foundation problems and it stands for $e_c < L/6$. If the eccentricity e_c is equal to or greater than $L/6$, the rock block is not bearing on its whole base but only on the front edge (Pantelidis 2010).

Based on the trapezoidal distribution of normal reaction below the rock block of Fig. 216.1, the normal stress at a given distance *x* on the discontinuity is:

$$t(x) = \frac{x}{L}\sigma_m + \frac{L-x}{L}\sigma_M = \frac{W\cos\beta_d}{L}\hat{t}(x) \tag{216.3}$$

with

$$\hat{t}(x) = 1 + \left(\frac{x}{L} - \frac{1}{2}\right)\frac{2e_c}{L/6} \tag{216.4}$$

Thus, the safety factor finally reads

$$F = \frac{cL}{W\sin\beta_d} + \frac{1}{\tan\beta_d}\frac{1}{L}\int_0^L \hat{t}(x)\tan\varphi(x)dx \tag{216.5}$$

The random field $\tan\varphi(x)$ is assumed Gaussian with mean value μ and Markovian covariance function

$$C(x, x') = \sigma^2 \exp\left[-\frac{2|x - x'|}{\theta}\right] \tag{216.6}$$

σ^2 is the point-variance of the random field and θ is the correlation length, also known as the scale of fluctuation (Fenton and Griffiths 2008) which describes the distance over which properties (in this respect, the friction coefficient) tend to be spatially correlated. For example, an infinitely large correlation length θ corresponds to a perfect correlation between any two points along the discontinuity. In this special case, the random field is reduced to a single random variable everywhere.

In the framework of the LAS method the discontinuity is subdivided into N intervals, corresponding to N vertical parts of the rock block and its normal reaction. The expression Eq. (216.5) is approximated by

$$F \approx \frac{cL}{W\sin\beta_d} + \frac{1}{\tan\beta_d}\frac{1}{N}\sum_{i=1}^{N}\hat{t}_i\tan\varphi_i \tag{216.7}$$

where $\hat{t}_i$ is the value of the function $t(x)$ at the center of the i-th interval, $i = 1, \ldots, N$, and it is given by

$$\hat{t}_i = 1 + \frac{3}{4}\left\{-\frac{1}{2} + \frac{i-1}{N-1}\right\}\frac{2e_c}{L/6} \tag{216.8}$$

and $\tan\varphi_i$ is the random variable representing the friction coefficient $\tan\varphi(x)$ in the i-th interval. The variable $\tan\varphi_i$ is the average of the random field $\tan\varphi(x)$ in the i-th interval of the discontinuity. The stochastic properties of the variable $\tan\varphi_i$ are, thus, completely determined by the stochastic properties of the random field and they are implemented in the simulations.

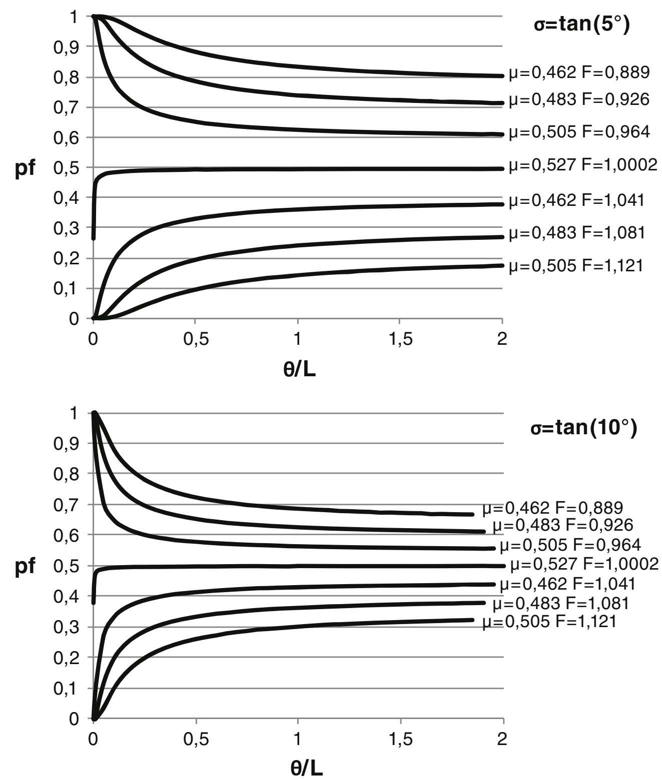

Fig. 216.2 Examples: p_f versus θ/L plot for the various F values and for $\sigma = \tan(5°)$ and $\sigma = \tan(10°)$

216.3 Application Examples

Two examples giving the probability of failure p_f for various μ and σ values of the field $\tan\varphi(x)$ against the normalized correlation length θ/L are given below; see Fig. 216.2. Cohesion is assumed constant along the discontinuity (deterministic value). Since different mean values μ correspond to different safety factor values F, the curves are labeled according to both the associated value of the deterministic safety factor F and μ. The following data stand for both examples: $\beta_d = 30°$, c = 40 kPa, L = 10 m, $e_c = 0.5$ m and W = 9000 kN/m. The results given in Fig. 216.2 correspond to 1,000,000 realizations of the random field and N = 4 subdivisions of discontinuity. It is important to be mentioned that, the minimum number of discontinuity subdivisions that fully and effectively describes the present problem is N = 4. Increasing the number of subdivisions only increases the computation time without having any influence on the results, whilst, as the number of realizations becomes greater, more stable results (smooth curves) are obtained.

The general features of the probability of failure curves shown in Fig. 216.2 can be described as follows. As the correlation length θ of the random field $\tan\varphi(x)$ becomes smaller the system tends to behave more in a deterministic way, that is, the probability of failure tends to 0 or 1 depending on whether the safety factor value is above or below the value 1. When the correlation length θ becomes comparable to the length of the discontinuity, then the entire random field $\tan\varphi(x)$ tends to behave like a single Gaussian random variable. Indeed, in the limit of large θ ($\theta \to \infty$) the covariance function $C(x, x')$ approaches *everywhere* the constant value σ^2; see Eq. 216.6. Thus, in the limit of large θ the probability of failure approaches an asymptotic value that depends only on the point variance σ^2 of the random field $\tan\varphi(x)$. The effect of greater variance σ^2, observed by comparing the two plots of Fig. 216.2, is to introduce stronger deviations from the deterministic answers for the probability of failure, 0 or 1, on the left part of the curves, and a stronger convergence towards the indecisive 0.5 value of the probability of failure, on the right part of the curves. For $\mu = \tan(27.75°)$, which corresponds to the just stable condition (F = 1), the probability of failure is essentially equal to 0.5 for all values of the correlation length θ. This is why the slightly different value we have chosen to consider in the case depicted in the middle curve of both graphs below deviates adequately from 0.5 only for relatively small values of the correlation length.

216.4 Summary and Conclusions

Soils and rocks are among the most variable of all engineering materials, and as such are highly amenable to probabilistic treatment (Griffiths and Fenton 2007, 2004). Acknowledging the significance of spatial variability of shear strength along a rock discontinuity, the LAS method of simulating random fields has been used to calculate the probability of failure of a rock slope. The friction coefficient along a discontinuity is treated as a Gaussian random field which is fully described by its mean value, standard deviation and spatial correlation length.

The examples presented herein highlight the strong influence of the scale of fluctuation of the friction coefficient on slope performance. Indeed, different correlation length values may correspond to totally different probability of failure values. Simplified probabilistic analyses, in which spatial variability is ignored by assuming perfect correlation, can lead to non-conservative estimates of the probability of failure. This effect is most pronounced at relatively low factors of safety or when the coefficient of variation $(\mathrm{COV} = \sigma/\mu)$ of the discontinuity strength is relatively high.

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